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# A PRECISION NON-LINEAR ANALOG TRANSFER FUNCTION GENERATOR*

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## Summary

A technique for generating driving waveforms having non-linear dependence on an independent variable is described. Forward biased high-conductance silicon diodes are used as feedback elements in an operational amplifier circuit. A FORTRAN time-sharing computer program has been written and used to aid in the selection of circuit components. A radio frequency program generator has been designed and built to provide the drive to a voltage-controlled oscillator as a function of the NAL main accelerator bend magnet current.

## Introduction

Diode function generators have long been used to obtain an output signal which is some non-linear function of an input signal. A popular technique is piecewise approximation of the non-linear function using an operational amplifier and a series of diodes and biasing networks to create breakpoints between which the function follows a straight line.[1] The accuracy of this approximation is determined by the number and placement of the breakpoints.

For many purposes, the breakpoint approximation is adequate, but the straight line segments do not provide a smooth function, and the generation of an extremely non-linear transfer function such as that shown in Figure 1 requires a great number of segments. The stability of the breakpoint function is proportional to the regulation of the reference voltages.

## General Characteristics

The function generator described herein was designed to provide the drive to a voltage-controlled oscillator as a function of the NAL main accelerator bend-magnet current to program the frequency of the rf system. It employs diodes as non-linear circuit elements, but in a manner that places them always in the forward conduction mode rather than in a reversed bias condition from which they are brought into conduction at a preset breakpoint level of the input signal. This mode of operation results in a low noise, smooth function without breakpoints that is insensitive to power supply variations. Since the forward I-V characteristic for a semiconductor diode is a function of temperature, the stability of this transfer function generator is dependent upon the circuit ambient temperature. For precision applications such as the example described here, the diodes in the circuit should be housed in a constant temperature oven.

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## Circuit Description

The basic circuit configuration is shown in Figure 2. Although this circuit is connected as an inverter to provide the particular function required in the example, non-inverting configurations can be used as well. In order to obtain the sharp knee required in the example, two stages of non-linear operation are used; the first due to the transformation of $V_{IN}$ to $V_D$ by the input diode, $D_{IN}$; the second by the non-linear feedback network around the operational amplifier.

In order for the diodes in the example circuit to always be operating in the forward mode as mentioned previously, it is required that $V_{OUT} \leq 0$, and that $V_{IN} \geq 0$, and that $V_{OUT} = 0$ for $I_0 + I_T = 0$. These restrictions can be accommodated by suitable translations of $V_{IN}$ or $V_{OUT}$ and the use of an offset current, $I_T$, into the summing junction of the op-amp.

## Analysis

This circuit may be analyzed mathematically to determine how well a given selection of components provides the required transfer function. A requirement for this analysis is a good determination of the forward I-V characteristic for the diodes to be used, either in tabular form or as an analytic expression. Figure 3 shows the forward I-V characteristic for typical 1N456 diodes, both at 25°C as specified by the manufacturer and at 65°C as measured in the component oven used for the example. Diodes were selected to be "typical" by having their forward voltage be close to that specified by the manufacturer at currents of 0.1 mA and 1.0 mA. A small number were then measured in the oven to yield the 65°C typical characteristic curve.

For a given circuit, the first step of the analysis is to find the op-amp input current, $I_0$, as a function of the input signal $V_{IN}$. This is accomplished by solving the set of equations:

$$I_D = I_{IN} - I_0$$

$$I_D = f(V_D)$$

$$I_{IN} = \frac{V_{IN} - V_D}{R_{IN}}$$

$$I_0 = \frac{V_D}{R_0} .$$

The output voltage can then be found by

solving the set of equations:

$$I_0 + I_T + \sum_{i=1}^{NB} I_{B_i} = 0$$

$$V_{OUT} = I_{B_1} R_1$$

$$V_{OUT} = I_{B_i} R_i + ND_i \times V_{D_i}, \quad 2 \leq i \leq NB$$

$$V_{D_i} = g(I_{B_i}); \; 2 \leq i \leq NB .$$

## Optimization

A FORTRAN program was written to perform the analysis described above, and a suitable circuit configuration to satisfy the required function shown in Figure 1 was obtained by starting with a guess and converging to an accuracy of less than 1% of the desired result by perturbing the circuit and noting the result. After a number of cases were run using the computer program to get a "feeling" for the technique, it was found that the design engineer could converge to an acceptable solution in a reasonable number of case studies using a time-share terminal. This technique would be most unsatisfactory in a batch-processing environment due to the long time lag between making the proposed change in the circuit parameters and viewing the effect. An obvious improvement in the procedure would result through the use of a computer program to automatically optimize the circuit, starting with some reasonable guess for the circuit parameters. A number of numerical minimization methods are available to accomplish this, but none have been employed to date in the solution of this problem, since the manual optimization technique yielded an acceptable result for the single example described here more quickly than the computer technique could be implemented.[2]

