NOISE IN PROTON ACCELERATORS*

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Summary

A high energy proton-electron-positron colliding beam system (PEP) will be described elsewhere in an invited paper. It is necessary that the proton beam be stored, tightly bunched in longitudinal space, for many hours. Since there is almost no damping of synchrotron oscillations (for protons), noise which can couple to this motion may cause the protons to diffuse so as to increase the synchrotron amplitude and destroy the tight bunches. This paper describes experiments performed on the Bevatron and attempts to correlate the results with theory. Initial theoretical studies of the effect of noise in the rf system are described in Ref. 1. The theory is extended to include the presence of a feedback control of the rf frequency. Although a properly designed feedback system can suppress coherent synchrotron oscillations of a bunched beam, it is found that the diffusion of particles within the bucket may be unaffected. An obvious source of noise is the rf voltage necessary to maintain the bunched structure. Other noise sources, such as collisions with residual gas, are included.

I. Introduction

In this work we describe experiments on the Bevatron in which noise was artificially introduced into the rf system. The theory presented here makes no attempt to explain the experimental results in detail. It merely provides a qualitative understanding of some of the observed effects. A few isolated experiments are selected in order to demonstrate that certain observed time scales are compatible with theoretical predictions.

During the experimental investigation, it became apparent that a very important noise source exists in the collisions of the beam particles with the residual gas. The theory takes this effect into account.

Two types of noise were introduced separately. These were a random variation of either the amplitude or frequency of the rf voltage. When amplitude variation was introduced, no measurable frequency variation occurred. However, due to circuit difficulties, frequency variation leads to amplitude variation. In addition, the feedback mechanism may intermix the two types of noise.

Throughout this work we discuss the spectral density of various quantities. The spectral density is defined to be the Fourier transform of the autocorrelation function, as in Eqs. (2.5) of Ref. 1. For our purposes, it suffices to note that the expectation value of the square of a stationary random variable f is related to the spectral density G_f by the equation

$$< f^2 > = \frac{1}{2\pi} \int_{-\infty}^{\infty} G_f(\omega) d\omega. \qquad (1.1)$$

If f were a voltage across a 1 ohm resistor, Eq. (1.1) gives the power dissapated.

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II. Phase Equation Including Random Errors

The Bevatron operates on the fundamental of the circulation frequency of the particles, and we will accordingly limit our treatment to a harmonic number unity. The phase Φ is defined by

$$\Phi = \theta - \omega_c t, \qquad (2.1)$$

in which θ is the azimuthal position angle and ω_c is the angular circulation frequency of a particle that is synchronous with a noise-free rf system. In the Bevatron experiments, $\omega_c = 4.6\pi \times 10^6$ rad/sec. By our definition, in what follows the synchronous particle has phase $\Phi = 0$. For any particle, we have

$$\dot{\theta} = \omega_c[1 - \eta(dE/E_o)], \qquad (2.2)$$

in which E_o is the energy of the synchronous particle and dE is the deviation about E_o. The quantity η is given by

$$\eta = \gamma_t^{-2} - \gamma^{-2}, \qquad (2.3)$$

with γ the energy of the synchronous particle and γ_t the transition energy in units of the rest energy of the particle. Approximating dE/dt by $(\omega_c/2\pi)$ dE/dn, with dE/dn the change in energy per turn, we obtain

$$\ddot{\Phi} \quad \frac{-\eta\omega_c^2}{2\pi E_o} \frac{dE}{dn} . \qquad (2.4)$$

We have

$$dE/dn = eV(t), \qquad (2.5)$$

with $V(t)$ the voltage on the cavity at the time t when the particle crosses the gap. We now consider the voltage $V(t)$ to be of the form

$$V(t) = - V_o[1 + u(t)] \sin [\omega_c t + \alpha(t)]. \qquad (2.6)$$

The quantities $u(t)$ and $\alpha(t)$ will be taken as stationary random variables, with $u(t)$ being the fractional variation of the voltage amplitude and $\alpha(t)$ the phase of the rf system (again relative to the phase of a noise-free system). We now expand Eq. (2.6) about a time $t = t_o$ when the synchronous particle crosses the gap and $\sin \omega_c t_o = 0$, $\cos \omega_c t_o = 1$. We retain u and α to first order only. To second order in $t - t_o$ we obtain

$$V(t) \approx -V_o\{\alpha + [\omega_c(1+u) + \dot{\alpha}](t-t_o) + [2\omega_c\dot{u} + \ddot{\alpha} - \omega_c^2\alpha] \times (t-t_o)^2/2\} . \qquad (2.7)$$

