BUNCHING OF INTENSE PROTON BEAMS WITH SIX-DIMENSIONAL MATCHING TO THE LINAC ACCEPTANCE

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Summary

A method has been developed for the determination of bunching parameters in the frame of a six-dimensional beam-matching to the linac acceptance. In the absence of space charge, initial solutions for the buncher distance, voltage and efficiency, which satisfy the matching requirements, are found analytically or with ancillary computer programs. Then an iterative calculation is applied to solve the more general bunching problem in the presence of space charge.

In the above-mentioned analysis, the beam is represented as an entity, described by the second momenta of particle coordinates and velocities. All the forces are linearized; the evolution of r.m.s. values depends primarily on the linear force component.

Introduction

In the design of beam transport systems, it is convenient to treat the beam as an entity (and not as a complex of many individual particles), because in this way the influence of various parameters of the transport system and their mutual relation appear in a clearer fashion. Continuous beams are specified with their covariance matrix in a four-dimensional phase space, whilst bunched beams are described correspondingly in a six-dimensional phase space.

In transport systems, where bunching of continuous beams occurs, there is a difficulty in treating conveniently the longitudinal phase plane problems in the transition region; the usual technique has been to apply here multiparticle programs, which are not well suited for design purposes.

In this paper a method is presented which permits one to treat the beam as an entity, even in the transition region, and which facilitates the determination of longitudinal matching parameters. The passage from a four- to a six-dimensional phase space takes place at the buncher, where the longitudinal beam emittance is formed via the non-linear energy modulation. As a first step, the space charge is neglected and solutions for the buncher voltage, efficiency and distance are found either analytically (single buncher) or with ancillary computer programs (double-drift buncher systems). These solutions are used later as first guesses in iterative computer calculations, dealing with general matching optimizations and having space charge included. The space charge is included in an approximate way, giving rise only to linear forces, which, in principle, suffice to calculate the evolution of the covariance matrix, provided the density distribution is ellipsoidal.[1] Multiparticle programs used to check the validity of the matching obtained by the above procedure[2] show a satisfactory agreement.

Determination of Bunching Parameters without Space Charge

Single buncher

The beam occupies, in the phase space, a volume limited by boundaries which are described by the quadratic form:

$$\sum_{1}^{i}\left(\gamma_i x_i^2 + 2\alpha_i x_i x_i' + \beta_i x_i'^2\right) = E\ , \qquad i = 1,\ 2$$

(if the emittances E in phase planes are not equal, the coordinates of phase planes can be scaled in such a way as to make them equal).

A preferable beam description uses second momenta of coordinates in phase space instead of emittance boundaries; the quadratic form in vector notation becomes

$$X^T \sigma^{-1} X = E_{rms}\ ;$$

X : column vector with components x_i, x_i' ($x' = dx/ds$);

σ : covariance matrix; the elements for each phase plane are

$$\frac{1}{E}\begin{pmatrix} \overline{x^2} & \overline{xx'} \\ \overline{xx'} & \overline{x'^2} \end{pmatrix}\ ;$$

the upper bars indicate mean values;

E_{rms} : r.m.s. value of emittance defined by

$$E_{rms} = \sqrt{\overline{x^2}\,\overline{x'^2} - \overline{xx'}^2}\ . \qquad (1)$$

The r.m.s. emittance is related to the marginal one by $E_{rms} = E/K$, where K depends on the particle distribution and on the number of dimensions of the phase space.

An important property of r.m.s. values is that their evolution depends primarily on linear force components,[1] provided the particle density distribution is of the ellipsoidal type. So, whatever the initial distribution may be, an equivalent uniform distribution yielding the same second momenta can be determined and linearized computer analysis applied for matching purposes. This is a standard procedure for the transverse phase planes; in what follows, an analogous treatment will be developed for the longitudinal plane.

