RESONANCE INJECTION IN THE SUPERCONDUCTING STRETCHER RING*

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Introduction

The shortest pulse length of the beam extracted from the Zero Gradient Synchrotron (ZGS) is of the order of 50 μs so that multiturn injection into the stretcher ring[1] is needed. The conventional method of using a programmed dipole bumper magnet to move the equilibrium orbit away from the septum magnet results in a considerable dilution of the radial phase space. This dilution is very much reduced if a sextupole field is added to the perturbation. Because of this nonlinear field, the radial phase space is divided into two regions. The region inside the separatrix is stable. The extent of this stable region is determined by the strength of the dipole and sextupole fields. Initially, the dipole field is large enough to cause this stable region to disappear completely. During injection, the dipole field is steadily reduced so that the stable region reappears and increases in size. By an appropriately located septum magnet, the particles are injected in the neighborhood of an "incoming" trajectory of the separatrix. Depending on the initial conditions, the particles are captured in the expanding stable region.

Transformation Equations

The perturbation magnetic fields can be written in the form

$$b_y(x, o, z) = b(z) + \frac{1}{2}\frac{\partial^2 b}{\partial x^2} x^2 \quad .$$

These fields occupy only a small fraction of the ring circumference ($=2\pi R$) so that the effects can be expressed in terms of a periodic θ function. With this assumption, one can write the equation of motion in the form

$$x'' + K(\theta)x = -(a + \frac{\varepsilon x^2}{2})\,\delta(\theta) \tag{1}$$

where primes denote differentiation with respect to θ, where $\theta = z/R$, z = the distance along the equilibrium orbit, $\delta(\theta)$ = a periodic Dirac delta function with period 2π. Note that $\theta = 0$ is chosen at the center of the resonance injection magnet. The quantities a and ε are given by

$$a = \int_{-\pi}^{+\pi} \frac{R^2}{B\rho}\, b\, d\theta = \frac{RL}{B\rho}\, b \quad ,$$

$$\varepsilon = \int_{-\pi}^{+\pi} \frac{R^2}{B\rho}\frac{\partial^2 b}{\partial k^2}\, d\theta = \frac{RL}{B\rho}\frac{\partial^2 b}{\partial x^2} \quad ,$$

where L = the effective length of the resonance injection magnets, $B\rho$ = the magnetic rigidity of the particle. Equation (1) can be rewritten in the form of two simultaneous first-order differential equations

$$x' = y$$
$$y' = -k(\theta)x - (a + \frac{1}{2}\varepsilon x^2)\,\delta(\theta) \quad . \tag{2}$$

It is convenient to examine the solution of Eq. (2) for $\theta = 2\pi n$, where n is an integer. The transformation equations are then given by

$$\begin{aligned} x_{n+1} &= (\cos\nu 2\pi + \alpha\sin\nu 2\pi)\, x_n \\ &\quad + \beta\sin\nu 2\pi\,(y_n - \frac{a}{2} - \frac{\varepsilon}{4}x_n^2) \\ y_{n+1} &= -\frac{a}{2} - \frac{\varepsilon}{4}x_{n+1}^2 - \frac{1+\alpha^2}{\beta}\sin\nu 2\pi\, x_n \\ &\quad + (\cos\nu 2\pi - \alpha\sin\nu 2\pi)(y_n - \frac{a}{2} - \frac{\varepsilon}{4}x_n^2) \end{aligned} \tag{3}$$

where ν = the number of betatron oscillations per revolution. The quantities α and β are defined by the transformation matrix for the unperturbed motion.[2]

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} \cos\nu 2\pi + \alpha\sin\nu 2\pi & \beta\sin\nu 2\pi \\ -\frac{1+\alpha^2}{\beta}\sin\nu 2\pi & \cos\nu 2\pi - \alpha\sin\nu 2\pi \end{pmatrix} \begin{pmatrix} x_n \\ y_n \end{pmatrix} .$$

In Eq. 3, setting $x_{n+1} = x_n$ and $y_{n+1} = y_n$ gives the fixed points. The coordinates of the unstable fixed point are

$$\begin{aligned} x_u &= -\frac{2\tan\nu\pi}{\beta\varepsilon} - \sqrt{\left(\frac{2\tan\nu\pi}{\beta\varepsilon}\right)^2 - \frac{2a}{\varepsilon}} \\ y_u &= -\frac{\alpha}{\beta}x_u \end{aligned} \tag{4}$$

and those of the stable fixed point are

$$\begin{aligned} x_s &= -\frac{2\tan\nu\pi}{\beta\varepsilon} + \sqrt{\left(\frac{2\tan\nu\pi}{\beta\varepsilon}\right)^2 - \frac{2a}{\varepsilon}} \\ y_s &= -\frac{\alpha}{\beta}x_s \quad . \end{aligned} \tag{5}$$

The stable and unstable fixed points coincide for

$$a = \frac{2}{\varepsilon}\left(\frac{\tan\nu\pi}{\beta}\right)^2 \quad . \tag{6}$$

In this case, the area of the stable region is zero.

