CLOSED ORBIT CORRECTION IN THE AGS*

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I. Introduction

In order to utilize efficiently the available aperture in the AGS and thereby achieve a high intensity accelerated beam, it is important that deviations in the horizontal and vertical equilibrium orbits be minimized. As has been emphasized[1], this is particularly true when multiturn injection is used. The AGS Conversion program has provided the synchrotron ring structure with 96 horizontal and 96 vertical dipole magnets located at the 2, 4, 6, 8, 12, 14, 16, 18 straight sections of each superperiod.[2] Each of these correction dipoles[3] is independently powered and capable of giving the beam an angular deflection of 0.15 mrad for the injection momentum (8.46 x 10^6 gauss in.). The position of the beam in the AGS is observed with the aid of 36 horizontal and 36 vertical electrostatic induction electrodes located azimuthally at the 3, 7, and 15 straight sections of each of the superperiods.

II. The Equilibrium Orbit

The equilibrium orbit, that is, the average position of a beam of particles in the AGS can be expressed in terms of normalized variables as[4,5]

$$\eta_{E.O.}(\psi_k) = \frac{\beta_{AV}^{\frac{1}{2}}}{\nu}\frac{\Delta p}{p} + \frac{1}{2\sin\pi\nu}\int_{\psi_k}^{\psi_k+2\pi\nu} d\tau f(\tau)\cos(\tau-\psi_k-\pi\nu), \tag{1}$$

where,

$\psi_k = \int_0^{s_k}\frac{d\tau}{\beta(\tau)}$, the betatron phase at the azimuthal position s_k,

$\nu = \frac{1}{2\pi}\psi(C)$, the number of betatron oscillations for a full circumference, $C = 2\pi R$, of the ring,

$\beta_{AV} = R/\nu$, the average beta function, $\beta(\tau)$, for the central superperiod-symmetric reference orbit,

$\Delta p/p$, the fractional momentum deviation from the momentum of the central orbit,

$f(\tau)=\beta^{3/2}(s)F(s)$, the normalized perturbing magnetic dipole distribution [F(s) = magnetic field increment at point s/magnetic rigidity of central particle], and

$\eta_{E.O.}(\psi_k)=\beta^{-\frac{1}{2}}(s_k)y(s_k)$, the normalized position variable corresponding to a transverse displacement at s_k equal to $y(s_k)$.

In the specific case of the correction dipoles, the perturbing field distribution consists of a set of 96 point dipoles and is best written in term of the Dirac delta function as

$$f(\tau) = \sum_{\alpha=1}^{96} F_\alpha\,\delta(\tau-\psi_\alpha) \quad . \tag{2}$$

Here,

$$F_\alpha = \theta(\psi_\alpha)\,\beta^{\frac{1}{2}}(s_\alpha) \quad , \tag{3}$$

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and represents the normalized angular deflection at the azimuthal location, ψ_α, of a dipole which deflects the particle trajectory through an angle of $\theta(\psi_\alpha)$ radians. Inserting Eq. (2) into Eq. (1), and restricting the ψ_k variable to the 36 pick-up electrode observation points, we derive the matrix equation

$$\eta_{E.O.}(\psi_k) = \eta_k = 1_k D + M_{k\alpha}F_\alpha, \quad k = 1,36,\ \alpha = 1,96 \ , \tag{4}$$

where 1_k is the vector with all components equal to one and multiplies the normalized momentum offset, $D = \beta_{AV}^{\frac{1}{2}}\Delta p/\nu p$. The rectangular matrix, $M_{k\alpha}$, has the elements

$$M_{k\alpha} = \frac{\cos\pi\nu}{2\sin\pi\nu}\cos(\psi_k-\psi_\alpha) + \frac{1}{2}\sin(\psi_k-\psi_\alpha), \quad \psi_k>\psi_\alpha \ , \tag{5a}$$

$$M_{k\alpha} = \frac{\cos\pi\nu}{2\sin\pi\nu}\cos(\psi_k-\psi_\alpha) - \frac{1}{2}\sin(\psi_k-\psi_\alpha), \quad \psi_k<\psi_\alpha \ . \tag{5b}$$

In writing this matrix relation we have adopted the notation that Greek letter subscripts correspond to the dipoles and run from 1 to 96, while Latin subscripts correspond to the pick-up electrode positions and have values 1 to 36. In addition we use the convention that repeated indices indicate summation.

