PARAMETER STUDY OF THE "INVISIBLE" LONG STRAIGHT SECTION FOR SYNCHROTRONS†

Philip F. Meads, Jr.

William M. Brobeck & Associates
Berkeley, California 94710

Summary

A symmetric inseretion containing an unusually long unobstructed drift space is described.[1] A consequence of its identity transfer properties in the radial plane (2π betatron phase advance) and its transverse reflection properties in the vertical plane (π betatron phase advance) is its *invisibility*, neither disturbing the betatron oscillation in either plane for particles of synchronous energy nor disturbing the closed (equilibrium) orbit properties for particles of different energy. Because of the different phase advances in the two planes, the insertion can be designed so that all of its six quadrupole magnets lie at the extreme ends of the insertion. This leaves *all* of the drift space, excepting only that portion required for coil terminations, etc., in a single unobstructed drift space. A thorough parameter study with linear and aberration properties through third order is reported.

Introduction

The π-2π *invisible* long straight section insertion differs previous long straight section insertions in two significant ways. It is *symmetrical* with regard to component placement relative to the midpoint whereas the other insertions are antisymmetrical. The π-2π insertion is *asymmetrical* with regard to the betatron phase advance in the two transverse planes. Because of this latter property, it is possible to place all of the quadrupole magnets at the ends of insertion, leaving all of the drift space—*uninterrupted*—in the center of the insertion. The insertion consists of two symmetrically placed asymmetrical quadrupole triplets designed to provide an identity transfer matrix in the radial plane, including the relative displacement of the off-energy equilibrium orbit, and a reflection matrix in the vertical plane (π phase advance). Given these conditions, there are, in addition to the lengths of the quadrupole magnets, three free parameters. These are best defined as (1) the total length of the insertion, (2) the length of the unobstructed central drift space L_c, and (3) the radial magnification m at the center of the insertion. In this study we treat a fixed total length of 15.736 meters with quadrupole gradients appropriate to 800 MeV protons. For purposes of brevity, the effective length of the quadrupole magnets is fixed at 0.3048 meter (12 inches) although different lengths, including unequal lengths, have been studied. The task is then to describe the insertion as a function of the parameters L_c and m. We restrict the discussion to the *normal permutation* which is defined as that possessing symmetry about the midpoint and for which the length of the central drift space exceeds twice that of the end drift spaces. Elements of such insertions may be permuted to provide nonsymmetric π-2π long straight sections that may be more appropriate for matching into a particular ring lattice.

Certain Properties

At the midpoint of the insertion the radial and vertical transfer matrices are

$$\begin{pmatrix} -m & 0 & 0 \\ 0 & -\frac{1}{m} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & F \\ -\frac{1}{F} & 0 \end{pmatrix} \quad \text{, respectively.} \qquad (1)$$

Here F is the focal length in the vertical plane of one of the triplets. Let β be the aspect ratio of the beam at a waist; i.e. β is the ratio of the maximum displacement to the maximum slope. (This is equivalent to the Twiss β function when the waist is in a closed lattice.) Let the subscript c denote a condition at the midpoint. Let the subscript 0 denote a condition entering the insertion. Then

$$\beta_c = m^2 \beta_0 \qquad \text{(radial plane); (2)}$$

$$\beta_c \beta_0 = F^2 \qquad \text{(vertical plane). (3)}$$

Optimal Matching

Let T be the transfer matrix from point 0 to some arbitrary point where the Twiss β function is to be minimized. We have

$$\beta = T_{11}^2 \beta_0 + T_{12}^2 \frac{1}{\beta_0} \qquad (4)$$

The condition on β_0 that minimizes β is

$$\beta_0 = T_{12} / T_{11} \qquad (5)$$

The minimum value of β is

$$\beta = 2 T_{11} T_{12} \qquad (6)$$

In a well-matched insertion with long central drift space, the maximum aperture occurs (neglecting the finite quadrupole length) at ends of the central drift space. This maximum β is minimized when there is a waist at the center of the insertion with $\beta_c = \frac{1}{2} L_c$. Then the maximum $\beta = L_c$. From eq. (2), we find the condition at the end of the insertion to be $\beta_0 = \beta_c / m^2 = L_c / 2m^2$. This is the general optimum radial matching condition. Because this represents a stationary condition, actual failure to match by as much as a factor of 2 will result in only a 12% increase in radial aperture.