When a continuous beam passes through a buncher, the non-linear velocity modulation (sinusoidal buncher voltage) creates a certain longitudinal emittance, whose r.m.s. value in the (z,z′) plane can be expressed by the above-mentioned formula. It is essential for our model to include in the longitudinal emittance only those particles which will subsequently be trapped in the linac longitudinal acceptance (bucket). In the absence of space charge, this problem can be solved analytically:[3]

We suppose that, just after the buncher, the accepted particles fill in real space an ellipsoid, see Fig. 1.

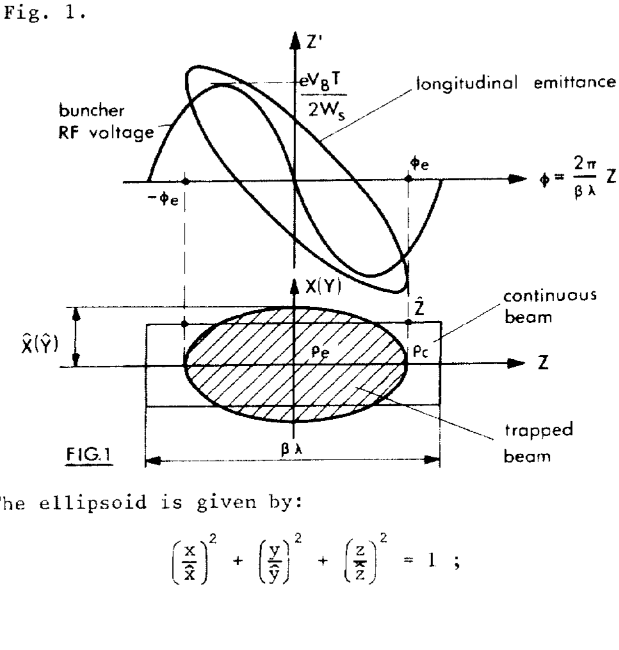

FIG.1

The ellipsoid is given by:

$$\left(\frac{x}{\hat{x}}\right)^2 + \left(\frac{y}{\hat{y}}\right)^2 + \left(\frac{z}{\hat{z}}\right)^2 = 1\ ; \qquad (2)$$

$\hat{x}$ and $\hat{y}$ are marginal values in the transverse phase planes obtained by keeping the r.m.s. values $\sqrt{\overline{x^2}}$ and $\sqrt{\overline{y^2}}$ of the continuous beam unchanged. The r.m.s. values in the (z,z') plane for a uniform density distribution in the ellipsoid are given by

$$\overline{z^2} = \frac{\hat{z}^2}{5} \qquad (3)$$

$$\overline{z'^2} = \frac{1}{8}\,\frac{eV_B T}{W_s}\left\{1 + 3\,\frac{\cos 2\phi_e}{(2\phi_e)^2} - 3\,\frac{\sin 2\phi_e}{(2\phi_e)^3}\right\} \qquad (4)$$

$$\overline{zz'} = \frac{3\beta\lambda}{4\pi}\,\frac{eV_B T}{W_s}\left\{\frac{\sin\phi_e}{\phi_e} + 3\,\frac{\cos\phi_e}{\phi_e^2} - 3\,\frac{\sin\phi_e}{\phi_e^3}\right\}; \qquad (5)$$

V_B, T and W_s are the buncher voltage, transit time factor and mean kinetic energy, respectively.

