MEASUREMENTS OF STACKING EFFICIENCY IN THE CERN INTERSECTING STORAGE RINGS (ISR)

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Summary. The efficiency of the stacking process, in which protons are accumulated into longitudinal phase space by means of a radio frequency (RF) voltage, was measured as function of different parameters such as energy, RF voltage, number of stacking cycles and

$$\Gamma = \sin \Phi_s,$$

Φ_s is the relative phase of protons in synchronism with the RF voltage.

The efficiency was found to depend predominantly on Γ and the number n of stacking cycles. Efficiencies higher than 70% were measured for values of $\Gamma \leqslant 0.5$ and $n \geqslant 30$ for both the repetitive and the non-repetitive stacking scheme. The agreement between theoretical predicted efficiencies, obtained from a computer simulation of the stacking process, and the measured efficiencies is very good for higher values of n.

From both the theoretical calculations and the measurements a rule of thumb could be derived which gives a good approximation of the stacking efficiency far above transition energy as function of Γ and n.

Definition of the stacking efficiency

Even under ideal conditions in a stacking process the density of protons cannot be increased (Liouville theorem). Hence the overall efficiency of an accumulation process in a storage ring can be defined as the ratio of averaged phase space density in the stack to the density at injection.

Several factors contribute to the overall efficiency which may therefore be written as

$$\eta_{tot} = \eta' \cdot \eta,$$

where

η' represents the reduction in phase space density resulting from the RF manipulations (trapping of the injected bunches, matching of the RF bucket to the bunch, reduction of the amplitude of RF voltage to the final value U etc.) during a stacking cycle, and

η stands for the stacking efficiency to which the following discussion is restricted, defined as

$$\eta = \frac{\text{number of protons in the ideal stack width } \Delta E_{id}}{\text{total number of protons in the stack}}.$$

The ideal stack width after n stacking cycles is

$$\left[\Delta E_{id}\right]_n = n\delta E_b,$$

δE_b denoting the RF bucket width in energy, averaged over the phase.

The stacking efficiency is essentially describing the reduction of phase space density of stacked protons due to the presence of moving RF buckets in or near the stack as occurs during subsequent stacking cycles.
In first approximation it is assumed to depend on the energy E, the RF parameters U, Γ used in the stacking process and the number n of stacking cycles, hence

$$\eta = \eta(E,U,\Gamma,n).$$

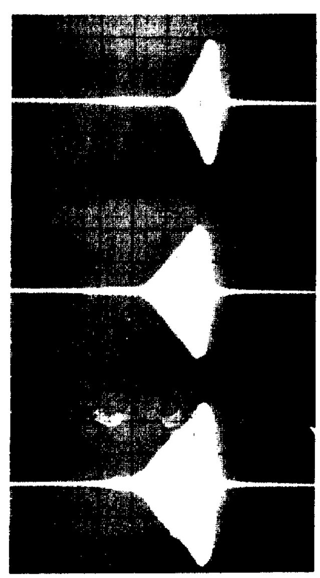
The measurement of the stacking efficiency

To measure the stacking efficiency many stacks were made at 22 GeV and 26 GeV with different sets of RF parameters (listed in Table 1) and varying number of stacking cycles. Both the repetitive and the non-repetitive stacking modes were investigated. In the repetitive stacking scheme particles are released from the RF buckets at the same energy for all stacking cycles, while in the non-repetitive scheme particles are deposited at decreasing energies, whereby the energy difference between successive stacking cycles is equal to δE_b.

The density distribution of the protons was measured for each stack by means of the RF scanning technique, whereby empty RF buckets, generated by the RF system, are swept through the stack. The signals, induced on an electrostatic pick-up electrode due to the passage of the empty buckets through the stack, are filtered, amplified and displayed on a memoscope. A voltage proportional to the RF frequency of the buckets is used as horizontal sweep for this oscilloscope, and density is thus obtained as function of energy. Typical density distributions obtained are shown in Fig. 1.