The cavity is located at $\theta = 0$, and from Eq. (2.1) we note that $t - t_o = - \Phi/\omega_c$. Inserting Eq. (2.7) into Eq. (2.5) and the resulting expression into Eq. (2.4) we have the phase equation

$$\ddot{\phi} + \omega_s^2 \left(1+u + \frac{\dot{\alpha}}{\omega_c}\right)\phi - \frac{\omega_s^2}{2}\left(\frac{2\dot{u}}{\omega_c} + \frac{\ddot{\alpha}}{\omega_c^2} - \alpha\right)\phi^2 = \omega_s^2\alpha, \quad (2.8)$$

in which the angular synchrotron frequency ω_s is given by

$$\omega_s^2 = \eta\omega_c^2 eV_o/2\pi E_o. \quad (2.9)$$

For the Bevatron experiments, $\omega_s = 2.6\pi \times 10^3$ rad/sec.

We shall not treat in detail the term in Eq. (2.8) proportional to ϕ^2, but will make a qualitative observation later regarding its implication.

III. Feedback

We now consider a quantity ϕ_b which can be regarded as the phase of the beam centroid. This is a quantity that may be determined experimentally and utilized to control the rf frequency by means of a feedback system. Neglecting the quadratic term in Eq. (2.8) and introducing a variable $\Phi \equiv \phi_b - \alpha$, we have from Eq. (2.8)

$$\ddot{\Phi} + \omega_s^2[1 + u + (\dot{\alpha}/\omega_c)]\ \Phi = -\ddot{\alpha}. \quad (3.1)$$

In obtaining Eq. (3.1) we have neglected terms of second order in u and α. We define the quantity ν to be the deviation of the rf frequency from the value ω_c. Apparently

$$\alpha = \int^t \nu\ dt\ , \quad (3.2)$$

so that $\dot{\alpha} = \nu$ and $\ddot{\alpha} = \dot{\nu}$. The value of ν at any time is determined by noise in the system (either inherent or artificially induced) and by the feedback system. We characterize the contribution from noise by the quantity Ω and write

$$\nu = \Omega + m\Phi, \quad (3.3)$$

in which the constant m is a characteristic of the feedback loop. Equation (3.1) may now be written as

$$\ddot{\Phi} + m\dot{\Phi} + \omega_s^2 \left(1 + u + \frac{\nu}{\omega_c}\right)\Phi = -\dot{\Omega}. \quad (3.4)$$

The quantity Φ determined by Eq. (3.4) characterizes coherent synchrotron oscillations of the bunch.

Equation (3.4) with u and Ω both random variables is indeed formidable. One consequence of this equation is that even with $\Omega = 0$, an amplitude variation u will drive Φ to some finite value, thus a frequency variation will result through the feedback system via Eq. (3.3). On the other hand, a frequency variation Ω will not result in an amplitude modulation via Eq. (3.4). However, an amplitude variation may result from the electronics of the rf system itself if the frequency is varied. For purposes of this work we shall neglect the u and ν/ω_c compared to unity in Eq. (3.4). We may then calculate the spectral density of ν by the following means: We take the Fourier transform of Eq. (3.4) and obtain the relation (the tilda over a quantity indicates its transform)

$$\tilde{\Phi} = i\omega\tilde{\Omega}/(\omega_s^2 - \omega^2 - im\omega). \quad (3.5)$$

Taking the Fourier transform of Eq. (3.3) and inserting Eq. (3.5) for $\tilde{\Phi}$ we obtain

$$\tilde{\nu} = (\omega_s^2 - \omega^2)\ \tilde{\Omega}/(\omega_s^2 - \omega^2 - im\omega). \quad (3.6)$$

