

STOPPING OF AN ELECTRON RING BY INDUCED IMAGE CURRENTS IN RESISTIVE WIRE LOOPS \*

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Summary

The possibility of trapping an electron ring in the Maryland ERA by use of a few resistive wire loops was investigated numerically. It was found that a resistive wire of 45 ohm resistance and 404 nh self-inductance was able to stop electron rings of  $10^{13}$  electrons with initial axial velocities up to  $\beta_{z0} = 0.05$ . A scheme, with two resistive wires and a small local mirror field, allowed the trapping and focusing of electron rings of small axial dimension. Electron rings with relatively large axial lengths were also studied.

Introduction

In the University of Maryland ERA scheme <sup>1,2</sup>, a hollow axially-directed electron beam is sent through a cusped magnetic field converting axial velocity to azimuthal velocity. After compression to its final radius, the beam forms a slowly drifting ( $\beta_{z0} = 0.03$ ), rotating electron shell. The extended ring then has to be stabilized against axial expansion. Two methods are being investigated to trap the ER for a short period of time and allow slow ion loading. (1) Trapping of the ER between two mirror coils, where the upstream coil is triggered after the ring has passed<sup>2</sup>. (2) Trapping by a resistive wire scheme similar to the Astron experiment<sup>3,4</sup>. This is the method that will be investigated in this paper. Some preliminary work on this has been done by Linhart<sup>5</sup>.

Theory

Consider the scheme shown in Fig. 1. A thin electron ring (ER) in a constant external field  $\underline{B}_0 = B_0 \hat{a}_z$  moves with axial velocity  $v_z = \beta_z c$  towards a resistive wire loop. The moving magnetic self-field  $\underline{B}_s$  of the ER induces an electromotive force in the resistive wire which drives a current  $I_c$ . The two loop currents are in opposite directions and the resulting magnetic repulsion decelerates the ER.

We take the incident ER to be an infinitesimally-thin charged current loop with major radius  $r_e$  and containing  $N_e$  electrons each with kinetic energy  $\mathcal{E}_T = m_e c^2 (\gamma - 1)$ . In the interaction, we make the approximations that  $r_e = \text{const.}$ ,  $\beta_\theta = \text{const.}$ ,  $\sqrt{\gamma^2 - 1/\gamma}$  and  $\beta_z \ll \beta_\theta$ . Then the equations of motion for the system are

$$\frac{d\beta_z}{dt} = \frac{e\beta_\theta}{m_e \gamma} I_c b_r(r_c, r_e, z) \tag{1}$$

$$V_c = R I_c + L \frac{dI_c}{dt}$$

where the induced voltage in the resistive wire is

$$V_c = -ec^2 N_e \beta_\theta \beta_z b_r(r_c, r_e, z) \tag{2}$$

and  $b_r(r_c, r_e, z)$  is the radial component of the B field of the resistive wire per unit current. The resistive wire loop has a major radius  $r_c$ , resistance  $R$ , and self-inductance  $L$ . If  $a_c$  is its minor radius then  $L$  can be calculated from  $L = \mu_0 r_c (\ln(8r_c/a_c) - 1.75)$ .

The electron ring loses axial kinetic energy to resistive heating in the wire, as shown by eliminating  $b_r$  from Eqs. (1) and (2).

$$\frac{d}{dt} \left( \frac{1}{2} N_e m_e c^2 \gamma \beta_z^2 + \frac{1}{2} L I_c^2 \right) + R I_c^2 = 0$$

Integration of the equations of motion can be carried out readily in some limiting cases. For  $r_e/r_c \ll 1$  we can approximate  $b_r$  by

$$b_r(r_c, r_e, z) = \frac{3}{4} \mu_0 \frac{r_e}{r_c} \frac{x}{(1+x^2)^{5/2}}$$

where  $x = z/r_c$ . This gives the following results:

(i) For a purely resistive wire ( $L=0$ ), the induced current is

$$I_c(x) = -\frac{3}{4} \mu_0 \frac{r_e}{r_c} \frac{ec^2 N_e \beta_\theta \beta_z x}{R (1+x^2)^{5/2}}$$

and the axial velocity as a function of position is

$$\beta_z(x) = \beta_{z0} - \beta \frac{R}{L} \left[ 1 + \frac{2}{\pi} \tan^{-1} x + \frac{2}{\pi} \text{sgn}(x) \right]$$

where

$$\beta \frac{R}{L} = \frac{45 \pi}{4096} \mu_0^2 \frac{r_e^2}{r_c^3} \frac{e^2 c N_e \beta_\theta^2}{m_e \gamma} \frac{1}{R}$$

$$g(x) = \frac{1}{1+x^2} + \frac{2}{3} \frac{1}{(1+x^2)^2} + \frac{3}{15} \frac{1}{(1+x^2)^3} - \frac{16}{5} \frac{1}{(1+x^2)^4}$$

and  $\beta_{z0}$  is the incident ER velocity. The resistive wire will then stop ER's at  $z \leq 0$  for all  $\beta_{z0} \leq \beta \frac{R}{L}$ .

(ii) For a purely inductive wire ( $R=0$ ) the induced current and axial velocity are given by

$$I_c(x) = \frac{3}{4} \frac{r_e}{r_c} \frac{ec N_e \beta_\theta \beta_z}{L} \frac{1}{(1+x^2)^{3/2}}$$

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$$\beta_z^2(x) = \beta_{z0}^2 - \beta_L^2 \frac{1}{(1+x)^2}$$

$$\text{where } \beta_L^2 = \frac{\mu_0^2}{16} \frac{r_e^2}{r_c^2} \frac{e^2 N_e \beta_\theta^2}{m_e \gamma^2} \frac{1}{L}$$

In this case the interaction is completely elastic. The ER is reflected for  $\beta_{z0} < \beta_L$  and transmitted for  $\beta_{z0} > \beta_L$ .

In practical cases, the resistive and inductive effects in the wire are both important. The inductance is given by the geometry and determines the maximum current in the resistive wire. The pure resistive wire calculation shows that the lower the resistance, the greater the energy loss. However, in the limit of  $R \rightarrow 0$ , the interaction becomes elastic. Intuitively, a good balance between the resistive and inductive effects is obtained by setting  $I_c (L=0) = I_c(R=0)$  at  $z = -r_c$ . This gives the requirement  $L/R = (2/3) r_c/v_{z0}$ . Alternatively, we can approximate  $b_r$  by a constant strength out to some distance  $|z_c| = 2r_c$ . This gives damped oscillatory solutions. Critical damping then gives the criterion

$$L/R = r_c/v_{z0} \quad (3)$$

This is similar to that obtained for the Astron Experiment<sup>3</sup>. In the next section we consider the results of numerical integration of Eqs. (1) and (2).

#### Numerical Integration

The equations of motion (1) and (2) were integrated numerically using a 4th Order Runge-Kutta method with the field  $b_r$  calculated from<sup>7</sup>

$$b_r(r_c, r_e, z) = \frac{\mu_0}{2\pi} \frac{z}{r_c \sqrt{(r_c+r_e)^2+z^2}} \frac{1}{2} \left[ -K + \frac{r_c^2+r_e^2+z^2}{(r_c-r_e)^2+z^2} E \right]$$

where K and E are the Complete Elliptic Integrals of the First and Second Kind with argument  $M = 4 r_c r_e / ((r_c+r_e)^2+z^2)$ .

The incident ER was taken to have  $W_T = 5$  Mev,  $r_e = 5$  cm, and  $N_e = 10^{13}$  electrons. The resistive wires had  $r_c = 8$  cm,  $R = 45 \Omega$ , and  $L = 404$  nh ( $a_c = 2$  mm). The following cases were considered.

(i) A single resistive wire loop at  $z=0$  stopped electron rings with  $\beta_{z0} \leq 0.05$ . The typical oscillation period was 85 ns, the damping time was 20 ns, and the rings were stopped around  $z = -8$  cm =  $-r_c$ . In the resistive wire ( $V_c$ ) max  $\sim 10$  KV and ( $I_c$ ) max  $\sim 100$  A. Fig. 2 shows the damped oscillations of the ER motion for different initial velocities  $\beta_{z0}$ .

