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PREBUNCHING WITH SPACE CHARGE

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Abstract A systematic study of single gap prebunching, including the effects of space charge, has been undertaken, using the second moment of the charge distribution as a measure of bunching action. Generalities of the bunching process are discussed.

This study describes the bunching produced by single gap cavities including the effects of space charge. The charged disc model of the beam was used to describe the effects of space charge.[1] The mono-energetic beam particles were assumed to receive an additional energy modulation in the prebunching gap and a one-dimensional ballistic model was therefore appropriate. The equations of motion used were:

$$\frac{d\gamma_i}{d\xi} = \frac{2I\eta e\lambda^2}{\pi m_o c^2 b^2 N}\sum_j^N \sum_n^\infty \left[\frac{J_1\left(\frac{\beta_{on} b}{a}\right)}{\beta_{on} J_1(\beta_{on})}\right]^2 \frac{\exp -\left|\frac{\beta_{on}\beta_w\lambda(\delta_i-\delta_j)}{2\pi a}\right|}{\gamma_j^2}$$

$$\frac{d\delta_i}{d\xi} = 2\pi\left(\frac{1}{\beta_w} - \frac{\gamma_i}{\sqrt{\gamma_i^2-1}}\right) \qquad \gamma_i(0) = 1 + \frac{e(V_o + V\sin\omega t)}{m_o c^2}$$

where $\beta_w = \sqrt{\gamma_o^2-1}/\gamma$ is an artifice which has the effect of being a phase reference in the drift space. Since (b/a) = (λ/a)/(λ/b), where b and a are the beam and pipe radii, respectively, there is less complexity in the problem than appears. N is the number of discs per wavelength into which the beam is divided; in the present case 21 was arbitrarily taken. A systematic investigation of the bunching process was undertaken for the cases a/b = 2.5, 3.0 and 5.0 for modulation indices from 0.1 to 0.3 and beam currents of 0.5, 1.0, 3.0 and 5.0 amperes at 10.5 cm wavelength. A preliminary study showed that for typical prebuncher cavity designs there are no perturbing effects owing to transit time of the cavity gap so that the assumption of an infinitesimal gap was justified; the gap phase delay, w d/v, had a negligible effect on the subsequent trajectories.

A typical plot of the particle orbits is shown in Figure 1. Examination of this figure reveals that it is very difficult to decide upon a useful measure of bunching. Arbitrary measures, such as the fraction of injected charge within one radian (as a function of drift distance) are often used. A conventional measure of dispersion, the second moment of the distribution has been used in this study

$$\nu = \frac{\sum f_i(\delta_i-\bar{\delta})^2}{N}$$

where f_i, the weighting factor, is the fraction of current associated with each trajectory and $N=\sum f_i$ is the total weight (or I_o). Thus, $f_i = I_o/n$ where n is the number of orbits and Eq (1) becomes, where the distribution is about $\delta = 0$,

$$\nu = \frac{1}{n}\sum \delta_i^2$$

To avoid a meaningless numerical value from Eq (2) the value obtained is normalized with respect to the original distribution at the gap (ν_o = 3.6).

In Figure 2 a plot of the second moment of the distribution is given as a function of drift distance for the example of trajectories shown in Figure 1. In addition, the fraction of injected current within one radian (total) of zero phase is shown (marked X). Obviously, the two measures are not equivalent; from the second moment diagram it is evident that the greatest charge concentration occurs at ξ = 2 rather than at ξ = 1.5, as appears otherwise. From the trajectory plot note that orbits are still converging toward zero phase after = 1.5, resulting in better overall phase compression even though the core of the bunch is dispersing. It is important to realize that the same phase (time) does not occur at the same value of , but that phase bunching must be measured along a slope $d\delta/d\xi = 2\pi/\nu_e\lambda$, which are simultaneous points on the orbit plot.

In Figure 3 a diagram is shown illustrating the effect of the beam/pipe-diameter ratio. The data are for a fixed current (5A) and fixed modulation (α = 0.1; only the size of the pipe is changed showing that space charge forces are minimized when the beam nearly fills the tubing, which is generally well-known. When the normalized second moment of the distribution exceeds unity (as for the case a/b = 5 at ξ = 0.8) the space charge density of the beam is worse than the original distribution for the continuous beam. This state of affairs can occur because the space charge trajectory program only considers one RF cycle of the beam. But, since such cases are of no interest to the designer, further study of the beam is not warranted.