The spectral density G_ν is thus given in terms of the spectral density G_Ω by the relation[1]

$$G_\nu(\omega) = (\omega_s^2 - \omega^2)^2\ G_\Omega(\omega)/[(\omega_s^2 - \omega^2)^2 + m^2\omega^2]. \quad (3.7)$$

The feedback system has the property that it reduces the spectral density of the random variable ν to zero at the frequency ω_s. Consequently the spectral density of α and $\dot{\nu}$ are also zero at ω_s, since $G_\alpha(\omega) = G_\nu(\omega)/\omega^2$ and $G_{\dot{\nu}}(\omega) = \omega^2 G_\nu(\omega)$. The significance of this will become apparent in the following sections.

IV. Expectation Value of Coherent Oscillations

We now calculate the expectation value, $< \Phi^2 >$, of the coherent oscillations. From Eq. (1.1)

$$< \Phi^2 > = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_\Phi(\omega)\ d\omega. \quad (4.1)$$

The spectral density G_Φ may be obtained from Eq. (3.5). We have

$$G_\Phi = \omega^2 G_\Omega/[(\omega_s^2 - \omega^2)^2 + m^2\omega^2]. \quad (4.2)$$

In the Bevatron the quantity m is determined by two devices. The first, which we call M, senses a value of Φ and converts radians to volts. The value of M is about one-half volt per radian. The signal from M is sent to the master oscillator, which we call K, that converts volts to frequency deviation. The value of K is about $5\pi \times 10^4$ rad/sec-volt. Thus $m \equiv MK$ has a value of about $2.5\pi \times 10^4$ rad/sec.

Noise in the form of a voltage from a random noise generator was introduced directly into K. This noise generator has a flat spectral density from some low minimum value of a few cycles out to a maximum frequency of about 2×10^4 cycles/sec. If we assume that the frequency variation artificially induced in this manner is much greater than the inherent frequency variation, we have $\Omega = KU$, with U the voltage from the noise generator. Furthermore, we have $G_\Omega = K^2 G_U$. From the equation for $< U^2 >$ analogous to Eq. (4.1) we find

$$G_U = \begin{cases} \pi < U^2 > /\omega_{max}, & \omega_{min} < \omega < \omega_{max}, \\ 0 & \text{elsewhere}, \end{cases} \quad (4.3)$$

with $\omega_{max} = 4\pi \times 10^4$. Inserting Eq. (4.3) into Eq. (4.2) and performing the integral in Eq. (4.1), the result is

$$< \Phi^2 > = 3.4 < U^2 >. \quad (4.4)$$

The experimentally observed rms values of Φ are generally somewhat higher than those obtained from Eq. (4.4), sometimes even a factor of two higher. This discrepancy is no doubt due in part to the approximate treatment of Eq. (3.4).

V. Incoherent Motion

The incoherent motion of particles within the bunch is characterized by the quantity $\psi \equiv \phi - \phi_b$, which obeys the equation

$$\ddot{\psi} + \omega_s^2\ [1 + u + (\nu/\omega_c)]\,\psi = 0. \quad (5.1)$$

A. Amplitude Variation

We first consider amplitude variation in the absence of frequency variation and set $\nu = 0$ in Eq. (5.1). If both u and ν are zero, the solution to Eq. (5.1) is $\psi = a \sin(\omega_s t + \delta)$. We make the assumption, which is justified by the results, that the amplitude, a, changes by a negligible amount during a synchrotron period (more rigorously, the amplitude, a, must change by a negligible amount over a correlation time of u) and write Eq. (5.1) in the form

$$\ddot{\psi} + \omega_s^2\psi = -\omega_s^2 ua \sin(\omega_s t + \delta). \quad (5.2)$$

A digression to noise theory is necessary at this point. From an equation of the form $\ddot{x} + \omega_s^2 x = f(t)$, with $f(t)$ a stationary random variable, it can be shown that the expectation value $\dot{x}f$, which we indicate with $< >$, is given by

$$< \dot{x}f > = G_f(\omega_s)/2. \qquad (5.3)$$

If we examine Eq. (3.1) with Eq. (5.3) in mind, it is obvious that the feedback system must reduce the spectral density of α to zero at ω_s in order to be effective in suppressing coherent oscillations.