III. Theory of Orbit Correction

The problem of correcting the deviations in an equilibrium orbit observed at discrete points is, in this case, that of finding 96 values for the dipole fields that can cause 36 deviations, and then introducing these dipole field values with a reversal of polarity into the ring. It is clear that the problem has many solutions depending on the additional constraints that are imposed. One solution suggested by E. Courant[6] is: Minimize the sum of the square of the 96 dipole field values subject to the constraint that the resultant orbit pass through the 36 observed deviations. Thus, letting

$$A = F_\alpha F_\alpha \quad ; \quad \alpha = 1,96 \quad , \tag{6}$$

we defined 36 constraints B_k:

$$B_k = \eta_k - M_{k\alpha}F_\alpha - 1_k D = 0, \quad k = 1,36 \quad . \tag{7}$$

We now consider the change of the quantities A and B_k for variations in F_α and D and obtain

$$\delta A = 2F_\alpha\,\delta F_\alpha = 0 \quad , \quad \text{and} \tag{8a}$$

$$\delta B_k = -M_{k\alpha}\,\delta F_\alpha - 1_k\delta D = 0 \quad . \tag{8b}$$

Since these variations are not independent we introduce the Lagrange multipliers, λ_k, and find

$$(F_\alpha - \lambda_k M_{k\alpha})\,\delta F_\alpha - \lambda_k 1_k\delta D = 0 \quad . \tag{9}$$

If we define the symmetric matrix $N_{\ell k} = M_{\ell\alpha}M_{k\alpha}$, we obtain for the required F_α values and the momentum offset, D:

$$F_\alpha = (N^{-1})_{k\ell}\,(\eta_\ell - 1_\ell D)\,M_{k\alpha} \tag{10a}$$

$$D = \frac{1_k\,(N^{-1})_{k\ell}\eta_\ell}{1_k\,(N^{-1})_{k\ell}1_\ell} \quad . \tag{10b}$$

It should be mentioned, at this point, that the above derivation is specifically concerned with the horizontal orbit for which a deviation from the center of the machine can be due to a momentum offset. For the vertical orbit, the derivation follows in a similar fashion except that the momentum offset equals zero, and only Eq. (10a) (with D = 0) is applicable.

IV. Analytical Model for Betatron Parameters

Because of the desire to simplify the analysis of the previous sections, we have used normalized variables wherever possible. However in applying the derived results, it is necessary to use specific values for the betatron parameters. These are conveniently represented by an analytical model of the AGS[7]; and, for completeness, we shall essentially list them. Thus for the betatron phase function at any azimuthal position s, we have

$$\psi(s) = \psi_L \left[\frac{s}{L} + \frac{\tau}{2\pi}\left(\cos\frac{2\pi s}{L} - 1\right) + \frac{\sigma}{2\pi}\left(\cos\frac{6\pi s}{L} - 1\right)\right] \tag{11}$$

where L is the length of a basic cell and $\psi_L = 2\pi\nu/60$. The two constants, τ and σ, take on equal and opposite values for the horizontal and vertical planes and are given by

$$\tau_H = -\tau_V = \frac{3}{8},$$
$$\sigma_H = -\sigma_V = \frac{1}{72}. \tag{12}$$

The beta functions which are consistent with this expression for the phase function are:

$$\beta_H(s) = \frac{R}{\nu_H}\left[\frac{1}{1 - \frac{3}{8}\sin\frac{2\pi s}{L} - \frac{1}{24}\sin\frac{6\pi s}{L}}\right], \quad \text{and} \tag{13}$$

$$\beta_V(s) = \frac{R}{\nu_V}\left[\frac{1}{1 + \frac{3}{8}\sin\frac{2\pi s}{L} + \frac{1}{24}\sin\frac{6\pi s}{L}}\right]. \tag{14}$$

V. Application and Results

An on-line program for the AGS control computer (Digital Equipment Corp. PDP-10) has been developed which, given a measured equilibrium orbit at injection energy, calculates the required magnetic field corrections. As can be seen from the block diagram, Fig. 1, the inputs are the measured orbit and the ν value. In addition to the output correction field values, the orbit positions at the 240 straight sections of the AGS are calculated and can be displayed. Figures 2 and 3 show a typical uncorrected and corrected vertical orbit. As can be seen the peak to peak distortion was reduced from approximately 2 cm to 2 mm. The very local deviation shown at the I15 pick-up electrode is indicative that the power supply limit was reached for those dipoles in that vicinity. Figures 4 and 5 are similar pictures for the horizontal orbit. Here again the limitation in correcting the orbit is due to the power supply limitation.

VI. Mid-Field Orbit Correction

It is of interest to mention that a method similar to that described in the previous sections for correcting the injection orbit has been developed for correcting the mid-field orbits. Because of the higher energy, it is necessary in this instance to move the main ring magnets. The basic calculation was modified such that those magnets to be moved were those which initially exhibited the largest physical displacements as determined by an optical survey of the ring. Thus by moving 153 of the 240 main ring magnets (the maximum horizontal motion was about 0.080 in.), we have reduced the peak to peak horizontal distortion from approximately 2 cm to 1 cm.

