LIMITATION IN SHORT BUNCHES PRODUCTION WITHIN ELECTRON STORAGE RINGS

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Abstract - Short bunches are required in electron storage rings. To get them shorter one is tempted to have machines working near transition energy. Then it becomes necessary to watch out for non linear terms because lower synchrotron and betatron stability limits arise. Typical results are given.

In order to shorten the length of the bunches, electron storage rings are designed to work near transition energy. With $\eta = (d\omega/\omega)/(dp/p) = 10^{-2}$, bunches, a few centimeters long, may be obtained.

In special cases, smaller η values may be required. As an example, for producing coherent synchrotron radiation in optical range, /1/ /2/ one needs to shorten bunches down to a fraction of a micron. Even using very high R. F. voltages, $\eta \leqslant 10^{-6}$ is needed.

In this range of η values, synchrotron motion becomes essentially non-linear /3/. Moreover, it may be necessary to care about modifications induced by betatron oscillation.

I. NON-LINEAR SYNCHROTRON MOTION

Let p be the momentum and $T = \frac{2\pi}{\omega}$ the revolution period. One has :

$$\frac{\Delta T}{T} = -\eta_o\left(\frac{\Delta p}{p}\right) + \eta_1\left(\frac{\Delta p}{p}\right)^2 + \ldots \text{ and } \frac{\Delta\omega}{\omega} = \eta_o\left(\frac{\Delta p}{p}\right) + (\eta_o^2 - \eta_1)\frac{\Delta p}{p} + \ldots$$

η_o and η_1 depend on chromatic closed orbit :

$$x_{c.o.}(s) = R.\, g(s).\left(\frac{\Delta p}{p}\right) + R.\, f(s).\left(\frac{\Delta p}{p}\right)^2 + \ldots$$

where s is the azimuth and R the mean radius of the ring.

When the length $\mathcal{C}$ of that closed orbit is expanded as follow :

$$\frac{\Delta\mathcal{C}}{\mathcal{C}} = \alpha_o.\left(\frac{\Delta p}{p}\right) + \alpha_1\left(\frac{\Delta p}{p}\right)^2 + \ldots$$

one gets :

$$\left\{\begin{array}{l} \alpha_o = \overline{g}_m \\ \\ \alpha_1 = \overline{f}_m + \frac{R^2}{2}\,\overline{\dot{g}^2} \end{array}\right. \quad \text{and} \quad \left\{\begin{array}{l} \eta_o = \frac{1}{\gamma^2} - \alpha_o \\ \\ \eta_1 = \frac{3}{2}\frac{\beta^2}{\gamma^2} + \frac{\eta_o}{\gamma^2} + \alpha_1 \simeq \alpha_1 \end{array}\right.$$

where $\overline{(\;)}_m$ represents the mean value calculated in the curved section and $\overline{(\;)}$ the mean value calculated over the whole ring. β and γ are the usual relativistic parameters.

The dominant contribution beeing due to $\frac{R^2}{2}\overline{\dot{g}^2}$, η_1 is positive. Computational results from different rings give η_1 values between some 10^{-1} and a few units.

Then, the synchrotron motion equations become :

$$\left\{\begin{array}{l} \frac{d}{dt}\left(\frac{\Delta p}{p}\right) = \frac{\Omega.\delta_{max}}{2}.\sin\varphi \\ \frac{d}{dt}(\varphi) = \omega_{RF}\left[-\eta_o.\left(\frac{\Delta p}{p}\right) + \eta_1\left(\frac{\Delta p}{p}\right)^2\right] \end{array}\right.$$

Ω being the angular frequency of small oscillations and $\delta_{max} = \frac{2}{\eta_o}\frac{\Omega}{\omega_{RF}}$

According to η_1 values, synchrotron diagram presents different aspects pictured in fig 1 to 4.

One notices the presence of a new stability zone centered around :

$\frac{\Delta p}{p} = \frac{\eta_o}{\eta_1}$; consequently, the number of bunches is doubled, provided that this momentum shift is accepted by the ring. When the synchrotron phase φ_s is not equal to zero, the synchrotron diagram is more complicated (fig 5) and the angular frequency of small oscillations becomes $\Omega\sqrt{\cos\varphi_s}$. The momentum acceptance is independant of φ_s.

