MEASUREMENTS ON BEAM-BEAM INTERACTION AT SPEAR*

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Abstract

Measurements of luminosity at SPEAR, which were performed at different operational conditions of the storage ring are shown and discussed. The parameters varied are: the current in both beams, the minimum betatron amplitude functions at the collision point, the energy and the vertical betatron frequencies. As a result of these measurements we found that the maximum achievable luminosity is much higher than predicted by the incoherent beam-beam limit using $\delta\nu = 0.025$. We also found that the maximum achievable luminosity is a strong function of the betatron frequencies. After computing the largest linear tuneshifts, we found that the quarter resonance seems to be the limiting effect for beam-beam collision.

I. Introduction

The goal of studying the luminosity at SPEAR under different operational conditions of the storage ring was to discover agreements and disagreements between measurement and theory. We expect the best agreement to occur at very low currents where the beam-beam effect is negligible. We made measurements at low currents to check the luminosity dependence on the energy and on the betatron function at the collision point. We also used these measurements to compute the natural beam cross section, and, from that, the coupling constant in the storage ring. All measurements have been done without any artificial beam enlargement.

II. Energy Dependence of Low Current Luminosities

Luminosities have been measured as a function of energy for a low current, where no beam-beam effect could be observed (Fig. 1). Before we check the energy dependence of the measured luminosity L_m we should take account of the finite bunch length. The well known theoretical formulas[1] assume a short bunch length compared to the minimum betatron function β_{y0}. This is not the case for SPEAR when we operate at a $\beta_{y0} = 10$ cm. Due to the variations of the betatron function along the collision length we get a degradation in luminosity.[2] If we adjust the measured (true) luminosity L_m for this degradation we get the idealized values L_0 in Fig. 1, which scale very well with the energy squared as they should.

The luminosities estimated before construction of SPEAR for low beam intensities are about a factor of 1.8 lower than the measured ones. An obvious explanation for this was that the coupling constant of K=0.1 used in the estimate was too high and the true value is about a factor of two lower. Therefore, the beam height or the beam cross section is a factor of two smaller and the luminosity correspondingly higher.

In order to make sure that this is the right explanation, we estimated the coupling constant in a more direct way by measuring the width of the $\nu_x - \nu_y = 0$ coupling resonance. It turned out to be $\Delta\nu_{min} = 0.004$ independent of the tune (Fig. 2) within the range of operation. The formulas for the eigenfrequencies (ν_I, ν_{II}) of a coupled ring accelerator[3] are:

$$\cos 2\pi\nu_I = \frac{1}{2}(\cos\mu_x + \cos\mu_y) + \frac{1}{2}\ (\cos\mu_x - \text{cox}\,\mu_y)$$

$$\cos 2\pi\nu_{II} = \frac{1}{2}(\cos\mu_x - \cos\mu_y) - \frac{1}{2}\ (\cos\mu_x - \cos\mu_y) \tag{1}$$

with $\mu = 2\pi\nu$, ν being the tune of the storage ring,

$$\kappa^2 = 1 + \Delta^2\,\frac{\sin\mu_x + \sin\mu_y}{\cos\mu_x - \cos\mu_y}\ , \qquad \Delta = \delta\sqrt{\beta_x\beta_y} \tag{2}$$

and $\delta = g_s\ell_s/(B\rho)$ where $g_s \cdot \ell_s$ is the integrated field gradient of an equivalent skew quadrupole which would produce the same amount of coupling. The betatron amplitudes at the skew quadrupole being β_x and $\beta_y \cdot B\rho$ is the beam rigidity. The coupling constant then is given for $\mu_x = \mu_y + \delta\mu$ by:

$$K^2 = \frac{\kappa^2 - 1}{2\kappa^2}\left[1 + \frac{1}{2}\,\delta\mu\,\frac{\cos\mu_y}{\sin\mu_y}\right] \tag{3}$$

For the limit $\delta\mu \to 0$ we find from Eq. (2) $\kappa^2 \cdot \delta\mu^2 \to \Delta^2$ and from Eq. (1),

$$2\pi\,|\nu_I - \nu_{II}| = 2\pi\,\Delta\nu_{min} = \Delta\ . \tag{4}$$

Thus, the amount of natural coupling in SPEAR is determined. We can get the coupling constant for the storage ring parameters used during the luminosity measurements from Eq. (3) and Eq. (2). We find a coupling constant of K=0.037. This is less than the value computed from the luminosity measurements, but they both are considerably smaller than the expected value of K=0.1. In fact, the agreement is rather good if we remember that the coupling constant in the presence of the distributed sextupole system of SPEAR depends sensitively on the closed orbit errors.

