NEUTRALIZATION OF BUNCHED PROTON BEAMS

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Summary

Electrons created by ionization of the residual gas can have initial velocities such that they survive several bunches and experience periodic focusing forces. Applying the AG synchrotron formalism to them shows the existence of regions of stable motion. In the frame of a linear theory one can expect partial neutralization for machines with only a few bunches above transition.

1. Introduction

Recently some attention has been paid to electron-proton instabilities of a coasting beam,[1,2] where threshold neutralizations were found to be rather low -- sometimes of the order of a few per mil.

This suggests the idea that even a small fraction of electrons could be the origin of some instabilities of bunched beams which are not yet satisfactorily explained.

2. Basic concept

Depending on its initial conditions, an electron created by ionization of the residual gas may survive the passage of several proton bunches, i.e. it experiences a periodic focusing force and, if there are more such electrons, it sees also their defocusing space-charge force. Roughly speaking, the electrons move in an F-D structure and one can apply the well-established formalism of AG synchrotrons to the electrons. Hence we can ascribe quantities like transfer matrices, a β function and an emittance to the electron "beam".

We assume one-dimensional motion of the electrons (parallel to the magnetic field) and restrict ourselves to a linear approach. We also neglect any envelope oscillation of the electron beam for the moment. Then it is easy to find the cos μ [3] for a square bunch, which is given for vanishing fractional neutralization η by

$$\cos\mu = \cos\alpha\sqrt{B} - \frac{1-B}{2B}\,\alpha\sqrt{B}\,\sin\alpha\sqrt{B}\ , \qquad (1)$$

where $|\cos\mu| < 1$ assures stability.

The parameters α and B characterize any particular machine and are given by

$$\alpha^2 = \frac{4\pi}{(h\beta)^2}\,\frac{2NRr_e}{(a+b)b} = \frac{\bar{K}}{T^2} \qquad (2)$$

where

$B < 1$ is the bunching factor,

R, r_e, a, b, N, h are, as usual, machine, classical electron and beam radii, total number of protons, and RF harmonic number, respectively,

$\bar{K}$ is the averaged focusing force per unit mass,

T is the length of the period (revolution time/h).

One solution of $\cos\mu = \pm 1$ is $\alpha\sqrt{B} = n\pi$, n = 0, 1, 2, ..., which suggests the existence of hyperbola-like bands of stability in the α versus B plane. This is illustrated in Fig. 1. The stable bands are computed for a parabolic bunch shape, and B is here understood as being the bunch length at the base over the period T.

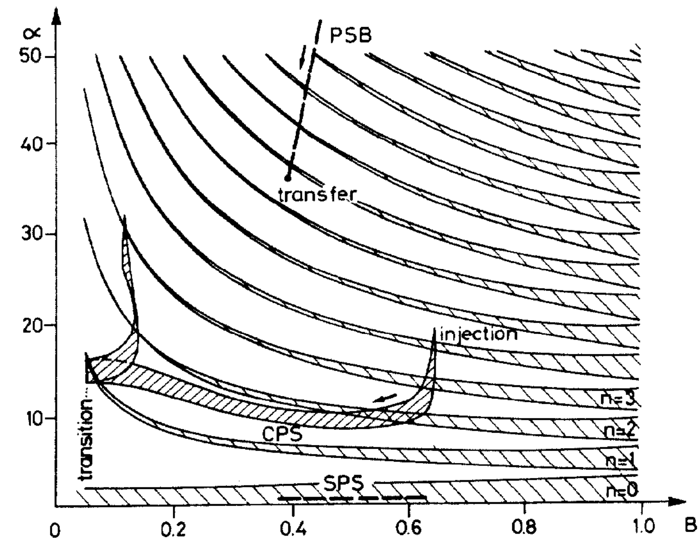

Fig. 1 Stability chart for vanishing neutralization with traces of CPS cycle and (estimated) PSB and SPS cycles.

3. Physical picture

Calculating and plotting the β functions of the electrons for bands n = 1, 2 illustrates what the stable electrons do. In Fig. 2 we also draw typical electron trajectories, at lower and upper band edges. Stable bands occur when the electrons undergo a nearly integral or half-integral number of oscillations during the passage of the bunch.