Where we are attempting to obtain a large central drift length, the end drift lengths are relatively short. Thus, assuming equal quadrupole apertures, the minimum aperture is obtained when $\beta_0 = \beta = L$ which yields $m^2 = \frac{1}{2}$. (7)

Another interesting case is that where the radial waist at the center is identical to that at the ends; i.e. m=1.

Vertical Plane

The previous remarks apply only to the radial plane. The maximum vertical aperture occurs in the second and fifth quadrupole magnets. From numerical work we find that both the optimum initial and the corresponding maximum value of β_y are nearly independent of m and decrease slightly with increasing L_c.

Two-Parameter Study

Based on the above observation, a number of π-2π insertions with four different values of m and with increments of 0.5 meter in the length of the central drift space have been studied. The parameters of interest for this study are contained in Table I. Three of these insertions are sketched in Figure 1. Here the radial beam profile is shown above the center line whereas the vertical beam profile is shown below the centerline. The insertions would fit well at the F-F symmetry point of an FDDF type lattice.

Maximal π-2π Long Straight Sections

We see from Table I that the objectives of both maximizing the length of the central drift space while minimizing the central radial magnification are constrained by the length of the drift space between the second and third quadrupoles. It is therefore instructive to set this parameter at some minimum acceptable value and then study the insertion as the single independent variable (which may be taken to be either L_c or m) is incremented. Such a study is shown in Table II where this critical drift length has been fixed at 0.2 meter.

As Lc increases the admittance decreases; the admittances shown are based on having identical quadrupoles with pole-tip fields equal to 10 kG. The product of the square of the magnetic field gradient and the β function is tabulated in columns 10 and 11. This indicates what is possible if one allows the quadrupole apertures to differ. Although the gradient of the first quadrupole gets enormous, the actual pole-tip field of a minimum-aperture quad increases only slightly with increasing L_c.

Rigorous analytic relationships for this insertion have been derived by Regenstreif.[2]

Aberrations

The particular symmetry of the π-2π insertion eliminates half of the normal aberration terms.[3] There are no generalized distortion nor coma terms in the displacement nor are there generalized spherical aberration nor astigmatism terms in the slopes. There are no second-order chromatic aberration corrections to the diagonal terms of the transfer matrices. The aberration corrections to the 12 matrix elements range from 5 $\Delta p/p$ meters to 40 $\Delta p/p$ meters. The smaller aberrations are associated with larger m and larger L_c. The rms geometric aberration displacement lies between 0.02 mm and 0.2 mm for the insertions shown.

References

[1] P. F. Meads, Jr., Nucl. Instr. and Meth. **96** (1971) 351.
[2] E. Regenstreif, Basic Relations for the Symmetric Insertion –to be published.
[3] P. F. Meads, Jr., Lawrence Radiation Laboratory Report UCRL-10807(1963).

† Work supported by the United States Atomic Energy Commission.