In the above formulae, ϕ_e (related to $\hat{z}$ by $\phi_e = 2\pi\hat{z}/\beta\lambda$) and V_B are not yet known. They can be determined by the condition that the longitudinal beam emittance matches the longitudinal linac acceptance, defined at a symmetry point by an energy spread $\pm\Delta W_A$ and a phase extension $\pm\Delta\phi_A$ (rad). In this case the second momenta (3), (4) and (5) have to satisfy the equations

$$\left(\Delta\phi_A\right)^2 = 5\left(\frac{2\pi}{\beta\lambda}\right)^2\left(\overline{z^2} - \frac{\overline{zz'}^2}{\overline{z'^2}}\right) \qquad (6)$$

$$\left(\frac{\Delta W_A}{2W_s}\right)^2 = 5\,\overline{z'^2}\,. \qquad (7)$$

The buncher has to be placed at such a distance from the symmetry point as to bring the emittance ellipse into principal axes:

$$d_{BL} = -\,\frac{\overline{zz'}}{\overline{z'^2}}\,. \qquad (8)$$

Introducing Eqs. (3), (4) and (5) into the right-hand side of Eq. (6), one finds that $\Delta\phi_A$ is a function of ϕ_e only; Eq. (6) determines therefore ϕ_e and hence the bunching efficiency η (ratio of trapped to total beam). Knowing ϕ_e, formula (7) is used to get the buncher voltage explicitly:

$$eV_B T = \left[\frac{2}{5}\bigg/\left(1 + 3\,\frac{\cos 2\phi_e}{(2\phi_e)^2} - 3\,\frac{\sin 2\phi_e}{(2\phi_e)^3}\right)\right]^{\frac{1}{2}}\Delta W_A\,; \qquad (9)$$

Eq. (8) is applied analogously for the buncher distance. The bunching parameters ϕ_e, V_B and d_{BL} are, in this way, uniquely determined for the zero space charge case. The definition of linac acceptance at a symmetry point makes the formulae simpler, but is not otherwise essential.

It should be mentioned that the non-linear bunching voltage has repercussions also in the transverse phase planes: the radial gap defocusing varies with the phase of the buncher RF voltage and the transverse emittances are slightly increased (formulae omitted due to lack of space).

The formulae (3), (4), and (5) are valid for a line distribution of particles inside the longitudinal emittance ellipse, see Fig. 1. In reality the particles occupy a finite area in the ellipse due to the variation of the transit time factor with radius, which brings about a radially-dependent energy modulation, see Fig. 2.

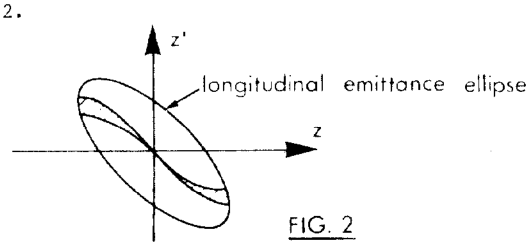

FIG. 2

One can refine the formulae for second momenta accordingly: the beam has at the buncher usually a nearly circular cross-section, so one can take as beam radius $R = \sqrt{\hat{x}\hat{y}}$ and describe approximately the radial variation of T as

$$T(R) \simeq T_a\left(1 + \frac{k_r^2}{4}\sqrt{\hat{x}\hat{y}}\right) \qquad (10)$$

T_a : value of T at r = 0

$k_r = \sqrt{(\omega/v)^2 - (\omega/c)^2}$

The calculation of second momenta in the (z,z') plane is now lengthy but straightforward; the integrals are of the type:

$$\int_{-1}^{1} (1 - u^2)^n \sin^2(\phi_e u)\,du, \qquad n = 1, 2, 3$$

$$\int_{-1}^{1} (1 - u^2)^n\, u\cos(2\phi_e u)\,du, \qquad n = 1, 2\,.$$

The results are as follows [for $\overline{z^2}$, Eq. (1) is still valid]:

$$\overline{z'^2} = \frac{1}{8}\,\frac{eV_B T_a}{W_s}\left\{I_1 + a\hat{x}\hat{y}\,I_2 + \frac{a^2}{3}\,\hat{x}^2\hat{y}^2\,I_3\right\} \qquad (11)$$

with $a = k_r^2/4$ and

$$I_1 = \left(1 + 3\,\frac{\cos 2\phi_e}{(2\phi_e)^2} - 3\,\frac{\sin 2\phi_e}{(2\phi_e)^3}\right)$$

