THE 'HEAD-TAIL' INSTABILITY*

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Introduction

In the ACO and ADONE electron-positron storage rings, a transverse instability was observed whose characteristics could not be explained by the 'classical' multiturn resistive-wall mechanism. This instability was independent of the betatron frequency and the threshold was a function of only the local current (i.e., number of particles in a bunch) not on the total circulating current. This prompted the development of a new theory, the head-tail theory, [1, 2, 3, 4] in which the head of a bunch drives the tail of the same bunch through some wake field, which need not be much longer than the bunch itself.

Over the last several years the head-tail mechanism has been used successfully in explaining many of the characteristics of transverse instabilities observed both in storage rings and synchrotrons. There are, however, several important discrepancies between theory and observation to which we should direct our attention.

The 'Head-Tail' Mechanism

The particles in a bunch trapped inside an RF bucket execute both synchrotron and betatron oscillations. A particle which leaves behind a wake field proportional to its instantaneous transverse displacement can increase (or decrease) the amplitude of a particle following behind, provided that there exists the appropriate phase relationship between the particle's betatron oscillations. During one synchrotron oscillation the particles will exchange positions, and if the phase difference in betatron motion is maintained correctly the particles will drive one another to increasing amplitudes. A simple mathematical model of this instability mechanism is given by Sands[1] and a more complete model by Sands[2] and Pellegrini.[3,4]

With the assumption that the 'wake' field from the head of the bunch changes only slowly on the time scale of the bunch length one finds that the particle amplitudes increase exponentially with a growth rate given by

$$\beta_n = \frac{NS}{\pi^2 c \gamma m_e} \frac{\xi}{\alpha} \frac{\ell}{(4n^2 - 1)}$$

where
N = number of particles per bunch,
S = strength of wake field,
ξ = $\partial\nu/\nu \;\; p/\partial p$ - chromaticity of the lattice,
α = the momentum compaction of the lattice,
ℓ = the bunch length,
n = the mode number.

In the n=0 mode the particles in a bunch move in phase, i.e., there is an oscillation of the center-of-mass of the bunch at the betatron frequency. In higher modes, n>0, the particles have a more complex phase relationship and we have bunch-shape oscillations. The n=0 mode is unstable, i.e., $\beta_0 > 0$, when the chromaticity is negative, and is stable when the chromaticity is positive. The dependence of growth rate on the sign of the chromaticity is reversed for the higher modes and the growth rate decreases rapidly for higher mode numbers. The effect of Landau damping has been investigated by Pellegrini[4] and Muelhaupt[5] and it is shown that although the relationship between the instability threshold and the magnitude of the chromaticity can be changed, the essential dependence on the sign of the chromaticity is unaffected.

Experimental Results

The transverse instabilities observed in the e^+e^- storage rings — ACO,[6] ADONE,[7] CEA,[8] and SPEAR[9] — have characteristics consistent with the above-described mechanism, when the chromaticity of the lattice is negative. In ADONE a feedback system is used to stabilize the n=0 mode, while in ACO, CEA, and SPEAR this mode disappears when the $\xi > 0$. In these rings where the chromaticity is a controllable parameter no higher modes have been observed when the chromaticity is positive up to more than one hundred times the local current density at which the n=0 mode appears with the $\xi < 0$.

The dependence of the threshold of the n=0 mode on the octupole content of the lattice, which controls the Landau damping, was studied at the CEA[8] and found to be in good agreement with the theory. The head-tail theory has also been applied to instabilities observed in the CERN PS and the NAL booster synchrotron.[10] In a proton synchrotron the dependence of the growth rate of the n=0 mode on ξ is reversed when the protons are below transition energy, and this is consistent with the observations. Zotter[11] has applied the head-tail theory to the ISR where in contrast to electron rings, there is a very large phase shift in betatron oscillations over the length of a bunch. (The ξ is positive and the $\Delta p/p$ is large.) He found that in this case the lowest modes n=0,1,2,3, were all stable. The behavior of the observed instability of the bunched beam in the ISR is not consistent with the theory and perhaps we can conclude that in this machine another mechanism is involved.

In the electron rings one finds that the threshold of the vertical and horizontal modes of the instability are very similar, in the absence of trapped ions, and this is difficult to reconcile with wake fields generated by the walls of the vacuum system or by plate structures within the system. In general, it has proved difficult to predict the thresholds of these instabilities from wake fields generated in known structures in the respective machines; the calculations underestimate the growth rates.

Conclusions

The 'head-tail' theory gives a good qualitative description of observed instabilities when the chromaticity is negative (above transition energy), but predicts far too low thresholds for higher modes with positive chromaticity.

It should be noted that notwithstanding the large differences in the beam environment in the existing electron storage rings, the instability thresholds measured in terms of the linear density (i.e., particles/cm) are quite similar when one scales to a common energy. This may be accidental but it re-emphasizes the fact that the greatest difficulty with the 'head-tail' theory lies in our lack of understanding of the source of the fields which drive the instability.

References

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