CALCULATIONS ON HIGH FIELD MAGNETS WITH IRON YOKES AND RECTANGULAR COILS

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Summary

Most studies on high-field bending magnets for superconducting synchrotrons or high energy beam transport in the last years concentrated on designs with a circular aperture and cos θ current distribution which is approximated by blocks of windings of stepwise decreasing current density. An alternative solution consists in the design of a magnet with square or rhomb shaped aperture which has the property that rectangular coils with only 2 steps of current density represent its exact solution. There exist also solutions with the same current density in all coils. The multipole field distortions of this class of magnets are studied in order to find suitable shapes of the outer iron shield.

Introduction

In the years 1964-1966 a type of high field magnet was proposed at CERN[1)2)3)] which is derived from the window frame magnet. It has flat poles in a square aperture and rectangular and square shaped coils but it yields good fields up to 3 Tesla while the good field ($\Delta B/B < 10^{-3}$) of a window frame magnet is limited to about 2.15 Tesla, the magnetization value of iron. The magnet in its simplest form is a superposition of the coils of two window frame magnets in one common square aperture as shown in Fig. 1. Either coil contributes one of the two perpendicular field components B_0 which add vectorially and yield a higher field in the aperture

$$B = \sqrt{2}\, B_0$$

than on the iron poles. If the flux density is 3 Tesla in the aperture, it is only $3/\sqrt{2} = 2.12$ T on the poles. The iron permeability at 3 Tesla is approx. $\mu = 3$, whereas at 2.12 Tesla it is still 30-50. Therefore, one obtains an order of magnitude of smaller sextupole field components in the 20-30kgauss range than with a similar window frame magnet.

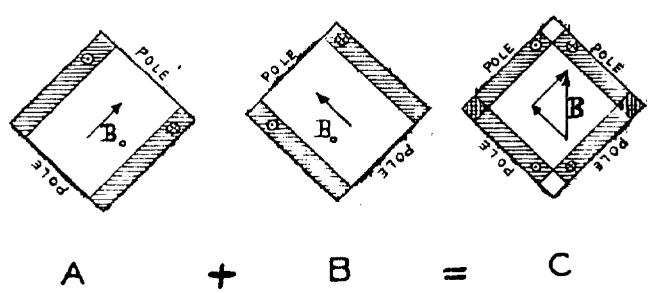

FIG. 1: PRINCIPLE OF THIS HIGH FIELD MAGNET

High field magnets generally require high current densities and still higher power consumption even in pulsed operation. However, with the rapid progress made in superconducting magnet design in recent years, current densities of 10.000A/cm^2 or more are perfectly feasible. Therefore, it seemed worth to reconsider the old design and model it on a computer in order to study quantitatively the effect of coil modifications and iron saturation, because such computer studies were not possible 9 years ago. Most other computer studies concentrated on superconducting magnets with circular aperture and a cosθ-azimuthal current distribution. These magnets yield good fields ($\Delta B/B < 10^{-3}$) up to about 4 Tesla. Nevertheless, even window frame magnets have been successfully built with superconductors[4)], although they require large sextupole field correction above 2.1T. The square high field magnet has similar coils but requires no sextupole field correction up to 3 Tesla.

Coil Configurations

a) Coils of two different current densities but same thickness.

When in Fig. 1 the two window frame magnets A and B are superposed, their coils overlap in the corners of magnet C. In the top and bottom corners current densities of opposite sign cancel. Thus there are no coils in these corners. In the right and left corner, there are square coils with twice the current density. Remember that in a circular aperture with coils of uniform thickness d a field H (Amp/cm) requires the current density

$$j = (H/d)\cos\theta \qquad (1)$$

which for $\theta = 45^o$ yields

$$j_1 = 0.707(H/d) \qquad (2)$$

This is the current density in the main rectangular coils of the square aperture magnet. The square coil block in the corner has the current density

$$j_2 = 1.414(H/d) \qquad (3)$$

Beth[5)] has shown analytically how the continuously varying cosθ current distribution can be approximated by N coil blocks with stepwise decreasing current densities. Here in the square aperture N = 2 represents already the exact solution. Other authors have approximated the cosθ current distribution by coil blocks of the same current density, but decreasing azimuthal dimensions or decreasing thickness. Such approximations are also possible in a square aperture:

b) Coils with same current density.