As a conclusion of this measurement we assume that the natural beam cross sections below the beam-beam limit correspond to the computed cross section using K=0.05 for the natural coupling in SPEAR. Also, since the bunch lengths at low currents correspond to the computed values we assume further that the storage ring behaves at low currents as theoretically predicted.

III. Variation of the Low Beta in the Collision Point

The next check of the behavior of the storage ring should be to measure the change in luminosity due to the change of the betatron function in the collision region. Neglecting bunch-length effect we would expect the luminosity to be proportional to $(\beta_{y0})^{-1/2}$ The experimental results are shown in Fig. 3. Since the finite bunch length affects the comparison for $\beta_{y0} = 10$ cm, here again we compute the idealized luminosity. For currents below any significant beam-beam effect the luminosities agree well with theoretical values.

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Knowing that two small colliding beams behave as expected, we assume deviations from theoretical values to come from the beam-beam effect. The first conclusion from Fig. 3 is that the luminosity seems not to be limited by the incoherent beam-beam limit of strength $\delta\nu = 0.025$. In fact, in this example, we observe an increase in the luminosity beyond this limit by a factor of about eight. We will see that this factor can be increased even more. There is negligible beam-beam effect up to about $\delta\nu = 0.05$. From this point on, the beam-beam interaction affects the beam cross section. The luminosity reaches a maximum value with increasing current but beyond a certain current it is impossible to get both beams into collision; one of them is destroyed.

IV. Variation of the Vertical Betatron Frequency

It has been assumed for a long time[4,5] that the higher luminosities would be obtained as the betatron frequency approaches an integer or a half integer from above. In this case the linear beam-beam effect diminishes the betatron function at the interaction point by a factor which increases as one approaches integer resonance from above. By this effect the maximum current and hence the luminosity can be increased although the maximum linear tune shift stays constant, e.g., $\delta\nu_{max} = 0.025$. This effect alone would have increased the luminosity for SPEAR at a tune of $\nu_y = 5.10$ by 54% over the estimated value using $\delta\nu_{max} \approx 0.025$.

There were other arguments[6] however, which show that in spite of the nonlinear field, from an optical point of view it should be possible to increase the maximum linear tune shift beyond the canonical value of 0.025. These arguments are as follows. We know that the electromagnetic field produced by a particle beam can be described by multipoles of the order 4n, e.g., a quadrupole, an octupole, a twelvepole, and so on.

If we look into this multipole representation of the field produced by the beam, we see that this field can be described later well by a quadrupole and an octupole field alone for amplitudes less than 2.5 to 3 units of standard deviation in the beam transverse dimension. In this amplitude range, which contains more than 90% of the particles, the octupole field describes very well the motion of the particles. For weak octupole fields or beam currents there is only little effect on the particle trajectories, since the separatrices are at large amplitudes. As the beam intensity increases the separatrices surrounding the fixed points reach smaller distances from the origin. As soon as these distances get smaller than 2.5 to 3 standard units in the beam transverse dimension a fast increasing amount of particles get captured around the outer fixed points thus reducing the beam density. If we now say the particles within 3 standard units of amplitudes should not be affected by these separatrices, this gives an upper limit for the octupole strength or beam current. It turns out that this maximum octupole strength corresponds to a beam current which gives a linear tune shift of

$$\delta\nu \approx \Delta\nu/p \qquad (5)$$

and because most storage rings are limited by the vertical beam-beam limit, $\Delta\nu$ is the distance of the vertical tune to the 1/4 resonance. p is the number of interaction points. For SPEAR, this would mean that the maximum linear tune shift for a tune of 5.10, for example, should be $\delta\nu \approx 0.075$ or three times as high as the originally assumed limit of 0.025. Since the permissible current goes up by the same factor, the luminosity should therefore go up by more than an order of magnitude.

The measurements of luminosity versus current for various vertical betatron frequencies are shown in Fig. 4. We see that the maximum luminosity achieved so far at 1.5 GeV is a factor of 25 higher than estimated with a linear tune shift of $\delta\nu = 0.025$.

