HIGH-INTENSITY EFFECTS IN THE LONGITUDINAL MOTION OF STORED PARTICLE BEAMS*

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Summary. A brief review is given of the various self-field phenomena associated with the longitudinal motion of particles in storage rings.

Introduction

Although there are some high-intensity phenomena for which the coupling of longitudinal and transverse motion is essential, such as, for example, the head-tail effect; the great majority of high-intensity phenomena primarily involve either longitudinal or transverse degrees of freedom. In this review, we restrict our attention to phenomena which are essentially longitudinal in nature.

It is convenient to consider separately the behavior of unbunched (coasting) and bunched (external RF system in operation) beams. Detailed experimental information on coasting beams has been obtained on the ISR, on the (old) CERN electron model CESAR, and on electron ring accelerators. All high-energy electron storage rings have bunched beams and, of course, so do synchrotrons, so that there are a large number of sources of experimental information about the longitudinal motion of bunched beams.

Unbunched Beams

The primary phenomenon, due to self-field effects, in the longitudinal motion of a coasting beam is spontaneous formation of longitudinal density variations. For a storage ring operating above the transition energy (so that df/dE is negative) the effect is physically very simple (the "negative mass instability"), and both growth rates in the linear regime and thresholds were theoretically derived[1] prior to the phenomenon being observed on a variety of machines.[2]

Although the original negative mass theory considered bunching arising from self-forces associated with a beam in a smooth, perfectly conducting, and essentially straight vacuum chamber, it was soon realized that the beam-surroundings played an essential role in the instability. In particular, unexcited RF cavities,[3] resistive walls,[4] and ceramic chambers[5] were shown to be able to greatly enhance the phenomenon (even to cause it below transition where the mass is positive), and hence the beam environment became of considerable concern to builders of storage rings.

It is convenient to introduce a longitudinal coupling impedance Z_n defined by[6]

$$Z_n = - \frac{2\pi R E_n}{I_n}, \tag{1}$$

where E_n is the $\underline{n}$th harmonic of the azimuthal electric field at the beam, I_n is the $\underline{n}$th harmonic of the beam current and R is the orbit radius. An approximate criterion for stability of a coasting beam against self-bunching at the frequency $n\,\omega_{rev}$ is[1,4,7]

$$\frac{|Z_n|}{n} \leqslant \frac{\eta\, \gamma\, U_0}{I_0} \left(\frac{\Delta E}{E}\right)^2, \tag{2}$$

where $\omega_{rev} = \beta c/R$ is the particle revolution frequency,

$$\eta = \left| \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} \right| ,$$

with γ_t the transition energy in units of m_0c^2 and γ the particle energy in units of m_0c^2 so that $E = \gamma\, m_0c^2$, $U_0 = m_0c^2/e = 0.511$ MV for electrons and 938 MV for protons, $I_0 = Ne\, f_{rev}$ is the circulating beam current, and $\Delta E/E$ is the full width of the beam energy distribution at half maximum. This stability criterion arises from the balance between the instability-driving forces characterized by Z_n and the Landau damping associated with energy (and hence frequency) spread.

At the present time a considerable body of knowledge exists concerning the coupling impedance Z_n. Theoretical calculations have been performed by a large number of workers for a variety of structures (see references cited in Ref. 6.), and methods have even been devised for measuring Z_n.[8]

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Recently, there have been detailed experimental studies of the onset of azimuthal instabilities.[9,10] The work on electron ring accelerators so-far provides only qualitative accord with the criterion (2), but the observations on the ISR are in remarkably good quantitative agreement with (2). Spontaneous beam bunching due to interaction of a coasting beam with an unexcited RF cavity has been observed on the AGS, and avoided either by increasing the beam energy spread or by de-tuning; i.e., reducing the cavity contribution to Z_n.[11]

The experimental observations are particularly interesting because the phenomenon of beam bunching can be observed in the nonlinear regime where the theory, presently, is far from complete.[12] Generally, it is observed[2] that the instability is self-stabilizing; i.e., a certain degree of bunching occurs, and, subsequently the beam again becomes longitudinally uniform (but with a larger energy spread than it had initially). Because this process leads to beam widening it is generally bad as it results in particle loss if there are aperture stops, and in the case of the electron ring accelerator to degraded ring quality.

