BEAM TERMINOLOGY

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Summary

An analysis of a thermionic electron beam is presented and some deficiencies in the concepts of brightness and emittance are pointed out. Radiance and Richtstrahlwert are found to be better terms.

Introduction

As the definitions of both brightness and emittance found in technical dictionaries are inappropriate,[1] the meanings of the terms must be gleaned from the current literature. There it is learned that brightness, as a crude analogy to what we would now call radiance, can refer to an average value of beam current density per unit solid angle or else to an average beam current density in a volume of four dimensional phase space.[2] Depending upon how that volume is prescribed, these definitions need not result in the same value for identical beams.

Emittance can also be interpreted in several different ways. It usually represents a quantity $1/\pi$ times the area of the smallest ellipse which encompasses a beam cross-section in two dimensional phase space. However, it may be normalized to take care of relativistic effects or else modified to include the beam energy, thereby making it possible to compare beams of different particles.

Therefore to be specific the general terms brightness and emittance must be avoided. In their place we will refer to the R-star and the effective emittance.

R-Star

For particle beams a quantity R-star can be derived directly from the thermodynamic concept of radiance. Let W* represent the differential quantity, in the direction of a chief ray, that measures the directional-energy-flux-density or radiance of a beam of charged particles or light. Then:

$$W^* = \left.\frac{\partial^2 P}{\partial A\,\partial\Omega}\right|_{\Omega\to 0} \quad \text{Watts m}^{-2}\ \text{sr}^{-1} \tag{1}$$

in which P represents the beam power, A the area and Ω the solid angle surrounding a chief ray of the beam.

Meanwhile for charged particle beams, in a region of uniform potential V, R* is defined as

$$W^*/V = R^* = \left.\frac{\partial^2 I}{\partial A\,\partial\Omega}\right|_{\Omega\to 0} \quad \text{A m}^{-2}\ \text{sr}^{-1} \tag{2}$$

where I is the beam current and R* is a property of a chief ray in that beam.

The significance of R* can be appreciated by considering an idealized, axially symmetric, non-relativistic beam of electrons which is emitted from a thermionic source with a Maxwellian velocity distribution. For simplicity it is assumed that space charge forces are negligible and that the beam, lossless and scatter free, is focused by an electrostatic lens with no serious aberrations. As a result the value of R* will be the same for all of the chief rays. Hence it can be considered as a characteristic property of such a beam.

By definition, all of the chief rays meet at a common point in the focal plane. Furthermore as a consequence of the uniform emission over the cathode surface, they arrive at the focal point uniformly distributed throughout the solid angle Ω_x which encompasses them at the focal point. Thus the total current density at the focal point J_{max} is the sum of the contributions from all of the chief rays, and the average value of current density over solid angle of arrival, at the focal point, equals R*, i.e.

$$R^* = \frac{1}{\Omega_x}\left(\frac{\partial I}{\partial A}\right) = \left(\frac{J_{max}}{\Omega_x}\right) \quad \text{A m}^{-2}\ \text{sr}^{-1} \tag{3}$$

If r'_x is the angle at the focal point between the most extreme ray and the beam axis, then assuming small angles,

$$R^* = \frac{J_{max}}{\pi\,(r'_x)^2} \quad \text{A m}^{-2}\ \text{sr}^{-1} \tag{4}$$

It has been shown elsewhere[3,4] that an electron beam emitted from a thermionic cathode with a Maxwellian velocity distribution will have a current distribution in the focal plane which corresponds to a Gaussian function of radius. As a mathematical consequence of such a distribution the total current in the beam can be expressed as:

$$I = J_{max}\,\pi\,(r_o)^2 \quad \text{A} \tag{5}$$

where r_o corresponds to the radius in the focal plane which encircles 63% of the total beam current.

Substituting Equation 5 into 4, we find:

$$R^* = \frac{I}{(\pi\, r_o\, r'_x)^2} \quad \text{A m}^{-2}\ \text{sr}^{-1} \tag{6}$$

An expression which corresponds to the definition of the Richtstrahlwert as used in German literature.[5]

A similar argument can be presented in the plane of a cathode image where the maximum current per unit solid angle coincides with the chief rays that are uniformly distributed over the surface of the image. Hence:

$$R^* = \frac{1}{A}\left(\frac{\partial I}{\partial\Omega}\right)_{\Omega\to 0} \quad \text{A m}^{-2}\ \text{sr}^{-1} \tag{7}$$

and in a manner that is analogous to Equation 5, it can be shown that:

$$\left(\frac{\partial I}{\partial\Omega}\right)_{\Omega\to 0} = \frac{I}{\pi\,\theta_o^2} \quad \text{A sr}^{-1} \tag{8}$$

in which θ_o corresponds to that angle which encloses 63% of the current that arrives at any point on the surface of the image.