The energy of the oscillator is $E = 1/2(\dot{x}^2+\omega_s^2x^2)$, and $dE/dt = \dot{x}f$. But also $E = a^2\omega_s^2$, so that the rate of change of $< a^2 >$ is given by $d < a^2 >/dt = G_f(\omega_s)/2\omega_s^2$. For the form of $f(t)$ on the right hand side of Eq. (5.2), we have

$$< \dot{\psi}f > = \omega_s^4 a^2 G_g(\omega_s)/2, \qquad (5.4)$$

with $g \equiv u \sin(\omega_s t + \delta)$. If u is a stationary random variable then g is also, and the phase δ is of no consequence in determining the spectral density of g. It can in fact be shown that

$$G_g(\omega) = (1/4)[G_u(\omega + \omega_s) + G_u(\omega - \omega_s)]. \qquad (5.5)$$

The contribution to $G_g(\omega_s)$ from $G_u(0)$ is spurious, resulting from the approximate treatment of Eq. (5.1). Clearly if u does not change in time, the amplitude of ψ does not increase with time.

From Eq. (5.4) and (5.5) together with the above discussion, we have (with a_o the initial amplitude of ψ)

$$\ln (< a^2 >/a_o^2) = t/\tau , \qquad (5.6)$$

in which the growth time τ is given by

$$\tau = 8/\omega_s^2 G_u(2\omega_s). \qquad (5.7)$$

The same noise generator discussed in Section IV was employed to produce an expectation value of $< u^2 >$ of 2.25×10^{-4} (i.e., about 1.5% rms voltage fluctuation). Although the signal from the generator has a uniform spectral density for $\omega < \omega_{max}$, the response of the final stage must be taken into account when calculating G_u. We have

$$G_u = \begin{cases} C\delta^2/(\delta^2 + \omega^2) & \omega_{min} < \omega < \omega_{max}, \\ 0 & \text{elsewhere.} \end{cases} \qquad (5.8)$$

In Eq. (5.8) C is a constant and $\delta = \omega_c/2Q$, with Q the quality factor (about 50 for the Bevatron). Analogous to Eq. (4.1) we determine C by the relation

$$< u^2 > = \frac{1}{\pi} \int_0^{\omega_{max}} G_u(\omega)d\omega = (C\delta/\pi)\tan^{-1}(\omega_{max}/\delta). \qquad (5.9)$$

We see that $\omega_{max} \sim \delta$, so that we have $C = 4 < u^2 >/\delta$. Since $\omega_s^2 \ll \delta^2$, from Eq. (5.8) we have $G_u(2\omega_s) \approx C$, and from Eq. (5.7) we obtain $\tau = 20$ sec, a result in rather good agreement with the experimental results.

B. Frequency Variation

The treatment of Eq. (5.1) for $u = 0$ and $\nu \neq 0$ is the same as that above. We simply replace the function $G_u(2\omega_s)$ in Eq. (5.7) by $G_\nu(2\omega_s)/\omega_c^2$, with G_ν given by Eq. (3.7). Although the feedback system insures $G_\nu(\omega_s) = 0$, $G_\nu(2\omega_s) \neq 0$. Unfortunately, when we follow this procedure, we obtain growth times several orders of magnitude longer than those observed experimentally. Two possible courses of this discrepancy present themselves. First our inability to treat Eq. (3.4) in detail casts doubt on the validity of Eq. (3.7) for $G_\nu(\omega)$. Also the possibility exists that a frequency variation gives rise to an amplitude variation that causes the observed spreading of the beam.

As a general comment, we note that the nonlinear term in Eq. (2.8) is of such a form as to give rise to a growth of Φ. This growth will be determined by the spectral density of α and/or u at ω_s and $3\omega_s$, since the term may be treated in the same manner as Eq. (5.2). That is, the driving term is taken as $\alpha a^2 \sin^2(\omega_s t + \delta)$, say, and the spectral density of this function at ω_s has terms proportional to $G_\alpha(\omega_s)$ and $G_\alpha(3\omega_s)$.