At this point it should be emphasized that in general it is not possible to choose just any values of ν_x and ν_y, because for special tunes the beam cross section is affected by higher order coupling resonances and the luminosity then is lower than one would expect from Fig. 4. Thus in addition to using lower values of ν_y it is important to choose the proper value of ν_x in order to maximize the luminosity.

There is another feature of the luminosity versus current curve which is appreciated very much by the users of the storage ring. Once the storage ring is filled to the maximum current the luminosity stays essentially constant for about a whole beam lifetime of the order of two or three hours, the beam adjusting itself to smaller cross sections as it decays. In this regime the beam cross section seems to be blown up proportionally to the square of the current.

V. Linear Tune Shift

It is now interesting to compute the linear optical tune shifts for these luminosity measurements. The data reduction is somehow complicated because of the fact that the betatron amplitude changes very much along the collision length. This variation is even larger when we include the change in the beam optics due to the linear beam-beam effect.[4,5] We certainly have to do this correction in order to get the linear optical tune shift. It is shown in the Appendix how we reduced the data.

With all these corrections we get, for the linear optical tune shift as a function of beam current, the values shown in Fig. 5 and we see that the maximum achievable linear tune shift depends very strongly on the vertical tune of the storage ring and is considerably higher than 0.025. The plot of the maximum $\delta\nu$-value versus the vertical tune of the storage ring is shown in Fig. 6. We see a linear dependence of the maximum linear tune shift versus the vertical tune. The interesting feature of this straight line is the slope. It turns out that the vertical tune plus twice the maximum linear tune shift leads just to a quarter resonance:

$$\nu_y + 2 \cdot \delta\nu_{max} \approx 5.25 \qquad (6)$$

We assume that the factor of 2 is due to the fact that we have two interaction points. From this measurement we conclude that the quarter resonance plays an important role in the beam-beam limit, although we do not yet completely understand the loss mechanism.

We should now compare these results with measurements on other storage rings. At the last International Accelerator Conference in Geneva, Prof. Amman[7] gave a review about electron-positron storage ring experiences so far. This article also contains an extended reference list covering the various aspects of electron-positron storage ring behavior.

In this talk we find a linear optical tune shift for ACO of $\delta\nu \approx 0.06$ and for ADONE a value of $\delta\nu \approx 0.03$ per crossing. In both cases the relation (5) is quite well fulfilled. Another measurement reported from ADONE[3] with only 2 interaction points result in a linear tune shift of $\delta\nu = 0.04$ at a betatron frequency of 5.15 which agrees again with the relation (5).

VI. Conclusion

As a conclusion of the luminosity measurements done so far at SPEAR we can state that the low current luminosity scales with energy and with low beta as expected. The coupling constant seems to be as low as 5% although there are large beam amplitudes due to the low beta section in SPEAR. There is a strong dependence of the maximum achievable luminosity on the vertical betatron frequencies and the maximum linear optical tune shift increases with decreasing vertical betatron frequency. It turns out that the total maximum linear tune shift is equal to the distance of the vertical tune to the quarter resonance. The maximum linear tune shift found so far at 1.5 GeV in SPEAR is

$$\delta\nu \approx 0.08 \quad .$$

VII. References

1. M. Sands, Report No. SLAC-121, Stanford Linear Accelerator Center (1970).
2. G. Fischer, Technical Note SPEAR-154, Stanford Linear Accelerator Center (1973); unpublished.
3. G. Leleux, Report No. 14-64 GL-FB, Orsay (1964); unpublished.
   G. Ripken, Report No. H5-70/7, DESY (1970) or Report No. SLAC-TRANS-134; unpublished.
4. M. Bassetti, Proc. V International Conference on High Energy Accelerators, Frascati, 1965; p. 708.
5. B. Richter, Proc. International Symposium on Electron and Positron Storage Rings, Saclay, 1966; p I-1-1.
6. H. Wiedemann, Report No. H5-71/6, DESY (1971); unpublished.
7. F. Amman, Proc. of the 8th International Conference on High Energy Accelerators, CERN, 1971.
8. F. Amman et al., Nuovo Cimento Lettere I, 729 (1969).

Appendix

In SPEAR the cross section of the beams varies along the collision length. Therefore we have to correct for this variation computing the linear tune shift from the measured luminosities. The luminosity is given by[1,2]

$$L = \frac{1}{4}\,\frac{1}{e^2 f}\,\frac{i \cdot i}{A_0} \cdot f\left(\frac{\sigma}{\beta}\right) \tag{A.1}$$

where f is the revolution frequency, i is the current, A_0 is the minimum beam cross section, and $f(\sigma/\beta)$ is the correction factor due to finite bunch length[2] (Fig. 7).