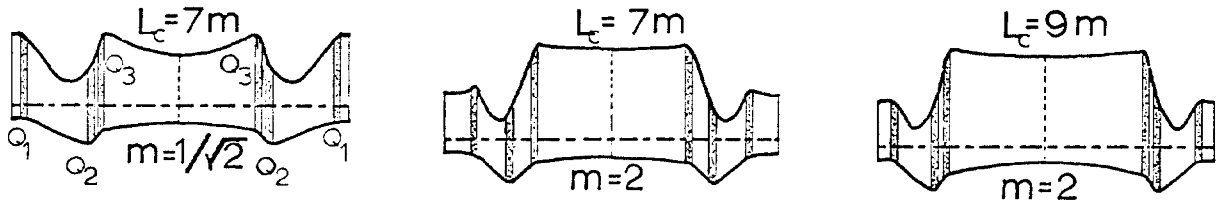

Fig. 1. Typical Beam Profiles for the π-2π Insertion

TABLE I. **Parameters of Selected π-2π Long Straight Sections**

Insertion length is 15.736 meters; Quadrupole gradients for 800 MeV protons with 0.3048 meter effective length quadrupoles.

End Drift meters	Q1–Q2 Drift meters	Q2–Q3 Drift meters	Center Drift meters	Q1 Gradient kG/cm	Q2 Gradient kG/cm	Q3 Gradient kG/cm	F meters	Beta y–max meters	Beta y–end meters
				m = 1/√2					
2.522	1.259	1.173	4.000	0.9772	-1.2660	1.2479	3.125	24.34	3.42
2.194	1.480	1.030	4.500	0.9107	-1.2570	1.2752	3.124	24.28	3.18
1.858	1.714	0.882	5.000	0.8533	-1.2683	1.3148	3.100	23.93	2.93
1.512	1.964	0.728	5.500	0.8026	-1.3036	1.3724	3.052	23.30	2.68
1.154	2.234	0.566	6.000	0.7572	-1.3726	1.4589	2.976	22.39	2.41
0.779	2.531	0.394	6.500	0.7158	-1.4984	1.5981	2.869	21.15	2.13
0.382	2.865	0.207	7.000	0.6773	-1.7482	1.8579	2.723	19.54	1.84
0.047	3.090	0.067	7.500	0.6889	-2.1195	2.2251	2.534	18.04	1.53
				m = 1					
2.556	1.027	1.371	4.000	1.1596	-1.2812	1.0584	3.144	24.09	3.41
2.309	1.179	1.216	4.500	1.1097	-1.2720	1.0991	3.154	24.01	3.22
2.057	1.336	1.061	5.000	1.0671	-1.2778	1.1475	3.147	24.09	2.94
1.799	1.497	0.907	5.500	1.0300	-1.2997	1.2065	3.123	24.00	2.74
1.536	1.665	0.753	6.000	0.9973	-1.3412	1.2807	3.080	23.70	2.52
1.266	1.842	0.596	6.500	0.9681	-1.4094	1.3780	3.016	23.16	2.30
0.987	2.030	0.437	7.000	0.9417	-1.5191	1.5142	2.929	22.37	2.08
0.698	2.234	0.272	7.500	0.9175	-1.7063	1.7255	2.814	21.30	1.84
0.396	2.461	0.096	8.000	0.8953	-2.0877	2.1291	2.666	19.91	1.59
				m = √2					
2.395	0.945	1.614	4.000	1.2964	-1.2609	0.8764	3.113	24.11	3.19
2.208	1.054	1.442	4.500	1.2699	-1.2571	0.9212	3.126	24.31	3.03
2.017	1.163	1.274	5.000	1.2495	-1.2648	0.9723	3.126	24.35	2.87
1.823	1.272	1.109	5.500	1.2341	-1.2845	1.0315	3.112	24.22	2.70
1.625	1.381	0.948	6.000	1.2231	-1.3178	1.1014	3.084	24.20	2.53
1.423	1.492	0.789	6.500	1.2157	-1.3680	1.1859	3.042	24.10	2.29
1.218	1.604	0.632	7.000	1.2118	-1.4411	1.2916	2.982	23.82	2.10
1.008	1.719	0.476	7.500	1.2109	-1.5482	1.4303	2.905	23.34	1.92
0.794	1.840	0.320	8.000	1.2130	-1.7132	1.6263	2.807	22.63	1.72
0.573	1.969	0.162	8.500	1.2180	-1.9980	1.9418	2.684	21.67	1.51
0.368	2.062	0.023	9.000	1.2568	-2.5000†	2.4651	2.534	20.57	1.31
				m = 2					
2.117	0.947	1.890	4.000	1.4118	-1.2215	0.7264	3.037	24.25	2.88
1.974	1.028	1.702	4.500	1.4085	-1.2238	0.7687	3.047	24.62	2.76
1.829	1.106	1.519	5.000	1.4111	-1.2349	0.8164	3.048	24.87	2.62
1.681	1.181	1.342	5.500	1.4192	-1.2552	0.8709	3.038	24.99	2.48
1.531	1.254	1.169	6.000	1.4326	-1.2860	0.9340	3.017	24.97	2.34
1.379	1.324	1.001	6.500	1.4511	-1.3291	1.0080	2.984	24.80	2.19
1.225	1.392	0.837	7.000	1.4749	-1.3876	1.0967	2.939	24.65	1.99
1.069	1.458	0.677	7.500	1.5040	-1.4668	1.2059	2.880	24.54	1.84
0.911	1.522	0.521	8.000	1.5390	-1.5760	1.3453	2.807	24.26	1.68
0.750	1.586	0.368	8.500	1.5806	-1.7329	1.5333	2.716	23.80	1.52
0.587	1.651	0.216	9.000	1.6295	-1.9779	1.8108	2.607	23.14	1.35