Finally, it should be noted that recently observations have been made on electron rings of RF signals without noticeable beam degradation,[9] which, perhaps, is related to a prior theoretical calculation on mini-instabilities.[13]

Bunched Beams

In a stored bunched beam there are two classes of high intensity effects; namely coherent bunch motion, and alterations in bunch size and shape.

Coherent Bunch Motion

The simplest coherent mode of a single bunch is the dipole mode ($m = 1$); i.e. rigid-bunch motion. Most of the analysis in the literature is confined to this mode, although some authors have considered higher order modes ($m > 1$).[14,15] For the simple case in which the phase of the bunch center ϕ_i for bunch i, may be treated in linear approximation and in which there is no external feedback we have[11,16,17,18]

$$\ddot{\phi}_i + \omega_s^2 \phi_i = \sum_{j=1}^{B} A_{ij} \phi_j , \qquad (3)$$

where B is the number of bunches, ω_s is the frequency of small phase oscillations, and the coefficients A_{ij} are given by

$$A_{ij} = - \frac{\omega_s^2}{V \cos \phi_s} \frac{\partial V_{ij}}{\partial \phi_j} . \qquad (4)$$

In Formula (4), V is the voltage gain per turn, ϕ_s is the stable phase angle, and V_{ij} is the voltage on bunch i caused by bunch j. Clearly V_{ij} can be expressed in terms of a coupling impedance.[17]

If all the bunches are equal then the A_{ij} are only functions of $i-j$ and the B normal modes are simply the B-roots of unity:[11,17,19]

$$\phi_1 = e^{i\omega_n^{(1)} t}$$
$$\phi_j = \phi_1 \, e^{in(j-1)2\pi/B}, \qquad (5)$$

where $n = 1, \cdots B$ characterizes the mode and

$$\omega_n^{(1)} \approx \omega_s - \frac{1}{2\omega_s} \sum_{j=1}^{B} A_{j1} \, e^{in(j-1)2\pi/B}. \qquad (6)$$

Clearly if $\mathrm{Im}\, \omega_n^{(1)} < 0$ the nth mode is unstable. If the nonlinear nature of the synchrotron motion is included, then a dispersion analysis shows[14,15,18] that the nth mode will be stable, even if $\mathrm{Im}\, \omega_n^{(1)} < 0$, provided

$$|\omega_n^{(1)} - \omega_s| < S/4 \qquad (7)$$

where S is the full-spread in the synchrotron oscillation frequency of particles in a bunch.

The analysis for unequal bunches has resulted in a condition for decoupling of the bunches; namely that the spread in individual bunch frequencies must be greater than the shift in frequency due to coupling. Stability requires, in addition, that a condition analogous to (7) be satisfied;[15] namely that for the mth order mode, having frequency $\omega^{(m)}$,

$$|\omega^{(m)} - m\omega_s| < \frac{\sqrt{m}}{4} S . \qquad (8)$$

Also, in the literature, are numerical studies of unequal bunches and analysis of the influence of beam control systems and active feed-back damping.[17]

Observations have been made on the CERN PS,[17] the AGS,[11] and the CEA,[20] as well as on the storage rings in Novosibirsk and Frascati. In general there is good agreement between the observations and theory, although in some cases the actual values of the A_{ij} have been larger than was a priori expected.

Bunch Size and Shape

It was first observed, at Orsay and Frascati, that the length of bunches in the storage rings was a function of the stored current. No such effect was observed at Stanford or Novosibirsk.[21] Recent observations at CEA,[20] SLAC,[22] and Frascati[19] have indicated that both bunch length and width increases with increasing stored current. The parametric dependence of bunch length, Δ, on beam current, I, beam energy, E, and RF voltage, V, is (approximately) for bunch length large compared to the natural (low-current) bunch length, Δ_0, (as has been summarized in Ref. 21):

$$\Delta(\mathrm{ns}) = \frac{0.46\, I(\mathrm{mA})^{1/3}}{E(\mathrm{GeV})^{7/6}} \left[\frac{30}{V(\mathrm{keV})}\right]^{1/6} \Delta_0(\mathrm{ns})^{2/3} . \quad (9)$$

