CORRECTING CLOSED-ORBIT DISTORTION IN THE NAL MAIN RING

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Summary

The position of the equilibrium orbit in the NAL synchrotron is measured by sensors immediately downstream of ring quadrupoles. The orbit distortion is corrected at low magnetic field by dc correction magnets located adjacent to the sensors and at high field by transverse displacements of the ring quadrupoles. The inconvenience of moving quadrupole magnets has favored making the minimum number of moves necessary to obtain adequate reduction of the maximum orbit distortion. A few familiar mathematical techniques have proved adequate to calculate the correcting moves, but the combination of these techniques is data dependent. A computer search program invariably gives a solution, but it has been useful to experiment with weighting and smoothing of the data and to compare alternative calculations to insure that the solution is optimum with respect to our priorities and constraints.

Introduction

The failure of the high-energy equilibrium orbit to pass through the centers of all the quadrupoles is due primarily to transverse errors in the position of the quadrupoles. Vertical orbit distortion is also caused by roll of bending magnets about their longitudinal axis. If the radio frequency and bending magnet excitation do not track perfectly the horizontal closed orbit contains an additive part proportional to the off-momentum orbit function x_p, which must be subtracted out to isolate that part of the magnetic field error caused by misalignment. From measurements of the closed orbit one can calculate compensatory displacements of the quadrupoles to reduce the distortion and thereby increase the usable aperture. This paper is specialized to the high-field case for which the practical difficulty in executing the corrections, i.e. in moving ring quadrupoles, places a premium on adequate correction from the fewest moves.

The NAL main ring has 108 electrostatic beam position detectors in each plane or about 5 per betatron wavelength. Operating experience is that measurement accuracy is about .05" vertically and .1" horizontally and that at any time 1 or 2% of the detectors may be inoperative. Because of ongoing activities in the tunnel the closed-orbit distortion can show significant changes in a few days. Besides installation of new equipment and an occasional quadrupole replacement we still have some differential settling of the tunnel to contend with. Therefore, we have not sought a single-step solution for closed-orbit distortion but instead a tolerable method to cope with frequent approximate adjustments. Our basic strategy has been to seek a minimum number of quadrupoles which will reduce the maximum orbit distortion by a factor of two to three. We observe as constraints that the vertical closed-orbit distortion be less than 1/8" at the extraction septum, that displacements not be so large as to require realignment of bending magnets or other devices from their surveyed positions, and that available quadrupole travel not be exceeded. After each set of moves the remeasured orbit is used to calculate the next step of correction. A rather limited set of analytical tools suffices, and it is reasonably straightforward to select the proper ones for a specific set of data.

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Computational Techniques

The high-field closed-orbit distortion x_i measured at the *ith* sensor is a sum of contributions proportional to quadrupole displacements δ_j:

$$x_i = \sum_j a_{ij}\delta_j, \qquad (1)$$

where to better than 1%

$$a_{ij} = \frac{\sqrt{\beta_i\beta_j}\;K_j}{2\sin\pi\nu}\cos\,(\mu_i-\mu_j-\pi\nu). \qquad (2)$$

In this formula, β_i and β_j are the Courant-Snyder[1] amplitude functions at the sensor and displaced quadrupole, respectively, μ_i and μ_j are the corresponding betatron phases, ν is the betatron oscillation number, and K_j is the strength of the *jth* quad.

Least Squares Solution

Because of the uncertainty in the data, x_i, and the desire to keep the number of correcting moves much smaller than the number of sensors, it is natural to regard Eq. (1) as the equation of condition on the fitting of the closed orbit. Our particular interest in the distortion at the extraction septa and in the maximum rather than just the rms distortion leads us to provide for placing extra weight on chosen sensors. This provision must be used carefully to avoid undue effects from sensor errors. The normal equations with arbitrary weights are conveniently expressed in the matrix form

$$A^T DA\Delta = A^T DX \qquad (3)$$

where X is the column vector of closed-orbit measurements, A is the matrix of the coefficients a_{ij}, Δ is the solution vector of fitted δ_j, and D is the diagonal matrix of the weights squared. The basis functions, Eq. (2), are far from orthogonal, but even in hundredfold fits there is generally no difficulty in solving the normal equations to useful precision.