To obtain precise measurements, calibration (energy per division) of the oscilloscope screen was necessary for each series of stacking efficiency measurements. This calibration was made by scanning single pulses accelerated to different energies, which were accurately determined by an RF frequency measurement.

To calculate the stacking efficiency, the total area A_{tot} of the density distribution and the maximum area $(A_{id})_{max}$ of the density distribution within the energy interval ΔE_{id}, given by the definition above,



n = 30
(2.2 Ampère)

n = 50
(3.3 Ampère)

n = 70
(4.8 Ampère)

Fig. 1. Density distributions in the repetitive stacking scheme with $\Gamma = 0.5$ and different numbers n of cycles.

are measured. The stacking efficiency is then obtained as

$$\eta = \frac{(A_{id})_{max}}{A_{tot}}.$$

The area measurements were carried out on enlarged photographs of the density distributions which made it possible to reduce the errors in the stacking efficiency measurement to less than 5%.

The measured efficiencies are given in Fig. 2 for different amplitudes U of the RF voltage, keeping Γ and n constant, and in Figs. 3, 4 and 5 as function of the number n of stacking cycles for different energies and phase parameters. The amplitudes U of the RF voltage and the resulting bucket areas in the $(E/m_0c^2, \phi_{RF})$ plane are given in Table 1.

Table 1

Γ	$\alpha(\Gamma)$	U/kV	A_{RF}
0.30	0.539	0.61	0.0256
0.50	0.333	1.60	0.0256
0.66	0.198	4.53	0.0256
0.80	0.099	16.00	0.0241

The computation of the stacking efficiency

A simulation of the stacking process on a computer[1,2] allows the stacking efficiency for various input parameters to be calculated. In the computer model the stacking region is subdivided into N labelled energy channels with equal width ΔE_c in energy, one of which is filled with test particles. The change in the distribution of these particles due to the moving RF buckets is computed in the form of a histogram constructed on the energy channels at each time t_i, the RF buckets cross the centre of an energy channel E_i. There are then N of these histograms. Assuming that all N histograms represent adequately the energy distribution of test particles at the times t_i and that the histograms depend only on the difference between E_i and the energy of the test particles, the effect on any distribution of particles can be studied by superposition. This is done by generating a matrix A from the normalized histograms. The effect of the moving bucket can then be described by the algorithm

$$V_n = A\,V_{n-1} + V_b,$$

with V_n the distribution of particles after the nth cycle, V_{n-1} the distribution before the nth cycle and V_b the distribution of the particles brought up in the buckets. With the definition of the ideal stack width, the computed stacking efficiency is found to be after n cycles

$$\eta_n = \frac{1}{n}\,\max_i \left\{ \sum_{k=i}^{i+\lambda n-1} (V_n)_k \right\}.$$

Through the operation $\max_i$, the absolute position of ΔE_{id} is not taken into account, which makes it easy to compare measured and calculated efficiencies.

The parameter λ, which has to be an integer, is the ratio of the width in energy of the RF buckets δE_b and the channel width ΔE_c.

All computations were performed for an energy around 26 GeV and typical ISR parameters. In order to reduce the computer time the computer model was scaled so that the trajectories in synchrotron phase space corresponded exactly to the trajectories in the ISR. The computed stacking efficiencies are plotted in Figs. 6 and 7.

Results

The results of the measurements indicate that the stacking efficiency is independent of energy for particles stacked well above transition energy (see Fig. 3). Furthermore, no dependence on RF voltage can be found (Fig. 2), which shows that the scattering of stacked particles due to the presence of moving RF buckets for Γ = constant is proportional to the area of the buckets.

The stacking efficiency is thus essentially a function of Γ and the number of stacking cycles.

For both stacking modes highest efficiencies are achieved for low values of Γ and large numbers of stacking cycles.