$$I_2 = \frac{4}{5}\left(1 + 15\,\frac{\sin 2\phi_e}{(2\phi_e)^3} + 45\,\frac{\cos 2\phi_e}{(2\phi_e)^4} - 45\,\frac{\sin 2\phi_e}{(2\phi_e)^5}\right)$$

$$I_3 = \frac{24}{35}\left(1 - 105\,\frac{\cos 2\phi_e}{(2\phi_e)^4} + 630\,\frac{\sin 2\phi_e}{(2\phi_e)^5} + 1575\,\frac{\cos 2\phi_e}{(2\phi_e)^6} - 1575\,\frac{\sin 2\phi_e}{(2\phi_e)^7}\right);$$

$$\overline{zz'} = \frac{3\beta\lambda}{4\pi}\,\frac{eV_B T_a}{W_s}\left\{\frac{\sin\phi_e}{\phi_e} + 3\,\frac{\cos\phi_e}{\phi_e^2} - 3\,\frac{\sin\phi_e}{\phi_e^3} + 2\,a\hat{x}\hat{y}\left(-\frac{\cos\phi_e}{\phi_e^2} + 6\,\frac{\sin\phi_e}{\phi_e^3} + 15\,\frac{\cos\phi_e}{\phi_e^4} - 15\,\frac{\sin\phi_e}{\phi_e^5}\right)\right\}. \qquad (12)$$

Introducing Eqs. (11) and (12) into (6), (7) and (8), the bunching parameters are determined with a better accuracy.

Double-drift harmonic buncher system

Figures 1 and 2 show that a single buncher fills the longitudinal beam emittance ellipse in an inefficient and rather non-uniform way. The situation is improved if other bunchers are added. The method described above for the determination of longitudinal matching parameters can, in principle, be applied also for systems including more bunchers. The longitudinal emittance is defined at the last buncher, after all of the energy modulation has taken place. However, at the last buncher, the modulation curve $z' = f(z)$ [or $z' = f(\phi)$] is no more a simple sinusoid, and the analytic determination of second momenta usually becomes impossible. This difficulty can be overcome with an ancillary computer program, which considers a certain number of macro-particles and calculates their respective energy modulation from the first to the last buncher; in a certain sense, one obtains a point by point representation of the modulation curve $z' = f(z)$ and the second momenta are then determined numerically.

The method and the computer program have been developed particularly with respect to a double drift harmonic buncher system. In contrast to a single buncher, here one has two more variables in order to fulfil the same number of conditions or constraints (the longitudinal linac acceptance is defined with three parameters); these variables are the distance between the bunchers and the voltage of the second one. To make full use of the possibilities of a two-buncher system, the additional variables may be determined by additional constraints: choose the distance and the voltage so as to make the longitudinal emittance a minimum for a certain bunching efficiency. Since, in our model, the area of the longitudinal emittance is given (it is equal to the acceptance), the additional constraints, in fact, maximize the bunching efficiency. In Fig. 3, the optimum ratio of buncher voltages V_{B_2}/V_{B_1} and the optimum value of ψ_{12} [$\psi_{12} = 180\, eV_{B_1} T\, d_{12}/\beta\lambda W_s$ (degrees), where d_{12} is the distance between the bunchers] are shown as functions of the bunching efficiency:

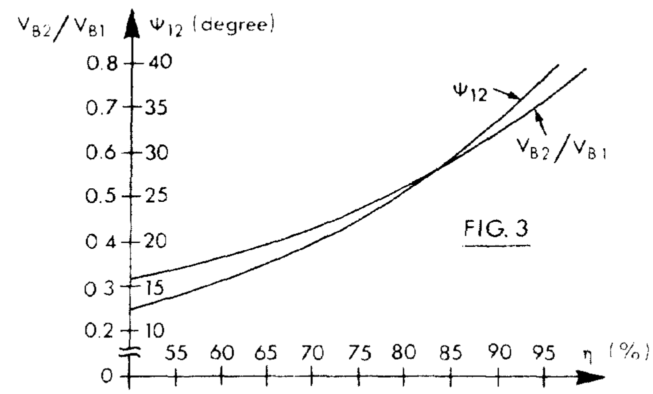

FIG. 3

Proceeding in this way, one gets the best filling of the emittance ellipse and the highest bunching efficiency of the double-drift system.