Figure 4 provides an answer to a recurrent question. Often on examing a trajectory plot such as Figure 1, it is suggested that the current density at a specified drift distance (the 'cross-over' or apparent phase waist of the beam) can be increased by increasing the gap modulation factor. For small beam currents increasing, the gap voltage will cause the cross-over to occur closer to the gap. But from Figure 4 it can be seen that the minimum of the second moment of the distribution occurs at the same drift distance for all degrees of modulation. This is true for the 5A case shown; however, at lower currents the well-known phenomenon regarding cross-overs occurs. For example, in Figure 5 it is evident that for increased modulation amplitude (gap voltage) the minimum of the second moment occurs nearer to the gap (ξ = o). The relation is somewhat complicated, involving both beam/pipe-diameter ratio and the current (space charge magnitude). From Figure 5 it is also obvious that for a range of current operation one should very the index of modulation or the drift distance at which the bunch should occur.

It is customary to use the bunching parameter,

$$X = \pi\left(\frac{s}{\lambda}\right)\left(\frac{c}{u_o}\right)\left(\frac{V_i}{V_o}\right)$$

giving the distances at which the 'best' bunch will occur for a given gap voltage (α = Vi/Vo, Vi gap voltage, Vo gun voltage, Uo/c normalized electron velocity) in terms of a measure, such as X = 1.84 for maximum fundamental component content with no space charge. It is evident that with space charge, the best bunch occurs in a shorter drift distance and higher values of gap voltage (modulation index) are desirable. The original restriction of about ten per cent modulation was because small signal (linear) theory would then be valid. With the availability of computers and a space charge model of the beam, there is no need of such restrictions.

A serious shortcoming of the data calculated above is that the model should include the effects of a solenoidal magnetic field which certainly would be used with any high current beam. With velocity modulation, one cannot exactly achieve Brillouin flow conditions in the beam; but this is a reasonably good approximation as a starting condition. Then the effects of beam scalloping on space charge forces can be included in the bunching program, as well as supply an estimate of the required beam pipe size to prevent scraping loss of beam. If the magnetic field is commenced before the modulation gap, the effects of the radial component of the magnetic field are eliminated and the equations of motion are much simpler. There is considerable improvement in the engineering design since the electron-optics of the lens system is reduced to that of merely entering the solenoid on the axis with nearly mono-energetic electrons.

One sort of question commonly asked is what injection phase interval will result in a specified bunch phase extent, and what will be the energy range of the particles. Clearly, the normalized relativistic energy at the end of the drift space is

$$\gamma = 1 + e(V_0 + V_g \sin \delta_0)/m_0 c^2$$

and the output phase

$$\delta = \omega t = \omega t_0 + \omega s/u_0$$

where

$$\frac{u}{c} = \sqrt{\gamma^2 - 1}/\gamma$$

These equations can be combined giving such a relation; the result is algebraically complicated, however. Moreover, from computed orbits with space charge it is evident that the answer is not accurate. The above ballistic equations do not consider the space charge interaction which seriously perturbs the particle energy.

Power Input to Prebuncher

It is of some interest to determine what the beam loading effects are in a cavity. The following remarks refer only to a single gap cavity.

The kinetic energy of the extrant particles per second is

$$\frac{1}{2} N m u^2$$

where N is the number that enter (or leave) the gap per second,

$$N = \frac{A}{F} \bar{I}_0$$

where A is Avogadro's number, F is Faraday's number and I_0 is the dc beam current. The exit velocity is, from small signal ballistic theory,

$$u \doteq u_0 \left(1 + \frac{V_1}{2V_0} \sin \omega t\right)$$

Integrating over one cycle and multiplying by the number of cycles per second,

$$P = \frac{1}{2}(A/F) I_0 f m u_0^2 \int_0^{1/f} \left(1 + \frac{V_1}{2V_0} \sin \omega t\right)^2 dt$$

$$= \frac{1}{2}\left(\frac{A}{F}\right) I_0 m u_0^2 \left(1 + \frac{1}{2}\left(\frac{V_1}{2V_0}\right)^2\right)$$