A graph of the calculated percentage difference between the circuit transfer function and the desired function after convergence is shown in Figure 4. The independent variable in the graph was chosen to be the op-amp input current as a convenience in expanding the horizontal axis near zero for studying the character of the error. As one can note, the error is contained within a ± 1% band over the required range of operation. The circuit which resulted from the manual convergence technique is shown in Figure 5. A function generator was built using the circuit as calculated and its measured transfer function is within 10 mV (0.5%) of the calculated function over the full range of operation. The 1N456 diodes, their series resistors, and 3 reference diodes used to provide stable sources for translation currents and voltages are housed in a small component oven operating at 65°C (chosen as optimum for the reference diodes); all other components are mounted outside the oven. Due to the technique used in characterizing the diodes for the circuit analysis, selection of diodes to be used in the function generator only required finding units that had forward voltages close to the manufacturer's specified value at two currents at room temperature. Long-term stability (> 30 days) is ± 2 mV in the output voltage for a given input voltage.

As one can note from Figure 1, the required function in the example approaches an asymptote for high values of magnet current, but the error curve in Figure 4 indicates increasing error due to the fact that the differential feedback impedance cannot be made infinitesimally small without an infinite number of parallel diode branches. Since the NAL accelerator is to be used at energies higher than the 200 GeV level corresponding to the last point on the curve of Figure 4, a voltage comparator was added to the input of the function generator to give constant output for inputs corresponding to magnet currents above 1,500 amps. The resulting error function is indicated by the circled points in Figure 4.


## Acknowledgements

The original concept on which the function generator described herein is based was provided by Quentin A. Kerns. The author wishes to thank him also for the many discussions and encouragements provided during the course of the project. I also wish to thank Allen Fogle for his assistance in construction and extensive testing of the frequency program generator.


## References


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[2] S. W. Director, Survey of Circuit-Oriented Optimization Techniques, pp. 3-10, IEEE Transactions on Circuit Theory, CT-18, No. 1, January, 1971.


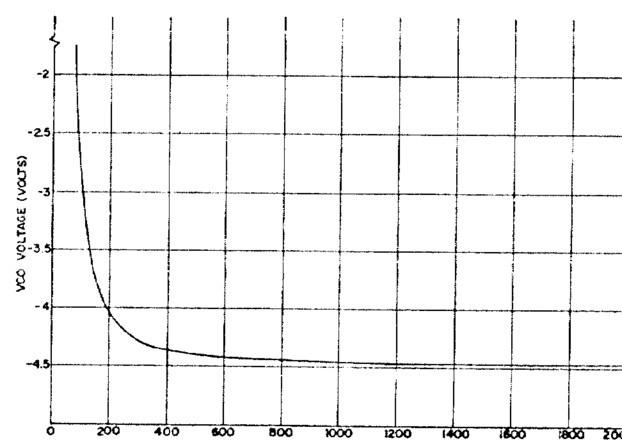


Figure 1. Radio Frequency Voltage-Controlled Oscillator Drive Required as a Function of Main Accelerator Bend Magnet Current.

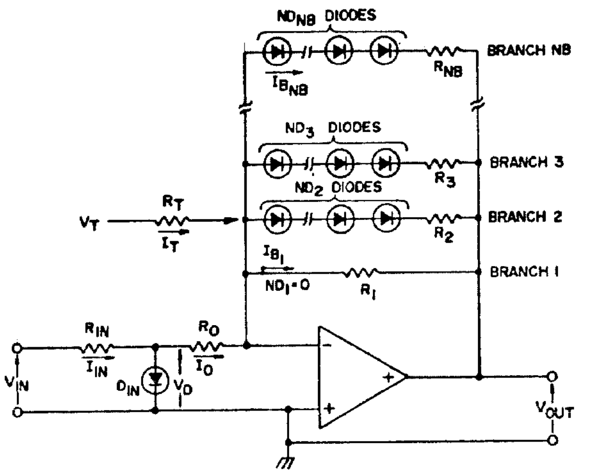


Figure 2. Basic Non-Linear Transfer Function Generator Utilizing Forward Biased Diodes.