For the beam-beam limit we have[1]:

$$\xi = \frac{r_e}{2}\,\frac{1}{\gamma}\int n^S(s)\,\frac{\beta_w(s)}{A^S(s)}\,ds = \frac{r_e}{2}\,\frac{N^S\beta_0}{\gamma A_0}\;F(\sigma_\ell, \beta_{w0}, \beta_{s0}) \;. \tag{A.2}$$

where r_e is the classical electron radius, $n^S(s)$ is the particle density of the strong beam, γ is the energy, N^S is the total number of particles in the strong beam, β_0 is the design minimum betatron amplitude, and $F(\sigma_\ell, \beta_{w0}, \beta_{s0})$ is the correction factor for finite bunch length effect.

To get the correction factor F we assume a Gaussian intensity distribution along the bunch. A particle in the center of the weak beam at a distance s from the interacting point sees the field of the other beam of relative strength

$$\frac{1}{\sqrt{\frac{\pi}{2}}\,\sigma_\ell} \cdot \exp\left(-\frac{(2s)^2}{2\sigma_\ell^2}\right)\,\frac{1}{\left(1+\frac{s^2}{\beta_{s0}^2}\right)^{1/2}} \tag{A.3}$$

which gives for F:

$$F(\sigma_\ell, \beta_{w0}, \beta_{s0}) = \int_{-\infty}^{+\infty} \frac{e^{-2(s^2/\sigma_\ell^2)}}{\sqrt{\pi/2}\cdot\sigma_\ell} \cdot \frac{\left[1+(s^2/\beta_{w0}^2)\right]}{\left[1+(s^2/\beta_{s0}^2)\right]^{1/2}}\,ds$$

$$= \sqrt{(\pi/2)}\;a_s \;\times$$

$$\times\left\{e^{a_s}\cdot i\cdot H_0^{(1)}(ia_s) + \frac{2}{\pi}\left(\frac{\beta_{s0}}{\beta_{w0}}\right)^2 \int_0^\infty \frac{e^{-2a_s\cdot x}\sqrt{x}}{\sqrt{1+x}}\right\}$$

$$= F_1(a_s) + \left(\frac{\beta_{s0}}{\beta_{w0}}\right)^2 F_2(a_s)$$

$$\left[a_s = \left(\frac{\beta_{s0}}{\sigma_\ell}\right)^2\right] \tag{A.4}$$

where β_{s0}, β_{w0} are the minimum betatron amplitudes at the interaction point for the strong and weak beam, respectively.

Both functions F_1 and F_2 are shown in Fig. 7.

There are two ways how to use the correction factor.

1. If we assume that only the betatron function of the weak beam is changed due to the lens effect of the strong beam, while the strong beam follows the unchanged betatron function, then we have $\beta_{w0} = \beta_{eff} \neq \beta_{s0}$.

2. We also could assume, since we always made sure that both beams were very equal, that both beams influence each other in the same strength. Then we would use $\beta_{w0} = \beta_{s0} = \beta_{eff} \neq \beta_0$.

Reducing the data in both ways results in values for ξ differing only by about 10%. Since the accuracy of the measurement is not better than this we can ignore the difference. We used throughout method 2.

The resulting values for ξ are used now to get the linear tune shift $\delta\nu$ by[4,5]

$$2\pi\,\xi = \sin(2\pi\delta\nu)\left\{1 + \cot(\pi\nu)\cdot\tan(\pi\delta\nu)\right\} \tag{A.5}$$

taking into account the change in the beam optics due to the beam-beam effect. This procedure has to be iterated several times until the assumed value β_{eff} to compute ξ via (A.2) agrees with minimum betatron amplitude of the changed beam optic due to the beam-beam effect.[5]