One notices that the stability zone may collapse when η_1 increases. Consequently, the beam lifetime (Touscheck effect and diffusion losses) may be drastically reduced. For usual storage ring parameters ($\eta_o \# 10^{-2}$), the ratio $\eta_o/2\eta_1$ (that defines momentum acceptance) may get down to 10^{-2} or less, say 10^{-3}, which is obviously too small.

II. BETATRON OSCILLATIONS INFLUENCE

Betatron oscillations around the closed orbit increase the revolution period and by this fact disturb synchrotron motion.

In smooth approximation, particles follow sinusoidal trajectories with a maximum amplitude $\hat{y}$ and a wave length $2\pi R/\nu$. The resulting trajectory lengthening is given by :

$$\frac{\Delta\mathcal{C}}{\mathcal{C}} = \frac{\nu^2}{4R^2}.\hat{y}^2$$

y representing either radial or vertical motion amplitude. In the actual case, the lengthening is well approximated by :

$$\frac{\Delta\mathcal{C}}{\mathcal{C}} = \frac{1}{4}.\left(\frac{1+\alpha^2}{\beta}\right).\frac{\hat{y}^2}{\beta}$$

where α and β are the usual Twiss functions. When β-function is deeply modulated (low-β), the smooth approximation result may be 3 or 4 times too small.

A trajectory lengthening $\Delta\mathcal{C}/\mathcal{C}$ changes the synchrotron motion equations into :

$$\left\{\begin{array}{l} \frac{d}{dt}\left(\frac{\Delta p}{p}\right) = \frac{\Omega\delta_{max}}{2}\sin\varphi \\ \frac{d}{dt}(\varphi) = \omega_{RF}\left[\frac{\Delta\mathcal{C}}{\mathcal{C}} - \eta_o\left(\frac{\Delta p}{p}\right) + \eta_1\left(\frac{\Delta p}{p}\right)^2\right] \end{array}\right.$$

One notices that $\Delta\mathcal{C}/\mathcal{C}$ may result from different mechanisms such as :

- betatron oscillations : $\Delta\mathcal{C}/\mathcal{C}$ always >0 and depending on particle amplitude ;
- off momentum tuning by R. F. frequency shift or ring dilatation : $\Delta\mathcal{C}/\mathcal{C}$ of any sign and not depending on particle amplitude.

The corresponding stability diagram is shown in fig 6. One finds the case $\Delta\mathcal{C}/\mathcal{C} = 0$ again by changing

$$\frac{\Delta p}{p} \text{ into } \frac{\Delta p}{p} - \frac{\eta_o}{2\eta_1}\left(1 - \sqrt{1 - \frac{\Delta \mathcal{E}/\mathcal{E}}{(\Delta \mathcal{E}/\mathcal{E})_{lim}}}\right)$$

and

$$\eta_o \text{ into } \eta_o \sqrt{1 - \frac{\Delta \mathcal{E}/\mathcal{E}}{(\Delta \mathcal{E}/\mathcal{E})_{lim}}} \quad \text{with } \left(\frac{\Delta \mathcal{E}}{\mathcal{E}}\right)_{lim} = \frac{\eta_o^2}{4\eta_1}$$

A change in energy compensates the trajectory lengthening and a decrease ($\Delta \mathcal{E}/\mathcal{E} > 0$) [respectively an increase ($\Delta \mathcal{E}/\mathcal{E} < 0$)] of η_o goes with a decrease, [respectively an increase] of the stability area.

The new angular frequency of small oscillations is :

$$\Omega \left(1 - \frac{\Delta \mathcal{E}/\mathcal{E}}{(\Delta \mathcal{E}/\mathcal{E})_{lim}}\right)^{1/4}$$

the value of

$$\frac{\Delta \mathcal{E}}{\mathcal{E}}_{lim} = \frac{\eta_o^2}{4\eta_1}$$

is obviously crucial. It lies roughly around 10^{-4} but decreases sometimes down to 10^{-5} . These orders of magnitude determine the required minimum geometric stability of the ring and demand a limitation of betatron oscillation amplitude because of :

$$\frac{1}{4}\left(\frac{1+\alpha^2}{\beta}\right)\frac{\hat{y}^2}{\beta} < \frac{\Delta \mathcal{E}}{\mathcal{E}}_{lim} = \frac{1}{4}\left(\frac{1+\alpha^2}{\beta}\right)\frac{\hat{y}^2_{lim}}{\beta}$$

Variation of synchrotron stability versus $\hat{y}/\hat{y}_{lim}$ is plotted in fig 7.