Fig. 2 shows the example of a magnet with 9x9 = 81cm^2 inner aperture and coils of d = 3.5cm. At 3 Tesla or 23873 ampere turns/cm the current density in the square coil is 9650A/cm^2. If one wants to have the same current density in the flat rectangular coils, it is sufficient to halve their thickness and gain space as shown in Fig. 3. The field around the center of a symmetric dipole can be expanded in even powers of the coordinates z = x + iy

$$B = B_c\,(1 + b_2 z^2 + b_4 z^4 + \ldots + b_{2n} z^{2n})$$

where b_2 is the sextupole, b_4 the decapole and b_{2n} the (4n + 2) multipole field term. The computer code MAGNET[6)] has been used to compute the field and make a harmonic analysis of the distortions which result from halving the coil thickness. It turns out that if the rectangular coil has half the thickness, it must be extended towards the empty upper and lower corners. Then the sextupole field coefficient b_2 can be eliminated. b_2 can also be eliminated (without lengthening) by an even smaller coil displacement towards the center. However, the best is to both lengthen and displace the coil by the amounts indicated in Fig. 3. Then both sextupole and 10-pole fields are eliminated ($b_2 = b_4 = 0$) and only 14-pole and higher multipoles are left:

$$b_6 = 1.0 \times 10^{-7}/\mathrm{cm}^6 \qquad b_8 = -3.2 \times 10^{-9}/\mathrm{cm}^8$$

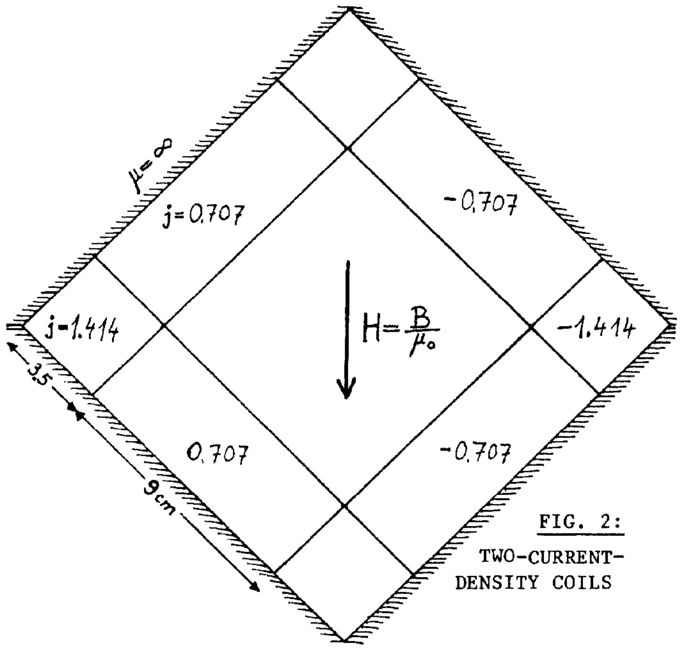

FIG. 2:
TWO-CURRENT-DENSITY COILS

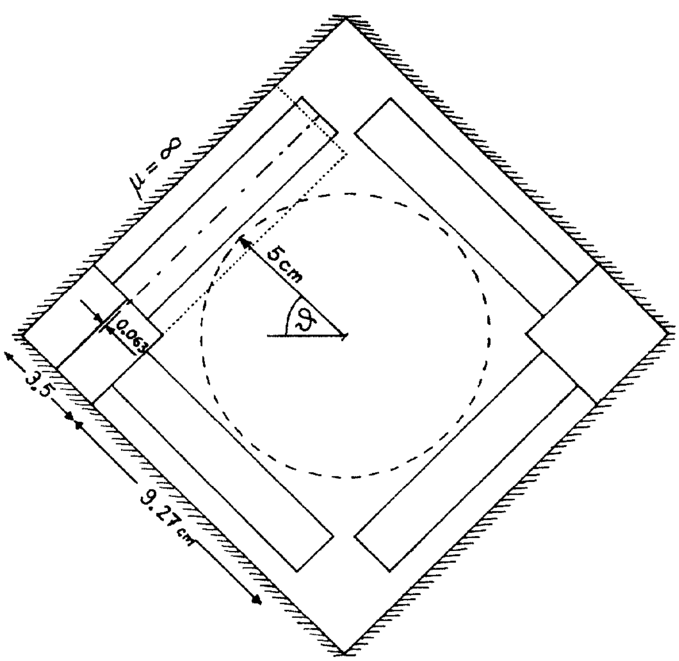

FIG. 3:
COILS OF EQUAL CURRENT DENSITY (NO SEXTUPOLE AND 10-POLE FIELD)

The field intensity in the 0°, 45° and 90° planes of the magnet in Fig.3 is plotted in Fig. 4 ($\mu_{iron} = \infty$).