Harmonic Analysis

The circular harmonics are an orthogonal basis with special usefulness in interpretation of the data. By expressing the closed-orbit distortion x as

$$x = \beta^{1/2} \sum_{n=-\infty}^{\infty} c_n e^{in\phi} \tag{4}$$

where β is the Courant-Snyder β function and

$$0 \le \phi = \int \frac{ds}{\nu\beta} \le 2\pi \tag{5}$$

one can derive from the differential equation for the closed orbit an expression for the field error

$$\frac{\Delta B}{B\rho} = \beta^{-3/2} \sum_{n=-\infty}^{\infty} \frac{\nu^2-n^2}{\nu^2} c_n e^{in\phi}. \tag{6}$$

Although the lumped ΔB is treated formally like x, a variable continuous in ϕ, we have position values sufficient to carry the sums only through about fiftieth order. The spatial resolution in a plot of ΔB is therefore inadequate to localize the errors precisely to individual quadrupoles. However, a single displaced quadrupole shows up as a ΔB symmetrical about the quadrupole location. Figure 1 shows (a) the orbit distortion $\beta^{-1/2}x$, (b) the thirtieth order Fourier fit, (c) the corresponding $\beta^{3/2}\Delta B/B\rho$ from Eq. (6); and (d) the closed orbit obtained using this ΔB to give the quadrupole moves. Although Eq. (6) is not a practical solution to the closed orbit problem because of the large number of quadrupole moves it involves, a plot of $\Delta B/B$ gives a way to select promising quadrupoles for a least squares correction according to Eq. (3).

Eq. (6) shows that Fourier components of the closed orbit data having frequencies very different from the ν value of about 20.3 are probably the result of sensor errors. Random error in the orbit measurements will contribute comparable amplitudes in all orders, and the effect of these errors can be mitigated by replacing $\beta^{-1/2}x$ with its Fourier sum to, say, thirtieth order. Furthermore, data points with large residuals in such a fit are probably in error; a better orbit correction can be obtained by excluding them.

Search Algorithm

All that is needed to get the solution according to Eq. (3) is a way to choose a set of quads to move for the correction. We have found most generally useful a systematic trial and error search like that used at the CERN ISR.[2] The *nth* step of this iterative cycle fits the closed orbit with every n-fold combination of quads consisting of the n-1 selected in the prior steps and one of those not selected. The n-tuplet which reduces the rms orbit distortion by the largest amount is the optimum set for this step. Thus, the first step, for example, finds the single quad which generates the closest fit to the observed distortion. On the second step that quad is paired with every other to find the optimum twofold fit, and so on. This algorithm creates a considerable amount of numerical work and is thereby limited for our most readily available computer[3] to about twelvefold fits. Typically, this search algorithm selects quadrupoles located near the middle of the peaks in the ΔB plot calculated according to Eq. (6). The accuracy with which the peaks are found improves with increasing number of search steps.

Typical Results

The closed-orbit distortion shown in Figure 2 has standard deviation of .6" and maximum of 1"; it is a test case generated by the displacement of all ring quads by random amounts with standard deviation .01". A 12-quad correction was found by the search algorithm for these data and the same data with .3" standard deviation of random noise added in. For both the perfect and noisy data, the calculation was made for the data and its 27th-order Fourier sum. The effect of the calculated moves in reducing the true orbit distortion is given in the following table.

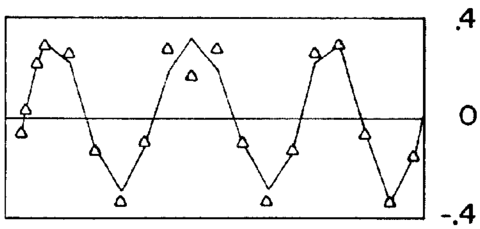
12-Quad Correction of
rms and Maximum Orbit Distortion

	rms	Maximum
Before Correction	.59"	1.04"
Corrected from Perfect Data	.04"	.14"
Corrected from 27th-order Fit to Perfect Data	.05"	.14"
Corrected from Perfect Data Plus σ = .3" Random Error	.19"	.62"
Corrected from 27th-order Fit to Sum of Data and Error	.17"	.45"

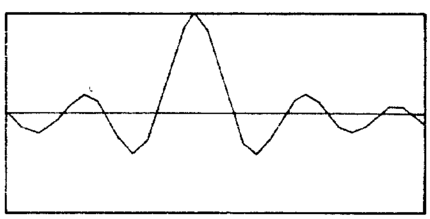
One sees in this example useful correction when the rms noise is comparable to the rms orbit distortion; the elimination of higher harmonics in the data improves the calculated correction slightly.

Figure 3 is a plot of $\beta^{3/2}\Delta B/B\rho$ calculated from the 27th order fit to the noiseless data. The positions of the qudrupoles selected by the search algorithm are indicated by numbers which give the order of selection. The ability of the algorithm to locate the ΔB peaks is evident, particularly after a couple of early misses.