Practically, vertical motion is disregarded and one restricts the amplitude of radial motion to $\hat{x} \leqslant (0.6 \text{ to } 0.7).\ \hat{x}_{lim}$ in order to keep a sufficiently wide synchrotron stability area. Consequently, a bunch width $L_x \leqslant (0.35 \text{ to } 0.40).\ \hat{x}_{lim}$ prevents bunches from too fast diffusion losses.

From this point of view, conventional rings seem to be safe, nevertheless, problems could appear during ring injection and before damping.

It may be interesting to notice that a $\Delta \mathcal{E}/\mathcal{E} < 0$ (a slight off-momentum tuning for example) may partly solve the problem by widening the synchrotron stability area.

III. CONCLUSION

With conventional rings, the value $\eta = (d\omega/\omega)/(dp/p) = 10^{-2}$ seems to be a lower limit under which non linear problems may appear. The obtaining of very short bunches ($L < 1\mu$) as necessary for coherent synchrotron radiation production in optical range, assumes a low energy ring (say 50 to 100 MeV). Then, the simultaneous decrease in width and energy spread of bunches moves the limit to $\eta \# 10^{-4}$. Consequently a 10 microns wave length seems to be the lowest feasible limit for coherent synchrotron radiation /2/. The uses of lower energy rings is not possible because of internal collisions within the bunch.

REFERENCES

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/3/ N. Vogt-Nilsen, "Theory of R. F. acceleration in fixed field circular accelerators", CERN-PS/ NVN-2 (1958).