C. Effect of Collisions with Residual Gas

Collisions with the residual gas (air) in the vacuum tank result in an average decrease in energy and also a spread in energy within the beam. The average energy loss merely leads to a small non-zero stable phase angle, while the spread in energy may become so great as to cause loss of particles from stable phase. From Eqs. (2.1) and (2.2) we derive the relation

$$\dot{\phi}^2 = (2\pi\omega_s/\omega_c)^2(dE/eV_o)^2. \qquad (5.10)$$

When averaged over a synchrotron period, Eq. (5.10) yields

$$a^2 = (2\pi\omega_s/\omega_c)^2(\delta E/eV_o)^2, \qquad (5.11)$$

in which a is the amplitude of the phase angle Φ and δE is the amplitude of energy oscillation arising from the collisions.

The distribution of δE after a time t was the subject of a thesis of K.R. Symon, the results of Symon that are pertinent to our problem are given in Sec. (2.7) of Ref. 2. A discussion of these results as they apply to Eq. (5.11) is beyond the scope of this work. As an example, we employ the theory in the limit that δE has a Gaussian distribution with a width given by Eq. (2.7.9) of Ref. 2, which may be written as

$$< \delta E^2 > = 4C(\gamma E_e)^2[1 - (\beta^2/2)] Dct, \qquad (5.12)$$

with $C = 7.5 \times 10^{-2}$ cm^2/gm for air, E_e the rest energy of the electron, D the density of the background gas in gm/cm^3 and c the speed of light in cm/sec. In writing Eq. (5.12) we have inserted a value $2\gamma^2\beta^2E_e$ for the maximum energy transfer in a collision.

At a pressure of 10^{-5} torr, $D = 1.7 \times 10^{-11}$ gm/cm^3, and for $\gamma = 3$ we obtain $< \delta E^2 > = 0.76 E_e^2 t$. Inserting this value in Eq. (5.11) with $eV_o = 5$ keV and $2\pi\omega_s/\omega_c = 3.55 \times 10^{-3}$, we obtain

$$< a^2 > = 0.09 \text{ t}. \qquad (5.13)$$

It must be pointed out that Eq. (5.12) is not valid for $t \approx 10$ sec, and in fact, δE is not Gaussian distributed until t is the order of a few minutes. For shorter times, Eq. (5.13) gives a value of $< a^2 >$ that is less than the accurate value. It is gratifying that experimentally at a pressure of 10^{-5} torr the beam decays in about 10 sec, and that the dependence of the beam decay time on pressure follows that predicted by Eqs. (5.11) and (5.12).

We can ascribe a spectral density to the collisional energy loss. It can be shown that the spectral density of the rate of change of $< \delta E^2 >^{1/2}$ can be found from the relation[3]

$$G_{\delta \dot{E}}(0) = < \delta E^2 >/t. \qquad (5.14)$$

The spectral density for the collisional energy loss is certainly a constant out to frequencies of the order of the reciprocal of the mean collision time, so that its value at $\omega = \omega_s$ is the same as its value at $\omega = 0$.

VI. Experimental Results

The loss of particles from stable phase is an exponential function of time as displayed in Fig. 1. (Particles are lost from the machine at a much slower rate.) The decay rate is a sum of two terms. For induced frequency variation, we may define a time $t_{1/2} \equiv K_T^{-1}$ in which the peak of the distribution in ϕ reaches one half its initial value, with

$$K_T = 20.8 < U^2 > + 1.56 \times 10^4 \ p, \text{ frequency} \qquad (6.1)$$

with p the background pressure in torr. For amplitude variation the equivalent quantity is given by

$$K_T = 7 < U^2 > + 1.56 \times 10^4 \ p, \text{ amplitude.} \qquad (6.2)$$

The first term is proportional to the spectral density of the induced noise, with a different proportionality constant for amplitude and frequency variation. The second term is proportional to the background pressure (as pointed out in Sec. V, the spectral density of the energy loss from collisions with the background is proportional to the background density.) The dependence of the decay rate on these two terms is exhibited in Fig. 2.