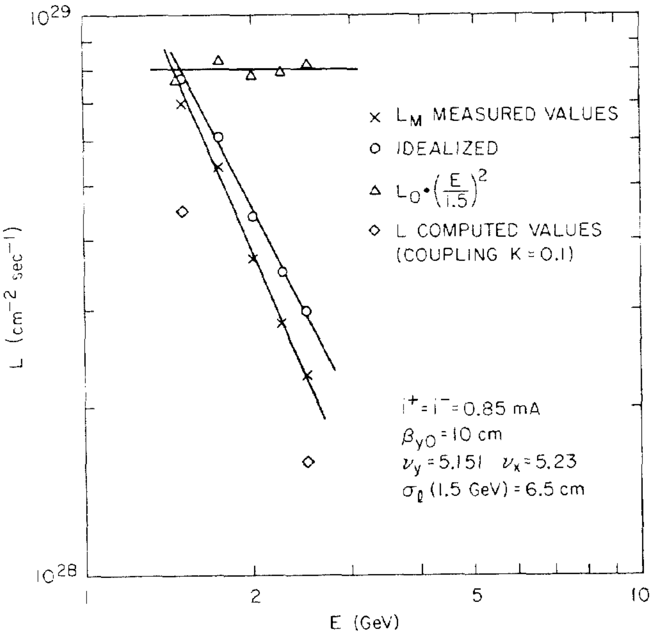

FIG. 1--Luminosity as a function of energy at low currents.

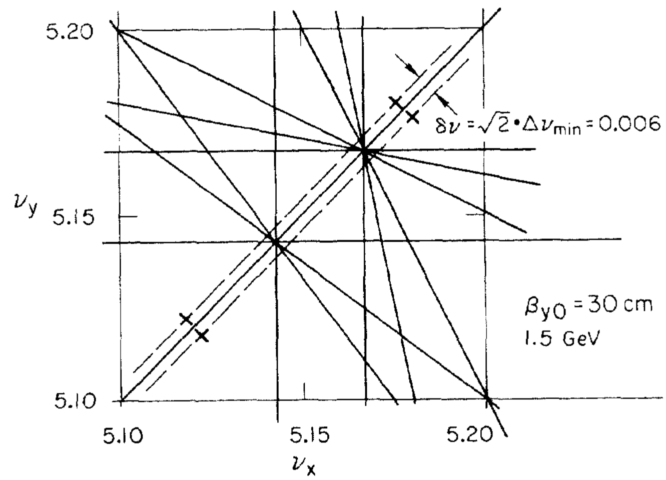

FIG. 2--Width of the linear coupling resonance.

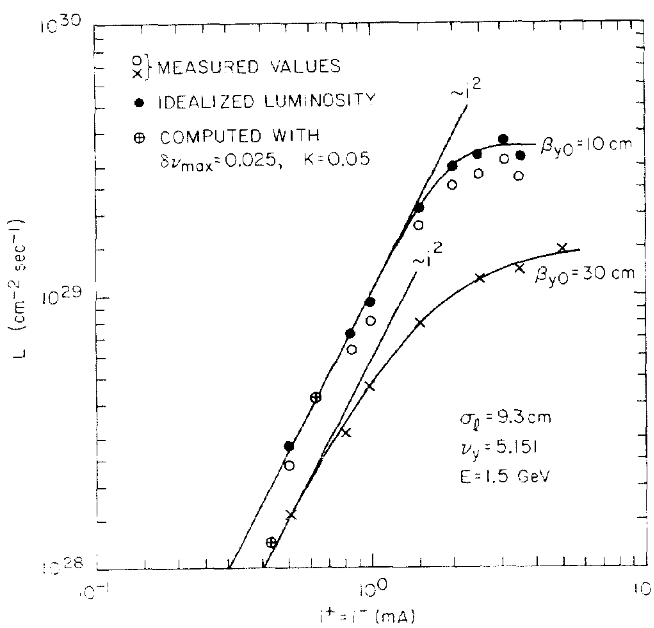

FIG. 3--Luminosity for different betatron amplitudes at the interaction point.

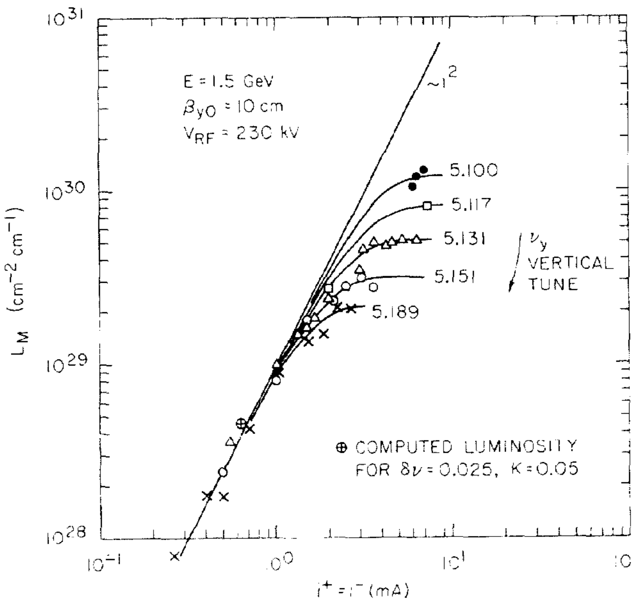

FIG. 4--Luminosity as a function of beam current for different vertical operating points.