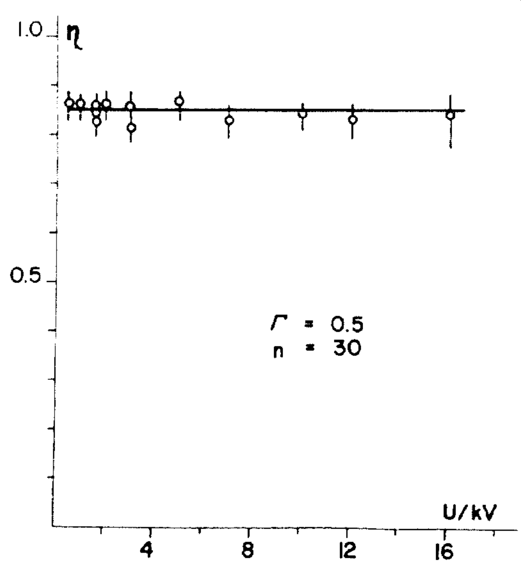


Fig. 2. Stacking efficiency measured at different amplitudes of the RF voltage in the repetitive scheme.

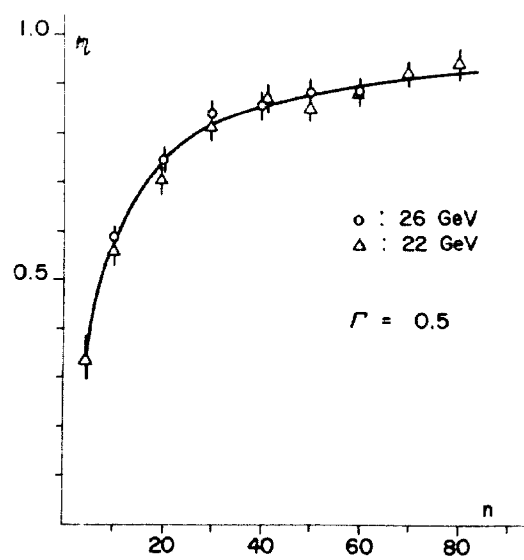


Fig. 3. Stacking efficiency measured at different energies in the repetitive scheme.

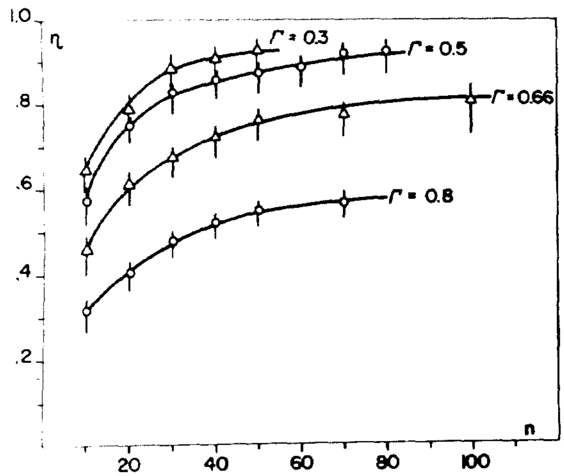


Fig. 4. Measured stacking efficiency in the repetitive scheme.

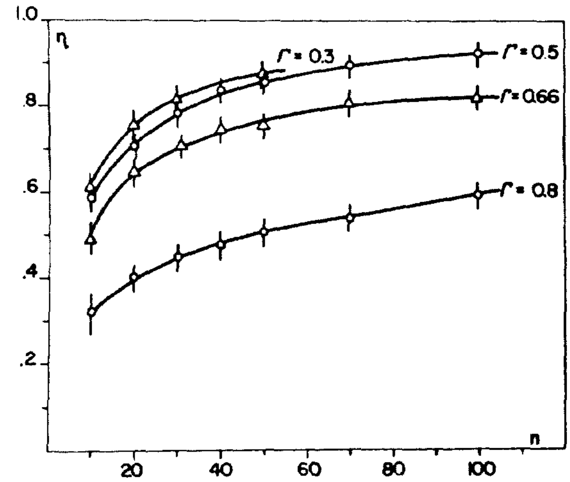


Fig. 5. Measured stacking efficiency in the non-repetitive scheme.