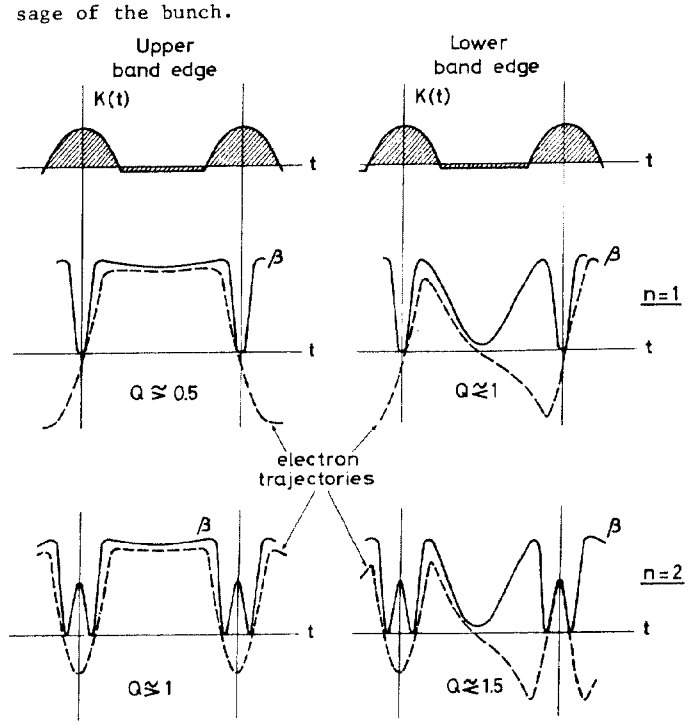

Fig. 2 Illustration of β function and electron trajectories at the edges of bands n = 1, 2.

For non-vanishing neutralization η, the bands of stability are shifted towards higher values of α. This is reasonable, because increasing neutralization weakens the average focusing force, which has to be raised accordingly to cause the same number of oscillations per period. This is demonstrated in Fig. 3a, which shows an η versus α plot of the $n = 2$ band, which is the most relevant one for the CPS.

4. How neutralization might build up

When tracing the cycle of a given machine, i.e. following a curve in the α versus B plane (as in Fig. 1), one may plot the occurring domains of stability. Figure 4 shows this for the CPS.

When starting from the interior of a band, neutralization increases with time by an effective production rate $\dot{\eta}_{eff}$, until it approaches an upper edge. There the β function and the envelope increase and some electrons are lost to the walls. Neutralization is reduced and the working point is carried back towards the band centre; we have a stable equilibrium.

We conclude from these considerations that only a band with positive slope $d\eta/dt$ allows a neutralization build-up starting from zero, provided that the slope $\dot{\eta}_{eff}$ of the production rate is steep enough.

This positive slope occurs only if during the cycle one crosses a band of stability $\alpha\sqrt{B} \simeq n\pi$ in the sense of increasing $\alpha\sqrt{B}$.

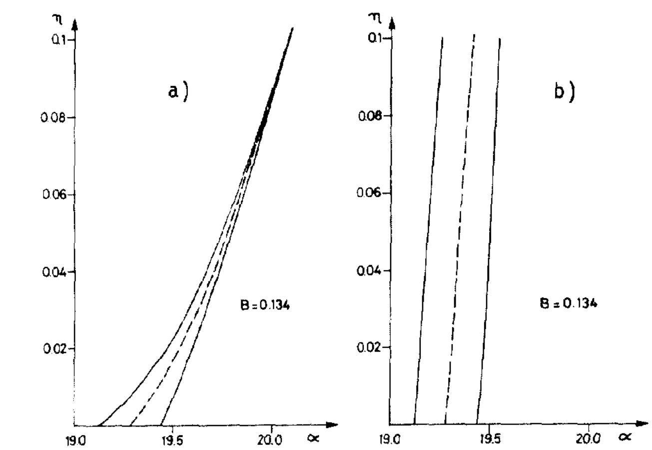

Fig. 3 a) $n = 2$ band for $B = 0.134$ and neglected envelope oscillation; electron and proton beams have the same dimensions.

b) same band, envelope motion included.

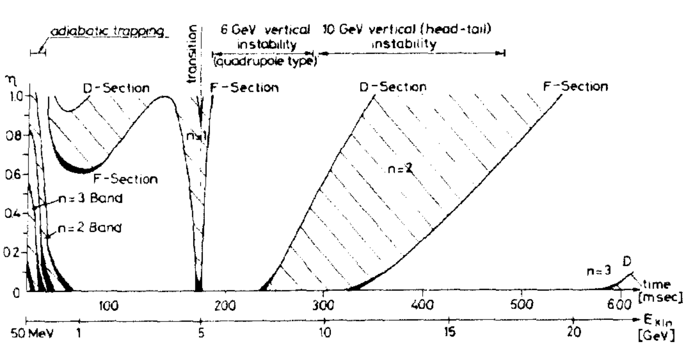

Fig. 4 Bands of stability in the course of CPS cycle for $N = 1.5 \times 10^{12}$. The hatched areas are covered by stable bands, of which the extreme ones are heavily shaded.