† linear programming constraint

TABLE II. **Parameters of Extreme π-2π Long Straight Sections**

Q2–Q3 Drift ≡ 0.200 meters (the constraint); Total length and gradients are as in Table I.

m	Center Drift meters	End Drift meters	Q1–Q2 Drift meters	Q1 Gradient kG/cm	Q2 Gradient kG/cm	Q3 Gradient kG/cm	beta x maximum meters	beta y maximum meters	B max radial kg^2/m	B max vertical kG^2/m	x admittance cm–mr	y admittance cm–mr
0.70124	7.0	0.3607	2.893	0.6712	–1.7596	1.8714	7.136	19.45	0.2451	0.6022	40.79	16.61
0.89900	7.5	0.5277	2.476	0.8295	–1.8055	1.8605	7.500	20.38	0.2596	0.6644	38.52	15.05
1.16134	8.0	0.6129	2.141	1.0304	–1.8627	1.8528	8.000	21.27	0.2746	0.7380	36.41	13.55
1.50451	8.5	0.6230	1.881	1.2818	–1.9287	1.8479	8.500	22.13	0.2903	0.8230	34.45	12.15
1.94690	9.0	0.5765	1.677	1.5794	–2.0036	1.8481	9.000	22.97	0.3074	0.9222	32.53	10.84
2.51183	9.5	0.4955	1.508	2.0032	–2.0910	1.8569	9.500	23.84	0.3276	1.0424	30.53	9.59
3.22972	10.0	0.3987	1.355	2.5432	–2.1975	1.8781	10.000	24.75	0.3527	1.1952	28.35	8.37
4.13989	10.5	0.2994	1.204	3.2893	–2.3329	1.9150	10.500	25.71	0.3850	1.3991	25.97	7.15
5.28912	11.0	0.2057	1.048	4.3615	–2.5117	1.9710	11.000	26.68	0.4273	1.6834	23.40	5.94
6.72528	11.5	0.1217	0.882	5.9718	–2.7574	2.0505	11.500	27.61	0.4835	2.0995	20.68	4.76
8.47714	12.0	0.0485	0.705	8.5378	–3.1126	2.1593	12.000	28.76	0.5595	2.7855	17.87	3.59
9.84886	12.5	–0.0041†	0.508	11.9367	–3.6700	2.3137	11.892	27.55	0.6364	3.7096	15.71	2.70
11.51324	13.0	–0.0692†	0.323	19.7437	–4.6263	2.5039	12.071	26.16	0.7562	8.0047	13.22	1.25

† virtual end drifts–included for completeness only