These figures assume $\eta_o > 0$

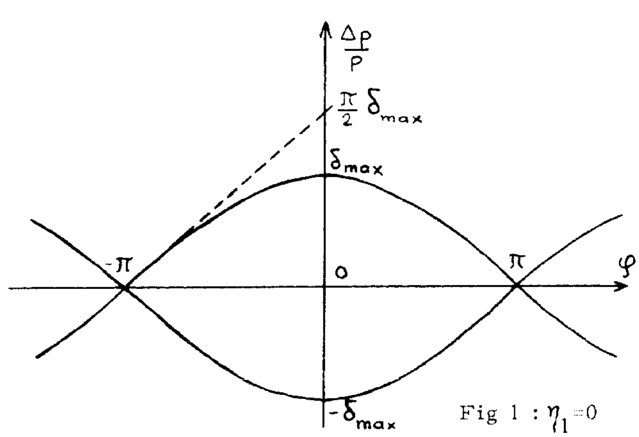


Fig 1 : $\eta_1 = 0$

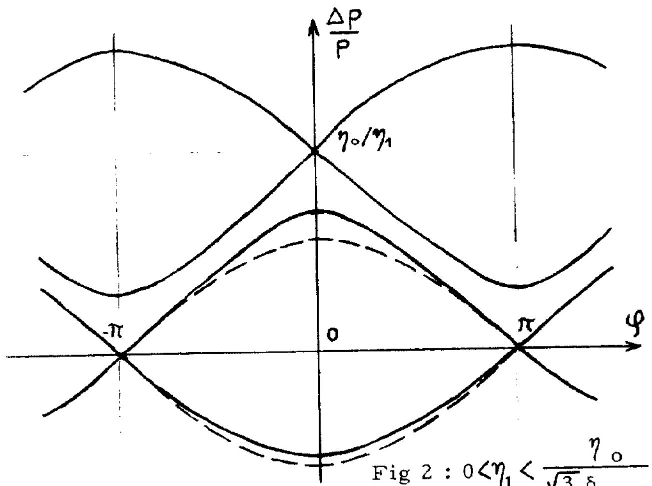


Fig 2 : $0<\eta_1<\frac{\eta_0}{\sqrt{3}\,\delta_{max}}$

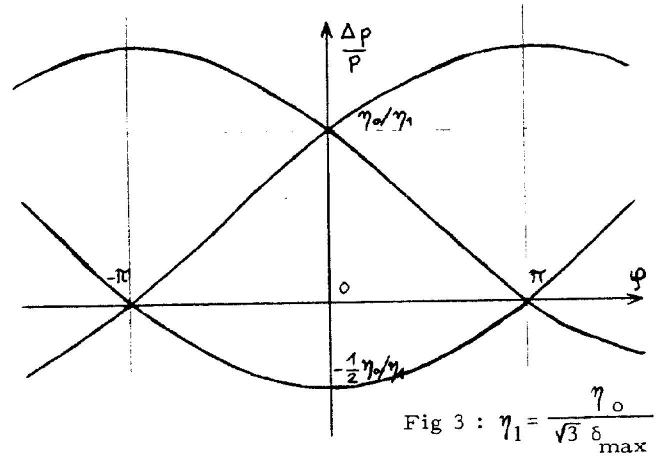
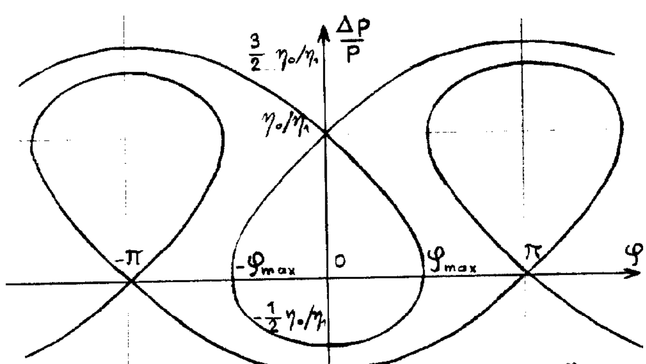


Fig 3 : $\eta_1=\frac{\eta_0}{\sqrt{3}\,\delta_{max}}$



$\cos\varphi_{max}=1-\frac{1}{2}\left(\frac{2\ \ \eta_0}{\sqrt{3}\,\eta_1\delta_{max}}\right)^2$

Fig 4 : $\eta_1>\frac{\eta_0}{\sqrt{3}\,\delta_{max}}$

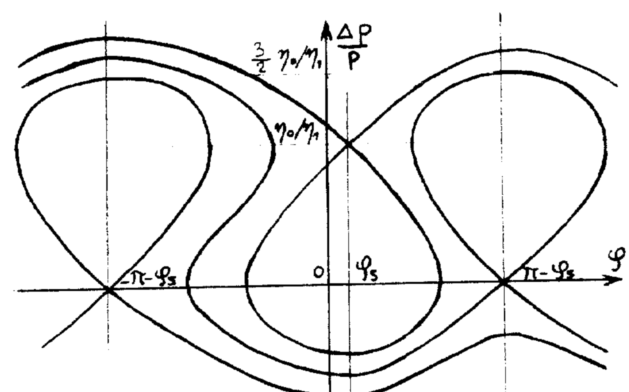


Fig 5 : $\varphi_s \neq 0$

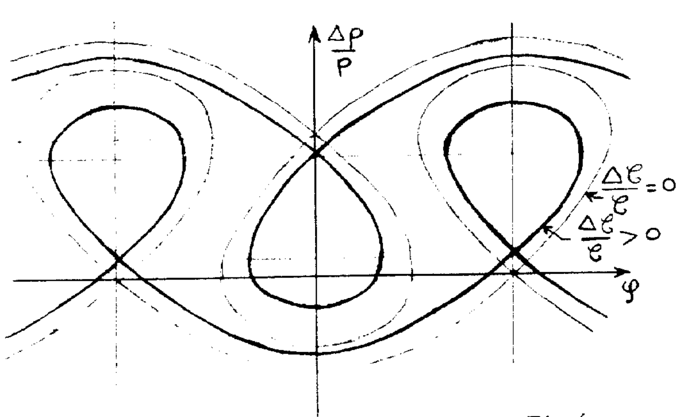


Fig 6

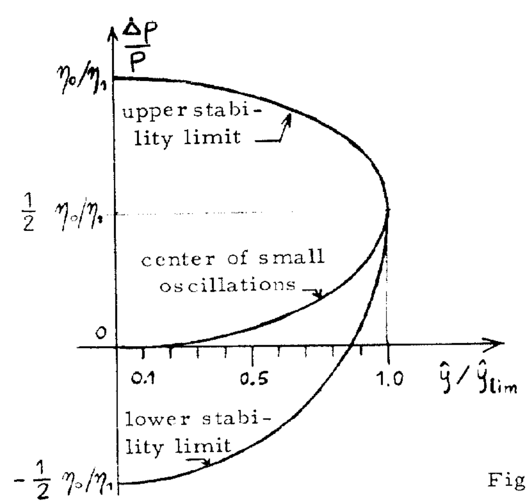


Fig 7