Since

$$\alpha^2 \sim T^2/[b(a + b)] \sim 1/(\gamma\beta^3)$$

and

$$B \sim \left[\left|\frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma^2}\right| \Big/ \gamma\right]^{1/4} = (\eta_s/\gamma)^{1/4},$$

$$\alpha^2 B \sim (\eta_s/\gamma)^{1/4}/(\gamma\beta^3) \qquad (3)$$

and the positive slope is connected to an increase of the quantity $\eta_s = |(1/\gamma_{tr}^2) - (1/\gamma^2)|$, which occurs only between transition and $\gamma_{tr}\sqrt{3}$. This is only a rough estimate and, as can be seen from Fig. 4, one should rather state: neutralization build-up starting from zero is only feasible above transition. The PSB is an example of a machine where one should not expect neutralization.

If there is some neutralization previously established (say, trapped during injection), it may persist for some time even below transition, steadily decreasing to zero in the course of time.

5. What neutralization can be expected

One obvious criterion is that the slope $d\eta/dt$ of the bands of stability must not be steeper than the effective production rate $\dot{\eta}_{eff}$. The latter is given by the ratio of the electron beam acceptance to the emittance of the production. Some simple considerations suggest that about 70% of the total production consists of electrons of transverse energy less than the ionization energy of the residual gas. We assume 10-20 eV for the ionization energy, corresponding to initial velocities $v_i \simeq 1.9 - 2.7 \times 10^6$ m sec^{-1}. A guess for the emittance of the production is then

$$E_0 \simeq 2b\, v_i \;. \qquad (4)$$

With 2b = 1.1 cm (CPS at 10 GeV), this gives $E_0 = 2.1 - 3 \times 10^4$ m^2 sec^{-1}. We will take $E_0 = 2.5 \times 10^4$ m^2 sec^{-1}. On the other hand, the emittance of the electron beam is given by

$$\frac{E}{\pi} = \frac{H^2}{\beta_{max}} \,, \qquad (5)$$

2H being the chamber height.

Figure 5 shows the electron beam ellipse and its intersection with the area of production. We call this intersection the instantaneous acceptance (of the electron beam with respect to the production), which is given by

$$A = \begin{cases} 4b\sqrt{\dfrac{E}{\pi\beta}} = \dfrac{4bH}{\sqrt{\beta\beta_{max}}} & \text{for } \sqrt{\dfrac{E}{\pi}\beta} > b \\ E & \text{for } \sqrt{\dfrac{E}{\pi}\beta} \le b \;. \end{cases} \qquad (6)$$

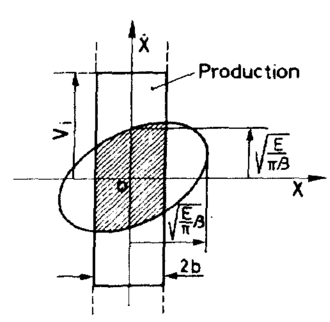

Fig. 5 Beam ellipse and production area.

The product of instantaneous production and of instantaneous acceptance has to be averaged over the period

$$\dot{\eta}_{eff} = 0.7\,\dot{\eta}\,\frac{1}{T}\int_0^T dt\,\frac{K(t)}{\bar{K}}\,\frac{A(t)}{E_0}$$

$$\simeq \dot{\eta}\,\frac{2.8\ H\ b}{E_0\bar{K}T\sqrt{\beta_{max}}}\int_0^T dt\,\frac{K(t)}{\sqrt{\beta(t)}}\ . \qquad (7)$$

We replace the average over $K/\sqrt{\beta}$ by a form factor F and obtain

$$\dot{\eta}_{eff} = \dot{\eta}\,\frac{2.8\ H\ b}{E_0\beta_{max}}\,F\ . \qquad (8)$$

The β function and F were calculated for the centre of the n = 2 band and CPS parameters around 10 GeV. We list some values for different neutralizations in Table 1.