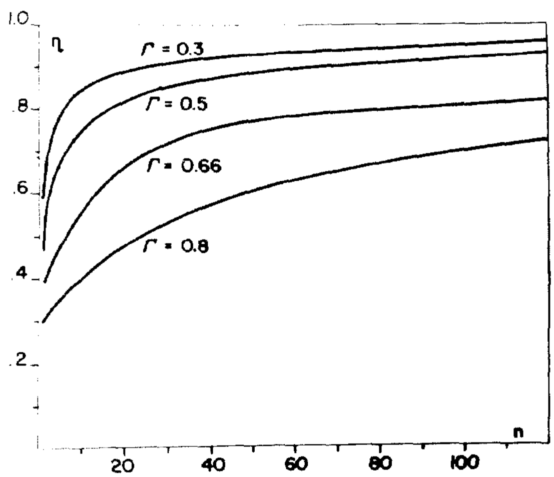


Fig. 6. Computed stacking efficiency in the repetitive scheme.

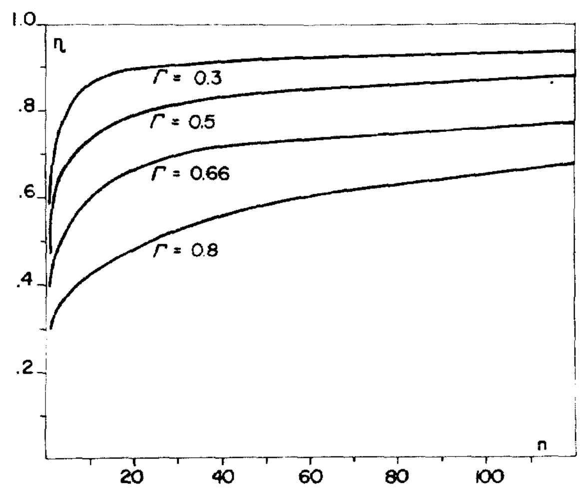


Fig. 7. Computed stacking efficiency in the non-repetitive scheme.

The agreement between theoretical and measured values of the stacking efficiency is very good for $n \geqslant 30$ as can be seen from comparison of Figs. 4, 6 and 5, 7 respectively. For lower values of n the measured efficiency is considerably lower than the theoretical value which comes mainly from the following effects.

(a) During the stacking process particles are released from the RF buckets at energies determined by a measurement of the momentaneous value of the RF frequency. The statistical fluctuations, as a result of system imperfections, in this frequency measurement, increase the stack width and hence reduce the efficiency. Clearly this effect is more pronounced for lower values of n.

(b) The empty buckets, swept through the stack to measure the density distribution, contribute to the scattering of the stacked particles as do the filled buckets during the stacking process. Obviously the scattering due to these empty buckets forms a larger percentage of the total scattering for low values of n.

Our measurements confirm qualitatively the results from the CERN electron storage ring.[3] A direct comparison is not possible, however, as slightly different parameters and definitions were used.

From phenomenological considerations[4] drawn from both the measurements and the computations a rule was derived describing the dependence of the stacking efficiency on Γ and n:

$$\eta(\Gamma,n) = \left[1 + \frac{2\Gamma}{3\sqrt{n}\cdot\alpha(\Gamma)}\right]^{-1} .$$

$\alpha(\Gamma)$ equals the bucket parameter.

With this rule a good approximation of the stacking efficiency is obtained for larger values of n for both the repetitive and non-repetitive stacking scheme.

Acknowledgements

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References

1. D.A. Swenson. "A study on the beam stacking process". International Conf. on High Energy Accelerators, Brookhaven, p. 187 (1961).

2. E. Keil and A.G. Ruggiero. "Computer programmes for RF accelerator studies", CERN internal report, ISR-TH/68-37 (1968).

3. CESAR Group. "The CERN Electron Storage and Accumulation Ring (CESAR, CERN report 68-8 (1968).

4. E.W. Messerschmid. "Wirkungsgrad der Akkumulation von Protonen in den longitudinalen Phasenraum der CERN-Speicherring", Thesis Bonn University 1972.