Prior to the observation of bunch widening, a general equilibrium theory of bunch length was developed.[23] This theory included the effect of coherent synchrotron radiation as well as the electrical interaction of a beam with various resonant structures that might be present in a storage ring. The parametric dependence of bunch length was found to be in good agreement with (9), and numerical estimates--based upon reasonable guesses for the characteristics of resonant structures--were in substantial agreement with (9). This theory, however, predicted no bunch widening which isn't necessarily a defect of the theory since the two phenomena may be unrelated.

An alternative, and older, theory suggested that the instability of internal coherent synchrotron oscillations could explain bunch lengthening.[24] However, the parametric dependence deduced in this work is not in good accord with observations, although the theory does predict both bunch lengthening and widening.

In some work which is still in progress, the present writer has, in the spirit of Ref. 24, developed a theory of the behavior of a beam whose natural energy width--resulting from the balance between quantum fluctuations and classical radiation damping --is less than that required for the stability of collective modes. It is shown that in this circumstance there is a turbulent equilibrium state in which the wave diffusion (due to stable collective modes with finite amplitudes) augments the quantum diffusion so as to yield a bunch of increased width and length. The theory, perhaps when combined with the equilibrium theory of Ref. 23, appears capable of explaining the observations, but it is too early to be sure that the proposed explanation is indeed correct.

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DISCUSSION

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B. Zotter: I would like to make a statement about a nonlinear effect we see in the blow-up of the bunches. The blow-up seems to fulfill a criterion given ten years ago by Dory, which is that the product of the final and initial momentum spreads equals the square of the critical momentum spread necessary for stability. In addition, there is the related phenomenon which is a sort of hysteresis, that if you turn up the coupling impedance of the cavity, then we get instability, and when we turn down the coupling impedance it disappears at a much lower value of coupling impedance.

Sessler: Pellegrini and I have been working on this problem and have a heuristic theory of nonlinear phenomena, and in this theory we did not get the Dory formula. We got a different formula, and it would be interesting for me to see the data, because in this theory what was involved was how many modes are unstable. Suppose we made a number of modes unstable by a cavity. Then, in this theory, one has the energy distributed over the unstable modes. The final energy spread arrived at was a function of over how many modes the initial energy is spread.

Zotter: We think that we have a single unstable mode. At least we tried to kill all higher modes in the cavity by damping.

Sessler: And in that case the final energy spread is greater than that you expect for threshold?

Zotter: Yes.

J. R. Rees (SLAC): This morning we heard Frank Sacherer talk about stability of bunched beams, and I was struck by the fact that the frequency spread required to stabilize bunched-beam instabilities varies as the square root of the mode number, while for continuous beams the required spread varies linearly Could someone explain physically why that is?

F. J. Sacherer (CERN): The criterion can be written as

$$S > \frac{1}{\sqrt{m}} \left| \Delta W_m \right|$$

but remember that the frequency shift ΔW_m also depends on m (m = 1) for dipole modes, m = 2 for quadrupole modes, etc.). For space charge $\Delta W_m \sim \sqrt{m}$ and the required spread is the same for all modes, just as for coasting beams. For cavities $\Delta W_m \sim 1/\sqrt{m}$ and $S \sim 1/m$, which again is analogous to the coasting beam case. The difference between coasting beams and bunched beams is not the dependence on mode number, but that frequency spread rather than energy spread is important, and that ΔW_m depends on bunched-beam parameters like bunching factor, RF voltage, etc.

Sessler: Let me remark that there are two different types of modes. Let us talk about just the dipole mode with B bunches; then there are B modes in which the particles move in rigid-bunch motion with different phase shifts between bunches. It is this mode which goes over into the continuous-beam case mode of order n and Z_n/n comes into the stability criterion. There is a different type of motion where you have quadrupole motion, etc., within one bunch, and I don't know how this goes over into the continuous-beam case. So you see these two types of modes are orthogonal and in one case you have the $\sqrt{n}$ in the criteria and in the other case you still get the Z_n/n.