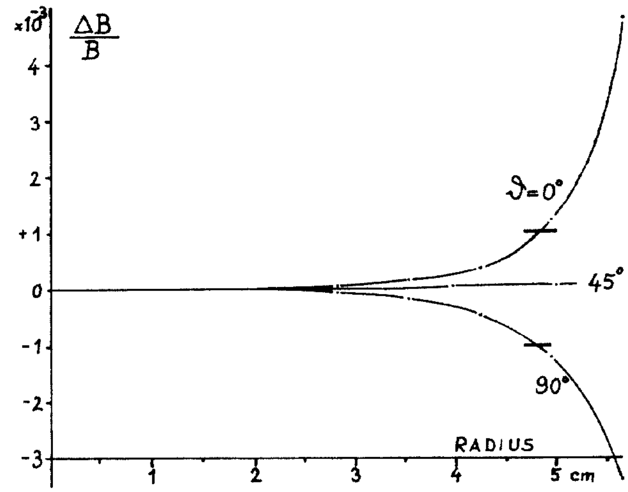

FIG. 4: FIELD HOMOGENEITY OF MAGNET IN FIG. 3

Effects of Pole Saturation

Let us assume that the coils in Fig. 2 are surrounded by a 1.8% Si-steel shield at 4.2°K which has the permeability curve published by Mc Inturff and Claus[7]. It is extrapolated to B > 22.3 kgauss assuming $B - \mu_0 H = 21.5$ kgauss. The steel extends up to a large outer radius of 25.5cm. Alternatively the high field magnet is converted (according to Fig. 1) into a window frame magnet with coils of the same thickness 3.5cm. In Fig. 5 the sextupole field distortions of the high field and the window frame magnet surrounded by the same large iron shield are compared at various levels. The two curves resemble, but the abscissa seems to be scaled by a factor 1.4, i.e. the sextupole term, due to pole saturation of the high field magnet at 3.4T, is the same as that of the window frame magnet at 2.47T and the sign of b_2 is reversed (45° coil rotation).

Almost the same result is obtained if the coils in Fig. 2 are replaced by the coils in Fig. 3 with equal current densities. Since the poles of this high field magnet are not opposed but adjacent, the return flux path is shorter and the iron on top and bottom can be removed and placed in the horizontal plane.

In this way one obtains a "figure of 8" iron shield of small reluctance which further diminishes b_2 (Fig. 6). More interesting than the sextupole term is the actual field at 30.1 kgauss in Fig. 7. One finds $\Delta B/B = \pm 6 \cdot 10^{-4}$ within 5.3cm aperture radius. This field homogeneity is obtained because the vertical sextupole field which reaches the maxima near the 4 corners is compensated by the high order multipole fields of the coils of equal current density (cf. Fig. 4).

These examples show that this type of an iron magnet can produce quite uniform fields up to 30 kgauss without a sextupole correction winding.

In this range the current density (of 97A/mm^2 in the last example Fig. 7) could be obtained either by superconducting or conventional water-cooled coils if the high power dissipation is accepted. Perhaps the fact that the coils have flat surfaces and right angles as in a window frame magnet may allow to manufacture and position them with higher precision.