A surprising aspect of the experimental results is the behavior of the "shape" of the bunch, that is, the distribution of particles in phase. As shown in Figs. 3 and 4, the shape does not change appreciably when noise is introduced, or as the number of particles in stable phase decreases. Theory would predict that rf noise and/or collision with the background gas would first lead to a spread in the amplitudes of synchrotron oscillations (and a "filling" of stable phase) followed by a loss of particles. This sequence of events occurs only when the feedback is disconnected. The constancy of the shape suggests that the particles are being lost primarily as a consequence of Eq. (5.1). Particles with large amplitude of synchrotron oscillations are most affected by spectral densities at $2\omega_s$, and are therefore more likely to be lost. However existing theory is based on linearized phase motion only, and extrapolation to non-linear motion is not justified.

By inducing noise with a narrow band width it was found that frequencies of ω_s, $2\omega_s$, and $3\omega_s$ are all destructive to beam lifetime. This result is qualitatively consistent with the theory in previous sections.

Band pass filters allowing a band width of 200 cycles/sec around ω_s and $2\omega_s$ resulted in reduction by an order of magnitude in the rf noise term of the decay rate. This is in contradiction to theory, because the introduction of such filters in no way alters the spectral density of the noise at ω_s and $2\omega_s$.

The effect of magnet noise was sought by turning off, separately and/or simultaneously, the passive voltage filter and the active field filter for the magnetic guide field. The two systems together reduce noise in the spectral range of interest by at least two orders of magnitude. These filters being on or off had no influence on beam lifetime.

Inherent amplitude modulation is negligibly small in the Bevatron. Inherent frequency variation is exhibited in Fig. 5. It takes a value of $< U^2 >^{1/2}$ of about 0.3 mV to double the signal, so we may conclude that the inherent frequency variation is of the order of that introduced by $< U^2 >^{1/2} = 0.3$ mV. Furthermore, the frequency content is roughly the same. If there were no other forms of noise present, this value of $< U^2 >$ in Eq. (6.1) would yield $K_T^{-1} \approx 5 \times 10^5$ sec. Such a long time indicates that even a conventional rf system would contain bunched protons for hours in a sufficiently high vacuum.

References

1. V.K. Neil, and R.K. Cooper, Particle Accelerators 4, 75, (1972).
2. High Energy Particles, Bruno Rossi, Prentice-Hall, New York, (1952). See especially Sec. 2.7.
3. Noise, Albert Van der Ziel, Prentice-Hall, New York, (1954).

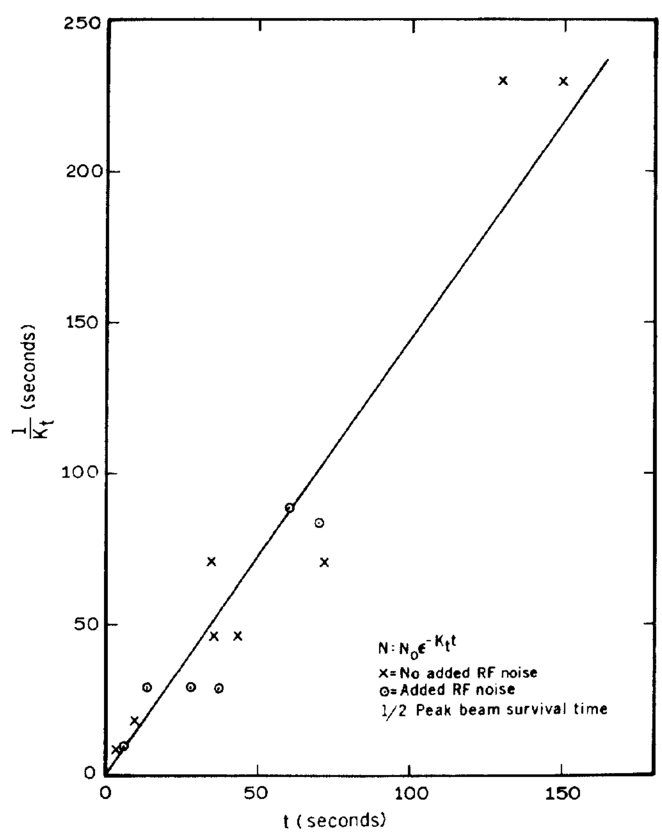


Fig. 1 - Exponential Behavior of Beam Loss.