Table 1

η	0	0.02	0.04	0.06	0.1
F	1.45	1.9	1.77	1.57	1.54
β_{max} [nsec]	89.9	202	416	742	2.06×10^3
$\dot{\eta}_{eff}/\dot{\eta}$	0.35	0.2	0.092	0.046	0.016
$\dot{\eta}_{eff}$ [$msec^{-1}$]	0.182	0.106	0.048	0.024	8.5×10^{-3}

The total production rate $\dot{\eta}$ was taken from another work[4] assuming an energy of 10 GeV and a pressure of 5×10^{-7} Torr, which gives a value $\dot{\eta} = 525\ sec^{-1} = 0.525\ msec^{-1}$. Comparing the values of $\dot{\eta}_{eff}$ with the slopes of the band n = 2 in Fig. 4 we see that this criterion allows neutralizations up to 0.12.

There is another criterion which restricts the neutralization even if α and B are kept constant with time: the statistical fluctuations in production and loss rate cause a fluctuation of the fractional neutralization. Obviously these fluctuations must be small as compared with the bandwidth $\Delta\eta$ under consideration. This criterion was not investigated further because the question of bandwidth cannot be properly answered within the framework of a linear theory. The following two sections will show that we are already beyond this frame.

6. Inclusion of envelope motion

We used a computer program to investigate (by an iterative approach) the influence of the envelope motion on the stable areas.

The behaviour of the β function was qualitatively shown in Fig. 2. We recognize there that near the upper band edge the envelope is close to its maximum value almost all the time, i.e. the beam extends vertically to the walls. This indicates that the majority of the electrons spend their time mainly out of the proton beam. Here evidently the linear theory is inadequate.

But it certainly remains true that this results in a weaker defocusing force, as compared with the (simplified) electron beam of the dimensions of the proton beam for the same number of electrons. A comparison of Fig. 3a and 3b shows the change in slope $d\eta/d\alpha$ due to that fact. One observes also a widening of the band. This can be explained by the waist of the β function at the lower band edge which results in an increased defocusing force and accordingly larger values of α.

7. Bunch-to-bunch fluctuations

In an accelerator of h machine periods, field errors create h commonly narrow stopbands. The fluctuations in population of h bunches cause a split-up of the ideal stable bands of the electrons into h narrow stable sub-bands separated by wide regions of instability. The width of the whole array of bands is several times wider than the unperturbed band. To illustrate the narrowing of the new sub-bands, we list in Table 2 the bandwidth $\Delta\eta$ of the (widest) sub-bands of the n = 2 band for typical CPS parameters, i.e. α = 19.6, B = 0.134, where the undisturbed band extends from η = 0.018-0.03, which gives a width $\Delta\eta = 1.2 \times 10^{-2}$. We do this for three different variances of the Gaussian distribution in particles per bunch.

Table 2

Variance (%)	Number of different bunches: 1 (ideal)	3	5	20
1	1.2×10^{-2}	7.7×10^{-4}	2×10^{-4}	2.2×10^{-5}
2.5	1.2×10^{-2}	1.6×10^{-4}	1×10^{-5}	1.4×10^{-10}
5	1.2×10^{-2}	4.9×10^{-5}	1.4×10^{-6}	3×10^{-13}

The maximum differences in bunch populations to be expected are about four times the variances. The table shows that for a machine like the CPS (20 bunches) bunch-to-bunch fluctuation of a variance of 1% is the upper limit where some neutralization is imaginable. Generally the fluctuations are believed to be larger and one can conclude that the linear theory allows significant neutralization for machines with only a few bunches.

There remains the puzzling coincidence of the occurrence of stable bands and of the "head-tail" instability in the CPS (see Fig. 4) and the question as to whether a proper non-linear theory will show a different stability behaviour of the electrons. There are many examples in mechanics of stabilization by nonlinearities.

An entirely different situation can occur, if the harmonic number increases by orders of magnitude as in the case of the SPS (h = 4620). Then the short period T makes α so small that the working points fall into the band n = 0 (Fig. 1). In this band the focusing properties are like those in a common accelerator and we have wide stable regions and narrow stopbands. The possibility of neutralization of the SPS beam has already been treated elsewhere.[5]

8. Acknowledgements

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References

1. D. Koshkarev and P. Zenkevich, Particle Acc. 3, 1 (1972).
2. E. Keil and B. Zotter, CERN-ISR-TH/71-58.
3. E.D. Courant and H.S. Snyder, Ann. Phys. 3, 1 (1958).
4. W.H. De Luca, IEEE Trans. Nuclear Sci. NS-16, No. 3, 813 (1969).
5. H. Schönauer and B. Zotter, Spring Study on Acc. Theory AMC-1 (April 1972), p. 135.