Thus at an image of radius r_i, $A = \pi\ (r_i)^2$ and:

$$R^* = \frac{I}{(\pi\ r_i\ \theta_o)^2} \qquad A\ m^{-2}\ sr^{-1} \qquad (9)$$

At a virtual image of the cathode assuming that the radius of the image is approximately equal to that of the cathode r_k it can be shown[4] that the angle of emission which encompasses 63% of the beam is equal to the ratio of the most probably transverse velocity resulting from the initial thermal energy of the source and the longitudinal velocity of the electron in the region of potential V. Hence at the virtual cathode:

$$\theta_o = (\frac{kT}{eV})^{1/2} \qquad rad \qquad (10)$$

in which k is the Boltzman Constant and T the cathode temperature. As a result:

$$R^* = \frac{I}{(\pi\ r_k)^2\ (\frac{kT}{eV})} \qquad A\ m^{-2}\ sr^{-1} \qquad (11)$$

It should be noted that normalization similar to that shown in equations 3 or 7 is not possible at any beam cross-section other than at a pupil (focus) or image.

Effective Emittance

Reviewing equations 6, 9 and 11, it can be seen that R* has a value which is invariant in the region of uniform potential. The current I, meanwhile, remains the same throughout the entire beam. Hence the denominators in all of these equations must be equal and can be considered as a volume in four dimensional phase space, or else as the square of π times an effective emittance E_e. Clearly the four volume and the emittance must also be beam invariants, as could have been concluded from the Liouville theorem. Thus the effective emittance can be expressed as:

$$E_e = \frac{1}{\pi}\ (I/R^*)^{1/2} = (r_o\ r'_x) \qquad m\ rad \qquad (12)$$

or

$$E_e = (r_i\ \theta_o) = r_k\ (\frac{kT}{eV})^{1/2} \qquad m\ rad \qquad (13)$$

and regardless of V,

$$E_e = \frac{1}{\pi}\ (\frac{P}{W^*})^{1/2} \qquad m\ rad. \qquad (14)$$

For a given source temperature E_e changes with r_k and therefore with the total current, where as R* and W* do not.

Conclusion

The effective emittance can only be defined for an ideal beam in which all of the chief rays have the same value of R*. As most beams are subject to lens aberrations, etc., this is rarely the case. Hence only specific threads or limited portions of a beam can be so described. In a region of uniform potential then, it is to be expected that the sum of the individual values of effective emittance for all of the component parts of a beam would be an invariant. Indeed these small assemblages when presented in phase space are what is meant by "the local density of points which remain constant" in accordance with the Liouville theorm. The area enclosed by any perimeter that includes these regions is not necessarily an invariant, and emittances, as usually measured, need not be invariant either.

We have no quarrel with the practical advantages of measuring emittance, nor with the application of that information toward the design of a subsequent piece of beam handling equipment. After all the change need not be great, but in a theoretical sense neither of the terms, brightness and emittance, are very satisfactory.

References

1. IEEE Standard Dictionary of Electrical and Electronic Terms, Wiley-Interscience, (1972).
2. A. van Steenbergen, IEEE Trans. on Nuclear Science, 12, #3, p. 746 (1965).
3. R. R. Law, Proc. IRE, 25, p. 954 (1937).
4. J. H. Fink and B. W. Schumacher, "The Anatomy of a Thermionic Electron Beam" to be published (1973).
5. B. von Borries and E. Ruska, Z. Techn Phys., 20, 255. (1939).

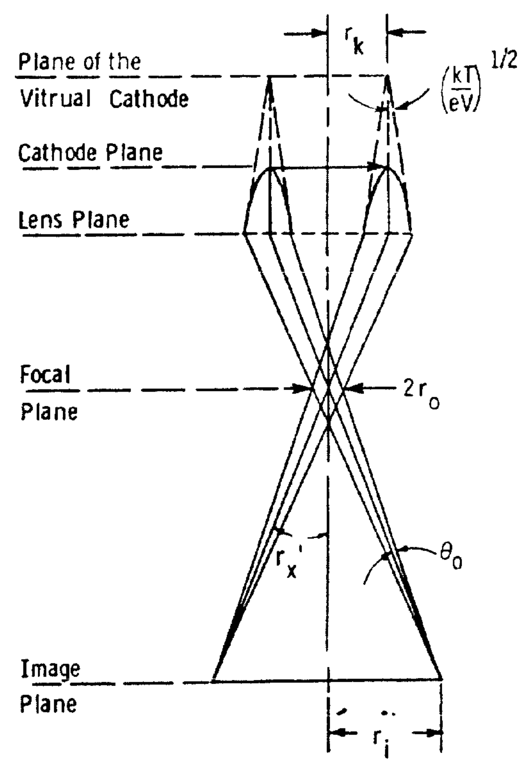

Figure 1—Electron Beam