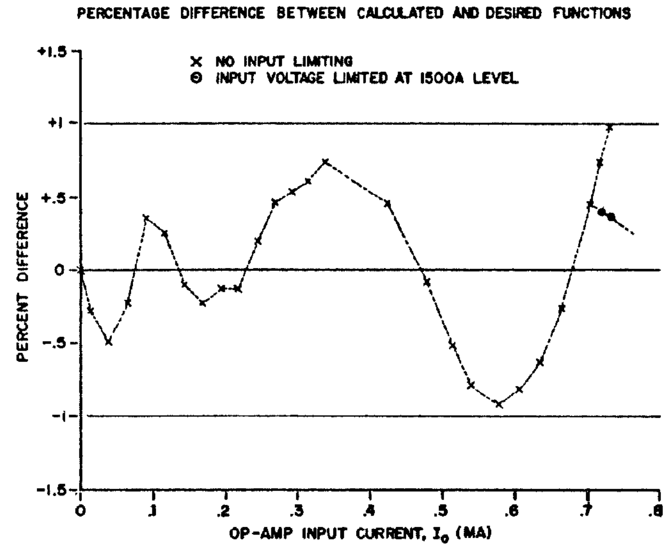


Figure 4. Percentage Difference Between the Circuit Performance and the Desired VCO Drive Voltage After Convergence.

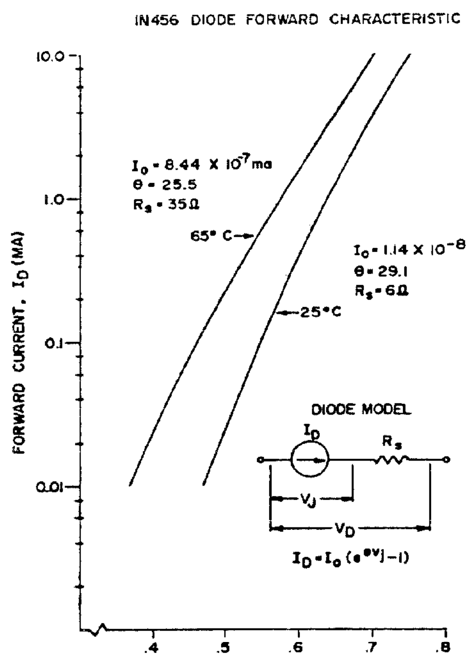


Figure 3. Forward I-V Characteristic For 1N456 Diodes with Diode Model and Fitting Functions Specified.

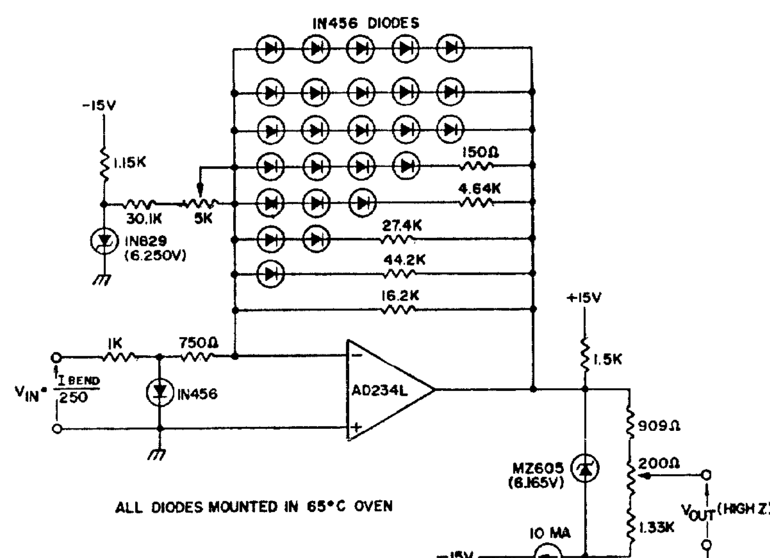


Figure 5. Function Generator to Provide Radio Frequency VCO Drive as a Function of Main Accelerator Bend Magnet Current.