Acknowledgment

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References

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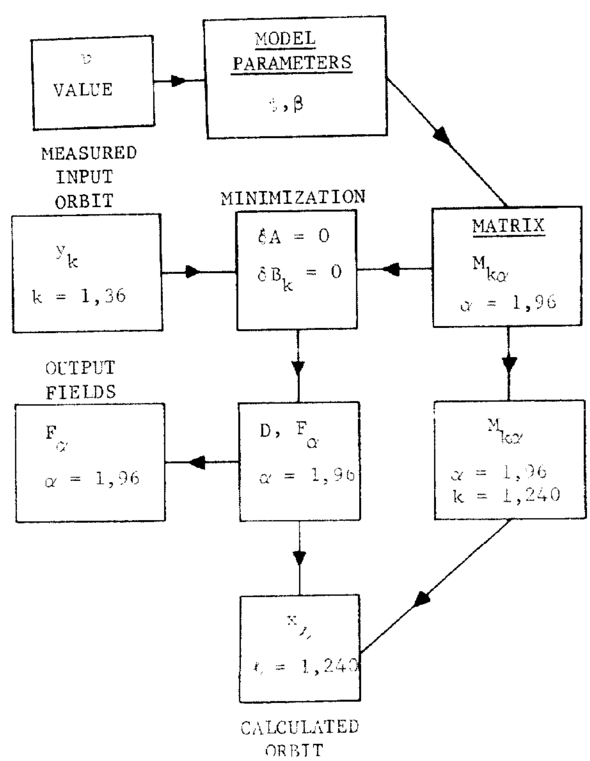

Fig. 1. Block diagram for orbit correction and display program.

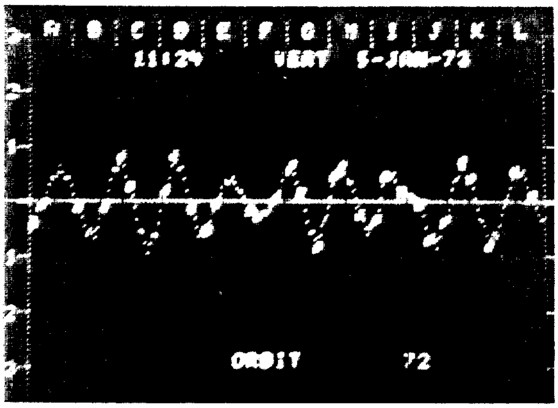

Fig. 2. Uncorrected injection field vertical orbit. The brighter dots are observed displacements.

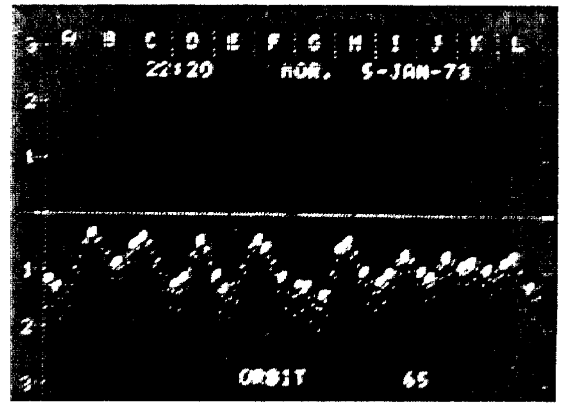

Fig. 4. Uncorrected injection field horizontal orbit.

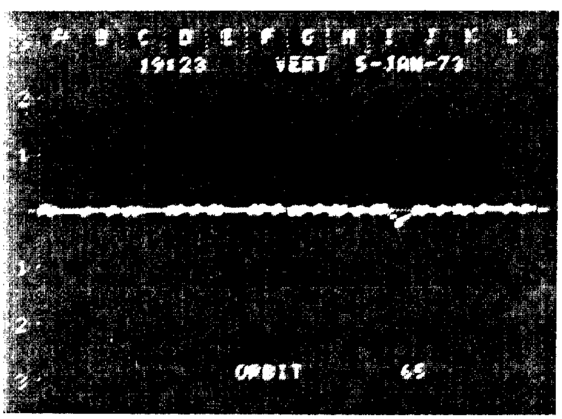

Fig. 3. Corrected injection field vertical orbit.

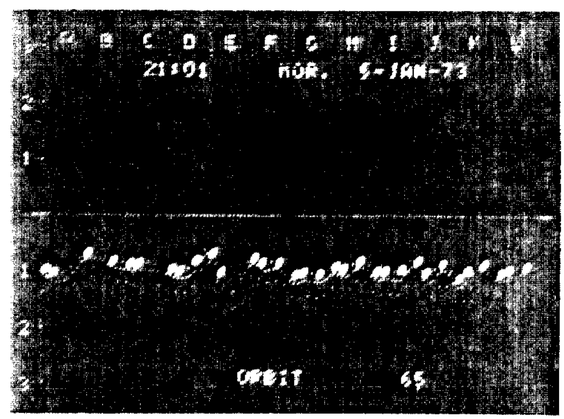

Fig. 5. Corrected injection field horizontal orbit.