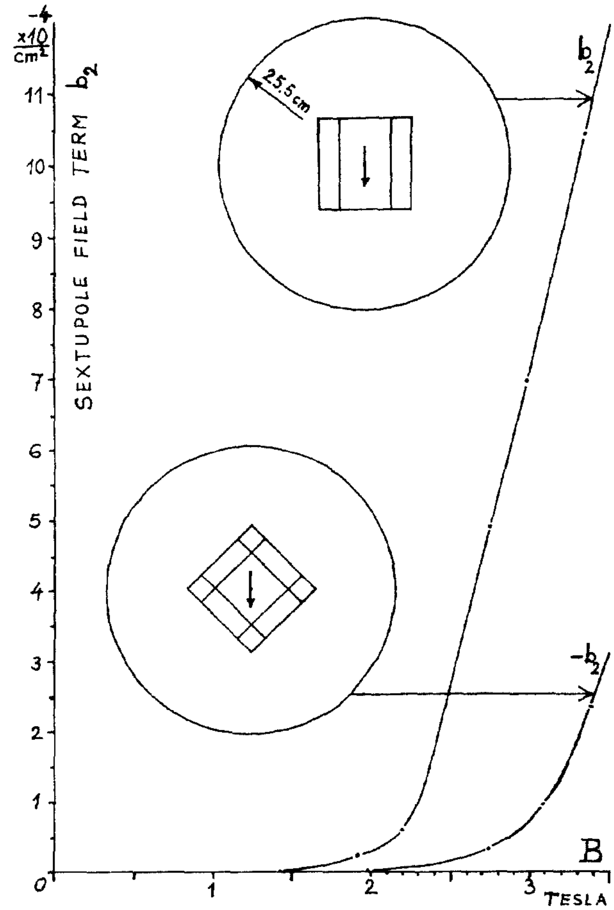

FIG. 5: COMPARISON OF WINDOW FRAME WITH HIGH FIELD MAGNET

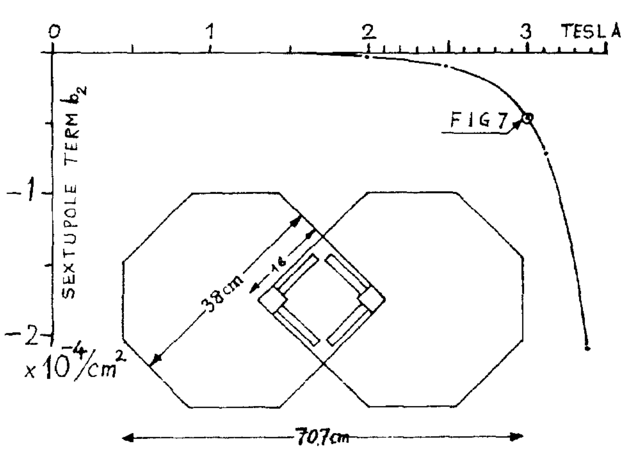

FIG. 6: SEXTUPOLE FIELD TERM OF HIGH FIELD MAGNET

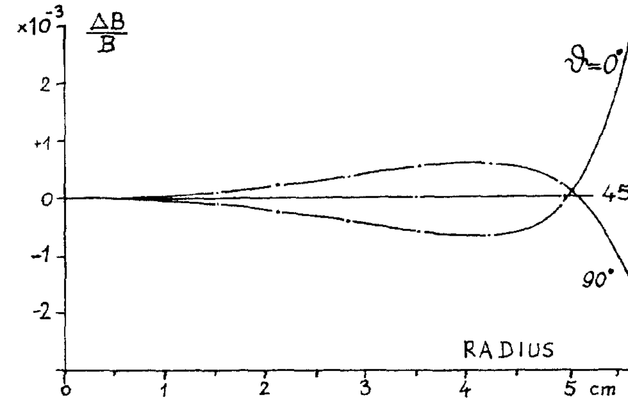

FIG. 7: FIELD HOMOGENEITY AT 30.1 KGAUSS

Outlook on Rhombic Apertures for Higher Fields

An essential feature of this high field magnet is that it has no poles on top and bottom but all 4 sides are poles on which the normal field component is smaller than in the aperture and uniform in order to achieve an approximately uniform magnetization (with the exception of the corners).

If the iron yoke is sufficiently large, it does not saturate before the flat poles. The flat rectangular coils which have an angle of inclination $\alpha = 45^\circ$ against the midplane bend the flux from the iron poles by this constant angle α. Thereby they concentrate the flux in the aperture. The square magnet is a special case of the more general lozenge or rhomb-shaped magnets[1,2,3]) which for $\alpha > 45^\circ$ achieve a higher flux density.

$$B = B_\rho / \cos\alpha \qquad (4a)$$

α can also be expressed by the aspect ratio of the vertical and horizontal diagonals of the rhomb aperture:

$$\frac{b}{a} = \mathrm{tg}\alpha$$

$$B = B_o \sqrt{1 + (b/a)^2} \qquad (4b)$$

We shall not discuss coil design but just mention that again there exists an exact solution[2]) which requires that the flat coils are rectangular and that the current density in the right and left corner is doubled whereas in the upper and lower corners it is decreased. If one assumes for example an aperture ratio 2:1 and operates such a magnet at 5 Tesla, then the flux density on the poles is $5/\sqrt{5} = 2.236$ Tesla and the permeability still > 20.

References

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7. A. Mc Inturff and J. Clauss - Proc. 3rd Int. Conf. Magnet Technol. (Hamburg 1970) p. 45, Table VI.