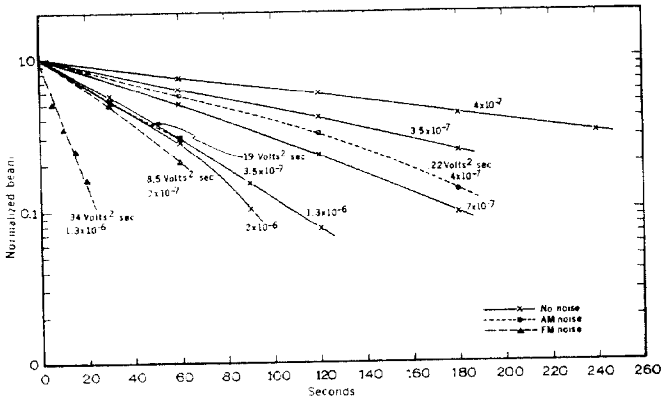
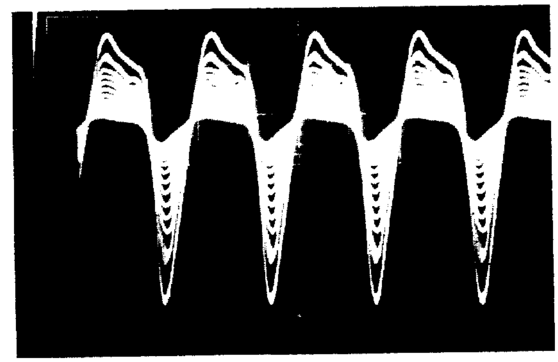
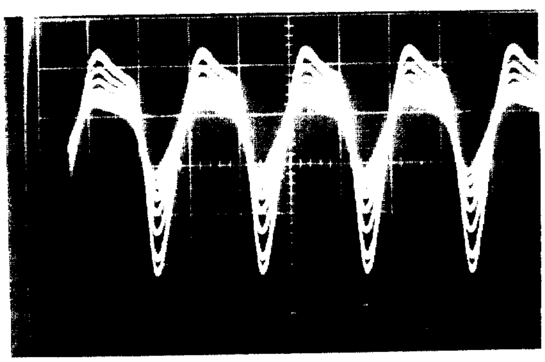


Fig. 2 - Beam loss with rf noise and pressure P(torr). For amplitude variation, the numbers in volts^2 sec are $< U^2 > V_o{}^2/f_{max}$, while for frequency variation these numbers are $< \Phi^2 > V_o{}^2/f_{max}$.



White Noise

Fig. 4 - Beam Bunch Shape.
Exposures taken at 5 second intervals.
$G(\omega) = 0.26$ volts^2 sec
$P = 10^{-6}$ torr.



No Noise

Fig. 3 - Beam Bunch Shape
Exposures at 15 second intervals. No noise added. $P = 10^{-6}$ torr.

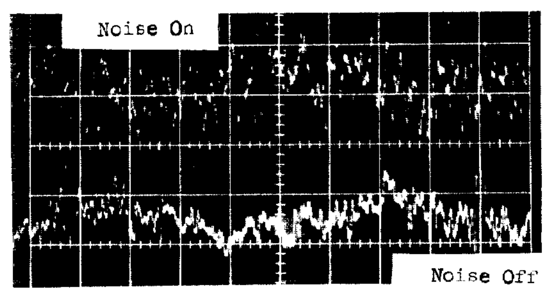


Fig. 5 - Comparison of residual FM noise to injected noise of 4.2×10^{-4} volts^2 sec
Sweep 1 millisec/cm.