Since the intrant beam power was 1/2 N m u_0^2, the power taken from the cavity is

$$P = (A/F) I_0 m u_0^2 \frac{1}{16}\left(\frac{V_1}{V_0}\right)^2$$

Since $u_0^2 = 2\,(e/m)\,V_0$ and since one ampere is (A/F)e coulombs per second,

$$P = I_0 V_0 \frac{1}{8}\left(\frac{V_1}{V_0}\right)^2$$

where V_1 is the gap voltage. Therefore, to apply a signal voltage V_1 will require a shunt impedance given by

$$V_1^2 = 2 R_L \left(P_0 + \frac{I_0 V_0}{8}\left(\frac{V_1}{V_0}\right)^2\right)$$

where P_0 is the input power to the cavity.

The beam loading will lower the shunt resistance and the Q of the cavity. The beam-loaded shunt resistance is

$$\frac{1}{R_{LB}} = \frac{1}{R_L} + \frac{1}{R_B}$$

where R_L is the coupling-loaded shunt resistance and R_B is the equivalent beam resistance to the cavity is eight times the beam impedance. Thus the coupling coefficient with beam will be

$$\beta' = \frac{R_B}{R_L + R_B}\beta = \frac{8 Z_0}{R_0/(1+\beta) + 8 Z_0}\beta$$

where is the open-circuit coupling coefficient.

Similarly the beam loading will lower the open circuit loaded Q to a value given by

$$Q_{LB} = \frac{(1+\beta) Q_L}{(R_L + R_B)/R_B + \beta}$$

From Equation (5) the required open-circuit coupling coefficient to obtain critical coupling with beam loading is

$$\beta = \sqrt{1 + \frac{R}{8 Z_0}}$$

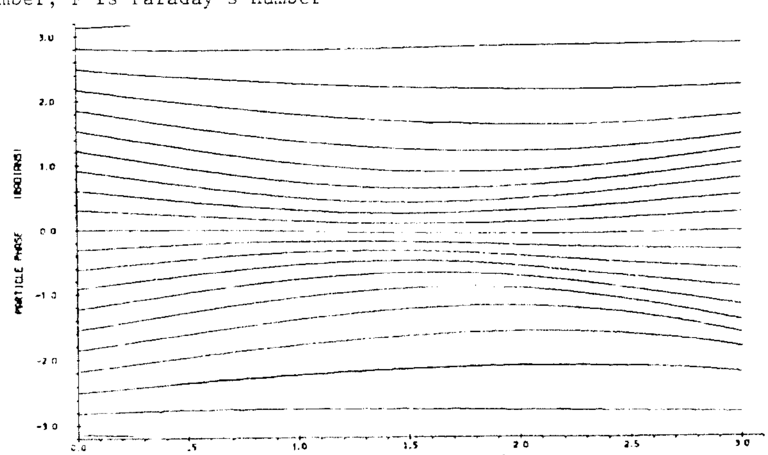

Fig. 1 Trajectories for 5 amp, 100 KV beam.

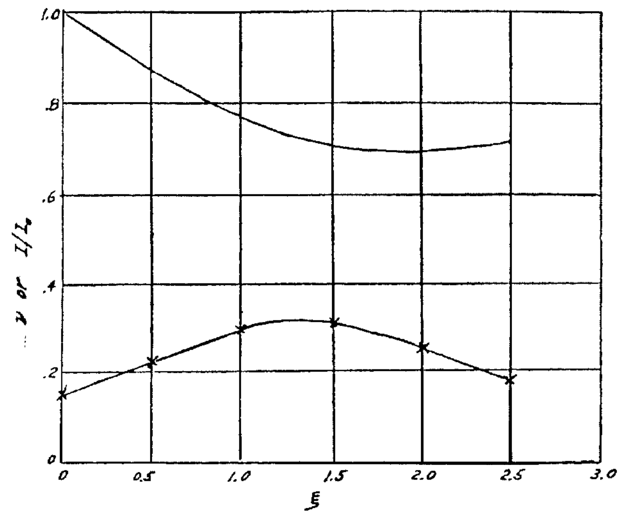

Fig. 2 Second moment of orbits shown in Fig. 1.