For a given  $\beta_{z0}$ , there is a range of R, L values which will stop the electron ring. This is shown by the shaded region in Fig. 3, for  $\beta_{z0} = 0.05$  and 0.08. The dashed line gives the optimum (Eq.(3)) L/R slope for  $\beta_{z0} = 0.05$ . In the computer runs, L/R was optimised for the average velocity  $\beta_{z0} = 0.025$ . Though Eq.(3) did not give critical damping for this velocity due to the complicated form of  $b_r$ , it still appeared to give optimum stopping for the ER.

(ii) A second identical resistive wire, with negligible mutual inductance, was added at  $z = -10$  cm. In addition a small current-carrying coil ( $r_M = 16$  cm,  $I_M = -200$  At) was placed between them at  $z = -5$  cm. This coil provided a mirror field for bringing together spatially, different velocity components  $\beta_{z0}$ . The results are shown in Fig. 4 for  $\beta_{z0} = 0.01$  to 0.06. This appears to be the best trapping configuration. Increasing the number of resistive wires would require a larger mirror field and correspondingly larger oscillation amplitudes and damping time.

(iii) To simulate an ER with a significant axial length  $z_L$ , the incident cylindrical ring was divided into thin ring sections which were integrated separately. Self interactions within the ER were neglected. It was found that with increasing length more particles were lost from the front of the beam<sup>4</sup>, but with the major fraction still being trapped. In addition, there was a slower damping of axial oscillations within the trapped ER.

(iv) For an extended electron ring in free-space the Budker condition for stability,  $f > 1 - \beta_\theta^2$  (where f is the fraction of ions in the ER), breaks down in the axial direction. This is due to the fact that the self magnetic field drops off faster with distance than the self electric field. The total axial force on an electron at the end of the ER was calculated by integrating over the ring using the EM fields<sup>7,8</sup> of an infinitesimally-thin charged current loop. In Fig. 5 we have plotted the minimum f (fmin) needed for axial self-pinching as a function of the radial half-width a and the axial length  $z_L$ . If  $f < fmin$  the ER can still exhibit stability in the sense that it can break up into two or more stable rings<sup>9</sup>. This was verified in computer runs where the thin ring sections making up the ER were allowed to self interact. These cases did not include nearby conducting walls which will improve the stability requirements on f.

#### Conclusion

The trapping of a drifting electron ring having some axial velocity spread appears feasible with the use of two resistive wire loops and a small mirror field. If the electron ring has a significant axial spread, there will be loss of particles from the front of the ring and the damping time will be longer. Also, in the absence of nearby conducting walls, a larger fraction of ions will be needed for ion focusing of the extended ring.

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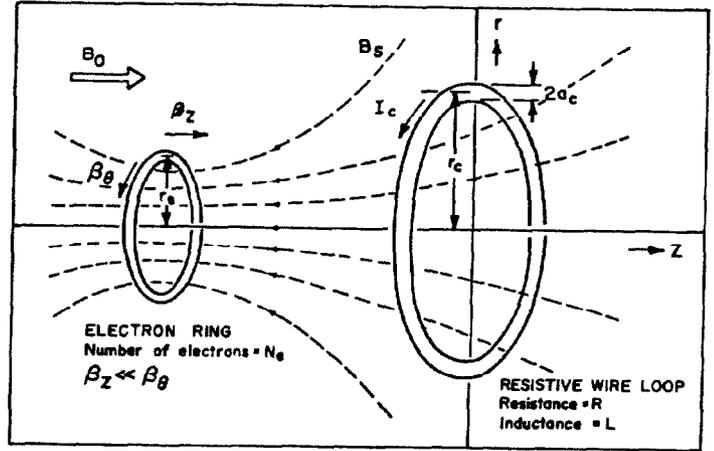


Fig. 1 Schematic of the interaction of a thin ER with a resistive wire loop.