Inclusion of Space Charge Forces

In order to apply the above treatment in computations dealing with general matching optimizations, space charge forces have to be introduced in the analysis. This can be done only approximately if one wishes to use linearized computer programs and consider the beam as an entity. It is essential that the influence of "accepted" as well as of "rejected" particles (outside the ellipsoid of Fig. 1) is taken into account. In addition, there must be a continuity of space charge forces at the buncher, where the transition from a four- to a six-dimensional phase space takes place. The space charge forces among "accepted" particles have also to increase progressively, as the beam bunching goes on.

A space charge model which satisfies these conditions is the one which deduces the space charge forces as coming from a combined action of an infinite cylinder with the density of rejected particles ρ_c and of an ellipsoid with the density $\rho_e - \rho_c$ (difference in density of accepted and rejected particles).[3] The density distributions ρ_c and ρ_e are supposed to be uniform. The formulae determining ρ_e and ρ_c are:

$$\rho_e = \frac{I\, T_{RF}\, \eta}{\frac{20\sqrt{5}}{3}\pi\sqrt{\overline{x^2}\,\overline{y^2}\,\overline{z^2}}} \qquad (13)$$

$$\rho_c = \frac{I\, T_{RF}\,(1-\eta)}{\frac{20\sqrt{5}}{3}\pi\sqrt{\overline{x^2}\,\overline{y^2}}\left(\frac{3}{5\sqrt{5}}\beta\lambda - \sqrt{\overline{z^2}}\right)} \qquad (14)$$

with I : pre-injector current

T_{RF} : RF period

η : bunching efficiency (ratio of trapped to total current)

$\sqrt{\overline{x^2}}$, $\sqrt{\overline{y^2}}$, $\sqrt{\overline{z^2}}$: r.m.s. coordinates of the ellipsoid containing the subsequently trapped part of the beam

The space charge fields due to the cylinder are:

$$E_x = \frac{\rho_c}{\varepsilon_0}\frac{\sqrt{\overline{y^2}}}{\sqrt{\overline{x^2}} + \sqrt{\overline{y^2}}}\, x \qquad (15)$$

and analogous for E_y. Fields due to the ellipsoid are:

$$E_x = \frac{\rho_e - \rho_c}{2\varepsilon_0}\sqrt{\overline{x^2}\,\overline{y^2}\,\overline{z^2}}\; I_x\, x \qquad (16)$$

and analogous for E_y and E_z, with

$$I_x = \int_0^\infty \frac{d\lambda}{(\overline{x^2} + \lambda)\sqrt{(\overline{x^2} + \lambda)\,(\overline{y^2} + \lambda)\,(\overline{z^2} + \lambda)}} \qquad (17)$$

and analogous for I_y and I_z.

The accuracy of the space charge model is the more limited, the more the particle distribution deviates from an ellipsoidal one. Nevertheless, the matchings obtained so far by this method, and tested with multiparticle programs, are satisfactory.[2]

General Matching Optimization

The methods for the determination of longitudinal matching parameters and space charge forces have been introduced into computer programs dealing with general matching problems in six dimensions:[2,3] two transverse phase planes and one longitudinal. The zero current solutions are used as first guesses and one starts with a relatively small current; each time solutions for a certain beam intensity are found, they replace the first guesses and the current is raised until the nominal value is reached.

The application of this computing technique to the pre-injector of the CPS Linac is reported in another paper submitted to this conference.[3]

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