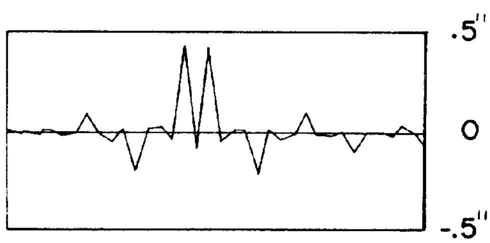
In the last year we have made five sets of quadrupole moves involving a total of 36 magnets. The improvement factor has been between two and three. The current closed orbit distortion (vertical: .1" rms, .2" max; horizontal: .3" rms, .8" max) is roughly a factor of five larger than what we can expect to obtain from the available sensors and techniques. We have yet to take advantage of the improved measure of field error to be obtained by intentionally lowering the tunes toward the integral value to increase the orbit distortion. Although the quadrupole moves we have made differ only slightly from those calculated by the trial-and-error computer search of the raw data, there have been important benefits from a broader approach. Among those have been the maintenance of acceptable error at the extraction septum by data weighting, the smoothing of data and elimination of bad data values by harmonic fitting, and the avoidance of extreme quadrupole displacements by substitution of a nearly equivalent move or moves for an excessive one chosen by the computer. The additional calculations have also helped in estimating the overall reliability of the data and the degree of correction attainable from it.



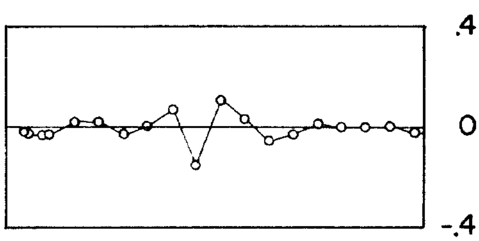
(a) Beam sensor data (Δ) divided by $\beta^{1/2}$ (~10) and 27th order Fourier fit.



(b) $\beta^{3/2}\Delta B/B\rho$ calculated from Fourier coefficients.



(c) Quadrupole moves calculated from above ΔB. The largest moves are the defocusing quads adjacent to the displaced quad.



(d) Closed orbit divided by $\beta^{1/2}$ after correction by above quadrupole moves.

Fig. 1. Closed-orbit distortion in one superperiod caused by 1" displacement of a focusing quadrupole.

Notes and References

1. E.D. Courant and H.S. Snyder, Annals of Physics 3, 1-48 (1958).

2. B. Autin and P.J. Bryant, Proc. VIII Int. Conf. on High Energy Accelerators, 515-520 (CERN, 1971).

3. Digital Equipment Corp. PDP-10 with KA-10 Processor and 1 μsec core.

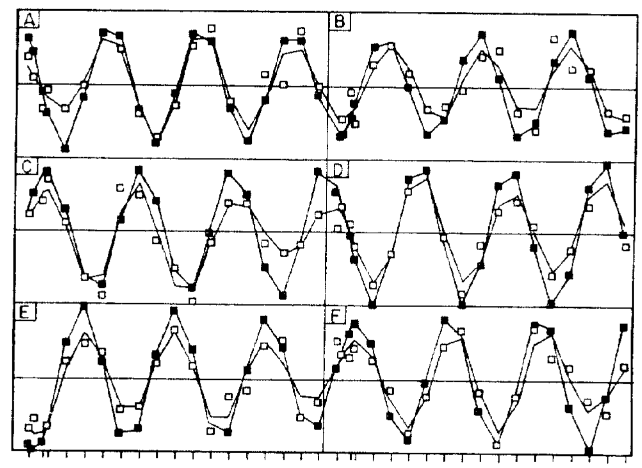


Fig. 2. Closed-orbit distortion (Max = 1", σ = .6") from random quad displacement (σ = .01"). Pure data ■; data plus σ = .3" noise □; curves are 27th-order Fourier fits.

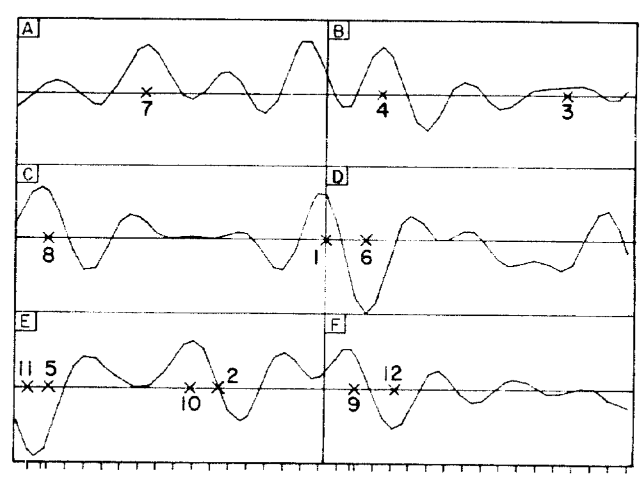


Fig. 3. $\beta^{3/2}\Delta B/B\rho$ from closed orbit data shown in Fig. 2. Order of selection of 12 quads by search algorithm is indicated.