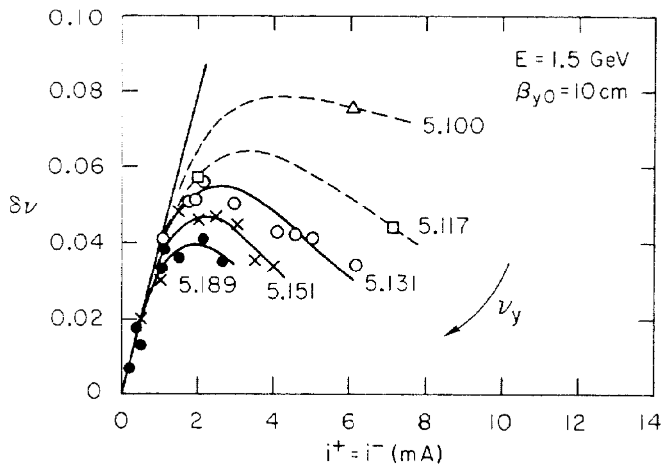

FIG. 5--Linear tune shift due to beam-beam effect as a function of beam current and operating points.

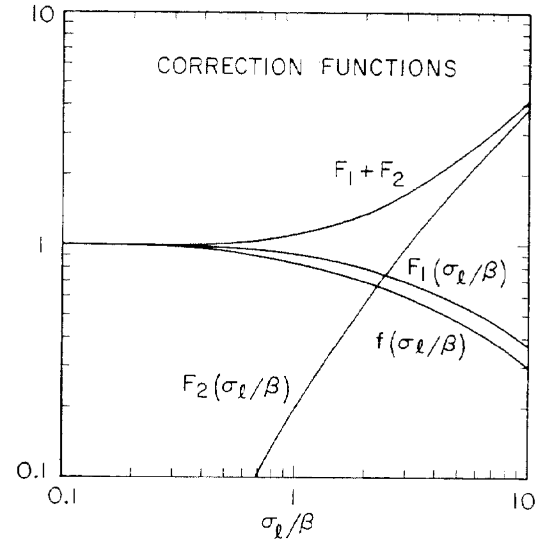

FIG. 7--Correction functions used to compute the linear tune shifts.

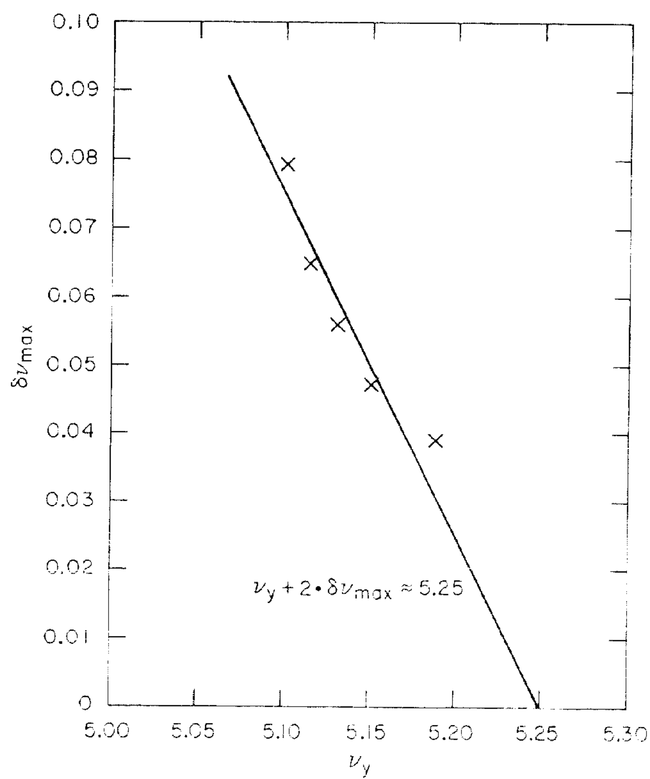

FIG. 6--Maximum linear tune shift as a function of vertical operating point.