BEAM DYNAMICS IN THE PRESENCE OF IONS AND ELECTRONS

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Almost 10 years ago coupled dipole oscillations of oppositely charged particles have been observed in the DCX fusion device [1]. Several Russian authors have described theoretical aspects of the same phenomenon [2]. Recently, interest in the subject has been revived because of its possible implications on electron-ring accelerators [3]. Experimental observations at the Bevatron and the CERN-ISR have been explained in terms of electron-proton interactions [4].

In this short introduction I shall limit myself to this one aspect, which has become known as "e-p" instability in proton accelerator jargon - although there are usually heavier ions involved in electron machines. In order to get a good physical understanding, I shall strip the actually complex situation to its bare essentials : consider a beam of charged particles moving in an external focussing field in the presence of stationary particles of opposite charge. We can disregard the effects of the surrounding structure (image forces) as well as "species-species" forces, which are not directly involved in driving the instability. The transverse motion is the governed only by two forces :

1) external focussing (only on the moving species) proportional to the displacement of a particle;
2) interaction with the other species, proportional to the difference between the displacement of a single particle and centre of charge of the other species.

The equations of motion for the two species then will have the form :

$$\left.\begin{aligned} \ddot{z}_1 + \omega_o^2 z_1 + \omega_1^2 (z_1 - \bar{z}_2) &= 0 \\ \ddot{z}_2 \qquad\qquad + \omega_2^2 (z_2 - \bar{z}_1) &= 0 \end{aligned}\right\} \qquad (1)$$

where ω_o is the betatron frequency, and $\omega_{1,2}$ the frequencies of oscillation of one species in the potential well of the other.

For an exponential time (and azimuth) dependence, and assuming that all particles of one species have the same oscillation frequencies, we get the dispersion relation

$$\left(\omega^2_o + \omega^2_1 - (\omega - n\Omega)^2\right)(\omega^2_2 - \omega^2) = \omega_1^2\,\omega_2^2 \qquad (2)$$

where n is the azimuthal mode number. This quartic determines the oscillation frequency ω, and has four real solutions when the RHS is sufficiently small. For larger values, two of the solutions become complex conjugate and hence one of them corresponds to a growing wave. The threshold can be expressed in terms of ω_1, or in terms of the neutralization. It is lowest for mode numbers near to $n \simeq \omega_2/\Omega$, and we thus expect to have a number of unstable modes around this value.

The amplitude ratio for the two species can become quite large - the electrons have several hundred times bigger amplitudes than the protons in the ISR. The electrons will thus reach the chamber wall and get lost, while the proton beam grows only little. Afterwards the neutralization will build up again by rest-gas ionization until the threshold is reached and the process repeats itself. If that process takes long enough, the proton oscillations will have smeared out, and their amplitudes add only quadratically. An estimate for the ISR yields reasonable agreement with observed loss rates [5]. Furthermore, the observed periodicity of the electron lines was well correlated with background spikes in the intersections. Installation of additional clearing electrodes brought the total neutralization below 1% and eliminated these problems.

The theory can be improved considerably by taking into account the finite spread in oscillation frequencies by distribution functions. In the slow wave approximation, and neglecting ω_1 compared to ω_o, we find the dispersion relation [6]

$$\int \frac{f_o\ (\omega_o)d\omega_o}{\omega_o-(\omega - n\Omega)} \int \frac{f_2\ (\omega_2)d\omega_2}{\omega_2 - \omega} = \frac{n\bar{\omega}_o\Omega^2}{\bar{\omega}_1^2\ \bar{\omega}_2} \qquad (3)$$

Curiously enough, for distribution functions of increasing width we first find a decrease of the threshold ("anti-Landau damping") followed by the expected increase when the two distributions overlap. In this regime we get the stability criterion

$$\frac{\bar{\omega}_1}{\bar{\omega}_o} \leqslant C\sqrt{\frac{\Delta_o}{\bar{\omega}_o} \cdot \frac{\Delta_2}{\bar{\omega}_2}} \qquad (4)$$

where C is a constant of order unity, depending on the specific distribution functions used. Inclusion of both image and species-species forces [7] yields a similar result, with adjusted definitions of $\omega_{1,2}$ and the spreads reduced by image terms.

The relative spread in betatron frequency is usually known, but the spread in ω_2 is not so easily found. In a strong focussing machine it may be simply due to varying beam dimensions around the ring, which will act as a spread if the instability is not too fast. In the ISR, this mechanism provides a relative spread in electron frequencies of about 20%.

A more direct source of spread is the amplitude dependence of the oscillation frequency. A simple nonlinearity has been considered recently [8], and may lead to a limitation of the amplitudes.

In conclusion, I should like to emphasize that agreement between theory and experiments of the e-p instability is quite satisfactory at the Bevatron but at the ISR it is at best sporadic. The presence of "electron lines" could usually be accounted for, but not their occasional absence under seemingly similar conditions. While the single frequencies seen were usually in rough agreement with theory, the number and spacing of lines were not. Some of the effects explained also by completely different mechanisms (e.g., Arnold diffusion) also dependent on neutralization. I think this is a good point to open the discussion.

References

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