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The Computer Simulation and Discussion of Results

The recursion Eqs. (3) have been programmed in the FORTRAN language for the SEL-810A computer. This has an 8000-word memory, teletype input, line printer output, a CRT display, and a 1,000,000-word disc pack. The CRT has an 8 x 8-cm storage screen with a 256 x 256 matrix of points. Sense switches on the computer can be used to interrupt the program to change selected variables.

The values of (x_{n+1}, y_{n+1}) are calculated for given values of the parameters B_0L, ΔBL, $B''L$, ν, α, and β starting from the initial point (x_0, y_0). The dipole field is programmed to decrease linearly with the number of turns n so that

$$BL = B_0L - n\Delta BL \quad .$$

Since the values of (x_{n+1}, y_{n+1}) are calculated at the center of the resonance injection magnet, another point $(x_{n+1,\theta}, y_{n+1,\theta})$ is calculated for an angle θ from (x_{n+1}, y_{n+1}). The transformation equations are

$$\begin{aligned} x_{n+1,\theta} &= (\cos\nu\theta + \alpha\sin\nu\theta)\, x_{n+1} \\ &\quad + \beta\sin\nu\theta \left(y_{n+1} - \frac{a}{2} - \frac{\epsilon}{4} x_{n+1}^2\right) \\ y_{n+1,\theta} &= -\frac{(1+\alpha^2)}{\beta}\,(\sin\nu\theta)\, x_{n+1} \\ &\quad + (\cos\nu\theta - \alpha\sin\nu\theta)\left(y_{n+1} - \frac{a}{2} - \frac{\epsilon}{4} x_{n+1}^2\right) \end{aligned} \tag{7}$$

for θ equal to a multiple of the superperiod, because α and β are periodic functions of θ.

The point (x_{n+1}, y_{n+1}) is calculated and plotted on the CRT where the origin and scale can be easily changed. Subsequent points are calculated and plotted until one of the coordinates has a magnitude $> 10^3$, or until the program is interrupted by setting a sense switch. This causes the program to pause until the operator types in the newly selected input variables. Any input variable not typed is not changed from the previous value. An option exists to print out the $(x_{n+1,\theta}, y_{n+1,\theta})$ values on the line printer. The stable and unstable fixed points can also be calculated and printed on the teletype.

Starting at a point close to the unstable fixed point, successive points are calculated and plotted (hundreds of machine turns) until a "solid" closed curve is produced as the separatrix, as shown in Fig. 1.

The "incoming" and "outgoing" parts of the curve are also plotted in this manner. The following parameters are constant for all of the figures.

$B\rho = 12.75$ GeV/c

$\nu = 5.1$

$B''L = 1.0$ (kG-ft)/in^2

$(BL)_c = 2.9784$ (kG-ft)

$\alpha = 0.9$

$\beta = 0.3$ (also $\beta = .15$ in Fig. 3)

$\theta = 3\pi/2$ ($\theta = 0$ for Fig. 1)

Fig. 1 Separatrix for $\theta = 0.0$

Figure 1 shows the form of the separatrix at the center of the resonance injection magnet for two different values of the dipole field. Figure 2 gives the same separatrix at the location of the septum magnet, 90° upstream of the resonance injection magnet.