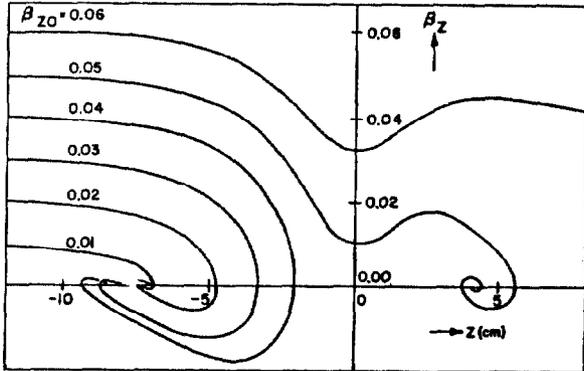


Fig. 2 Electron ring motion for different initial velocities  $\beta_{z0}$  under the influence of a resistive wire at  $z = 0$ . ( $W_T = 5$  Mev,  $r_e = 5$  cm,  $N_e = 10^{13}$ ,  $r_c = 8$  cm,  $R = 45 \Omega$ ,  $L = 404$  nh).

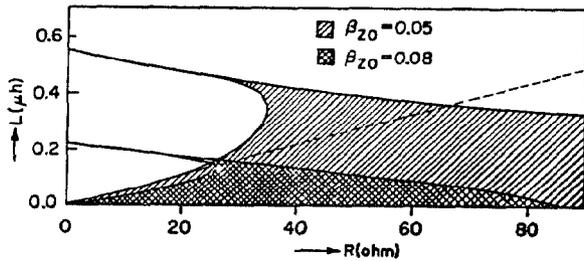


Fig. 3 The range (shaded) of R, L values of the resistive wire that will stop an ER with initial velocity  $\beta_{z0}$ .

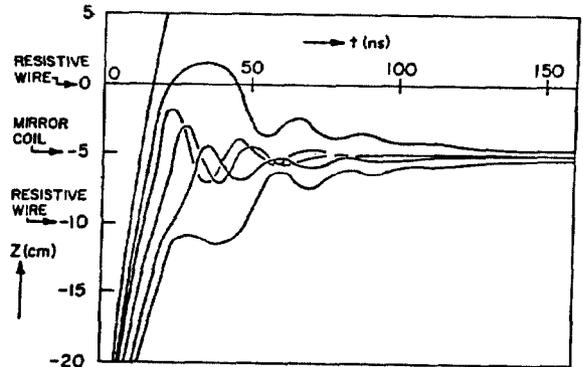


Fig. 4 Trapping and focusing of electron rings with different initial velocities  $\beta_{z0}$ .

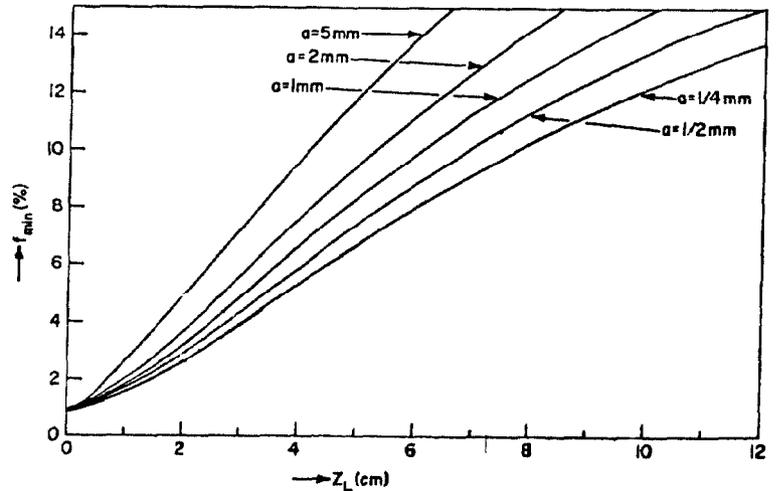


Fig. 5

The minimum fraction of ion ( $f_{min}$ ) needed for axial self-pinching for an extended electron ring of axial length  $z_L$  and radial half-width  $a$  ( $W_T = 5$  Mev,  $r_e = 5$  cm).