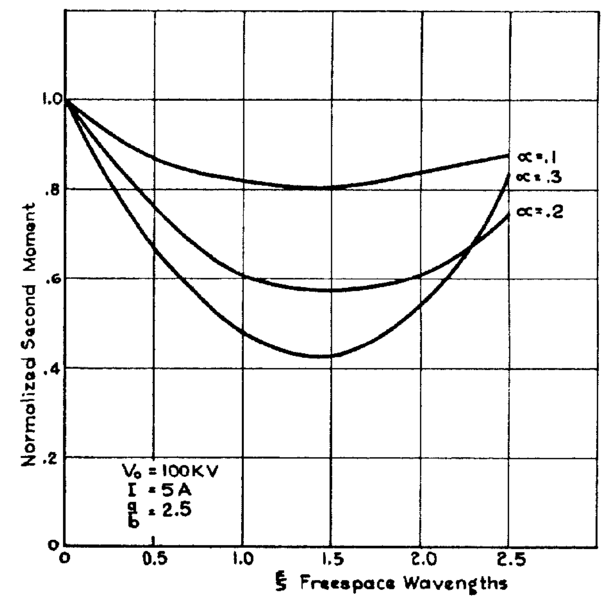

Fig. 4 Second moment of trajectories of 5 amp, 100 KV beam showing effect of modulation index.

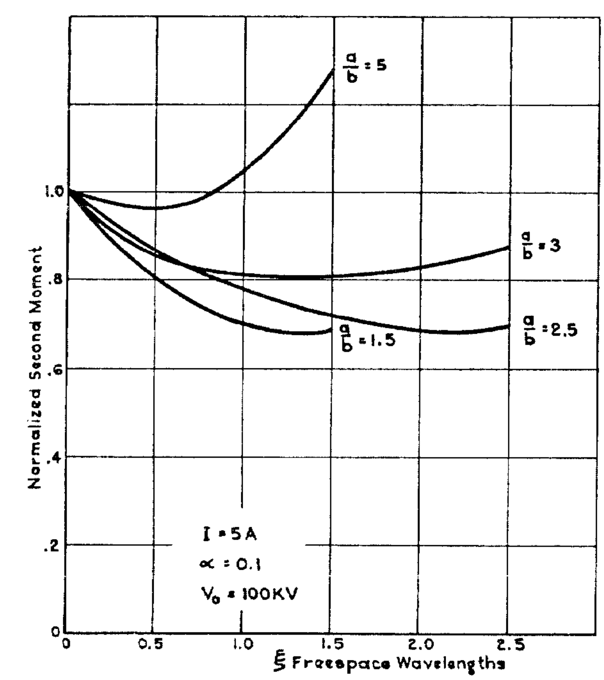

Fig. 3 Second moment of trajectories of 5 amp. 100 KV beam showing effect of beam/pipe diameter ratio.

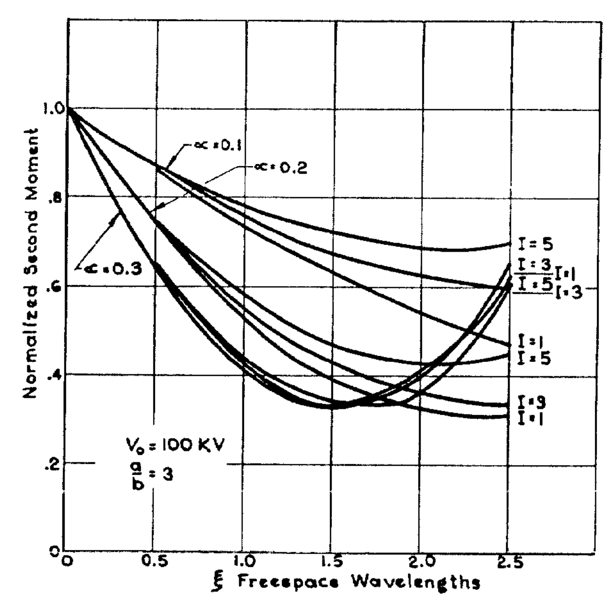

Fig. 5 Second moment of trajectories of 100 KV beam at various currents and modulation indices.