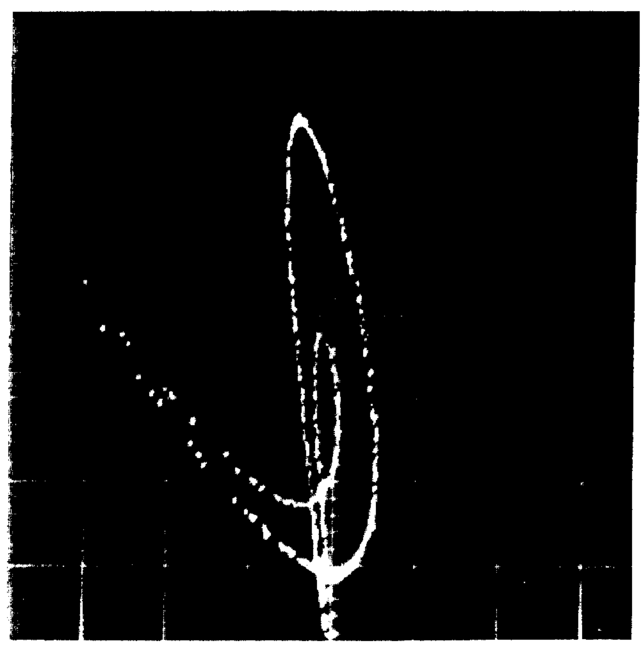

Fig. 2 Separatrix for $\theta = 3\pi/2$

Figure 3 shows the "jump" per turn as a function of the location of the particle in the septum magnet after the jump. Note that for a given x, the jump is independent of the dipole field value. This is necessary for a successful injection since the position of the injected beam in the septum magnet is fixed. Figure 4 shows the maximum acceptable divergence as a function of x for a 1% dipole field change per turn and several initial values of the dipole field. Figure 5 shows the required slope of the injected beam. This figure shows that the field of the septum magnet must be programmed to follow the change of slope. Figures 6 and 7 give the same results but for a dipole field change of 2% per turn. Figure 8 gives the instantaneous acceptance for $\Delta x = \pm 0.5$ in; $x = 2.50$ in as a function of the initial value of the dipole field.

Conclusion

The "strip" of injected particles is wound upon the emerging stable fixed point. Overlapping of the "strips" is not possible since one fixed trajectory could not be obtained from two different initial trajectories. Therefore, the phase space density of the injected beam will be, at best, identical to the original density, but more likely smaller. However, the dilution will be much smaller than in the case of the conventional multiturn injection.

References

[1] Superconducting (Beam) Stretcher Ring for the Zero Gradient Synchrotron, Argonne National Laboratory Proposal (December, 1971).

[2] E. D. Courant and H. S. Snyder, "Theory of Alternating Gradient Synchrotrons," Annals of Physics (1958), Vol. 3, p. 1.

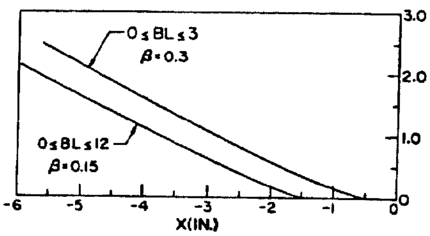

Fig. 3 Jump Vs. Position

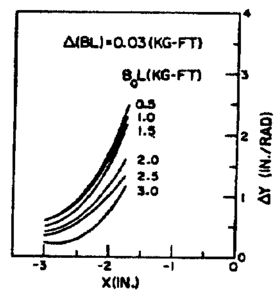

Fig. 4
Maximum Acceptable Divergence Vs. Position of the Beam (1% Dipole Field Change per Turn)

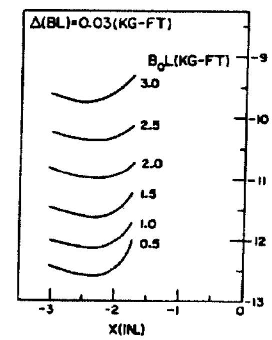

Fig. 5
Slope Vs. Position of the Beam (1% Dipole Field Change per Turn)

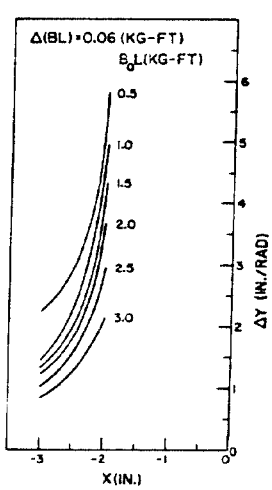

Fig. 6
Maximum Acceptable Divergence Vs. Position of the Beam (2% Dipole Field Change per Turn)

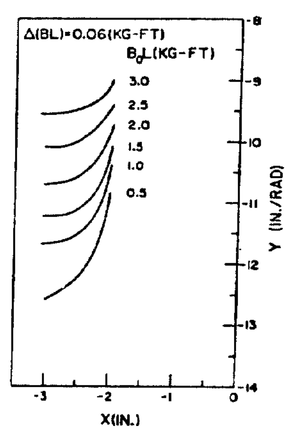

Fig. 7
Slope Vs. Position of the Beam (2% Dipole Field Change per Turn)

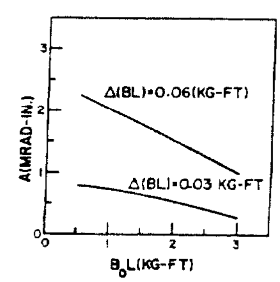

Fig. 8
Instantaneous Acceptance Vs. Initial Dipole Strength