

ACCELERATION OF POLARIZED PROTONS IN THE ZERO GRADIENT SYNCHROTRON (ZGS)\*

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Introduction

With the advent of high current (6-12  $\mu$ A) and high polarization (75%) proton sources,<sup>1</sup> it became attractive to consider the acceleration of polarized protons to high energy (12 GeV). Such a source has been procured and a second preaccelerator with a beam transport line to the 50-MeV linac has been installed. A description of this system will appear in the conference proceedings.<sup>2</sup> Since acceleration of polarized protons to 50 MeV has been previously accomplished at the Rutherford Laboratory linac,<sup>3</sup> we will not go into this topic in the present paper. We will concentrate on explaining those factors which make the ZGS an excellent accelerator for minimizing the depolarization of an accelerating beam of polarized protons.

In accelerating to 12 GeV/c in the ZGS, calculations<sup>4,5</sup> show that there are ten major depolarizing resonances. As suggested by D. Cohen,<sup>5</sup> pulsed quadrupoles can be used to provide rapid tune shifts to avoid depolarization. Such a system has been designed and constructed for the ZGS and is described in the conference proceedings.<sup>6</sup>

In Section I, we describe the relevant ZGS parameters and the corrections required to minimize depolarization. In Section II, we describe the procedure to be used to measure the polarization of the extracted beam.

Section I:

The ZGS as an Accelerator of Polarized Protons

We will first list the features of the ZGS which contribute to reasonably small depolarization effects and then show how they quantitatively enter into the transition probability of depolarization.

1. The ZGS is an essentially weak focussing zero gradient machine. This assures us that the gradient is small enough so that the effect of interactions between the gradient and the magnetic moment of the particle on orbit can be neglected.
2. The change in vertical tune  $\Delta v_y$  as a function of radius is less than 0.01 at all fields and indeed has been made as small as 0.003. This results in a small nonlinear depolarization.
3. The value of the vertical tune  $v_y$  is less than one and, as we shall see, this enters into the transition probability as  $[1 - \exp(-\text{constant } v_y^{\frac{1}{2}})]$ . Thus, the resulting probability for spin flipping is small.
4. The ZGS has straight sections of different lengths. There are four long straight sections and four short straight sections. As we shall show, this leads to cancellation terms which again reduce the spin flip transition probability.

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The motion of the spin in the rest frame of the particle is given by

$$\frac{d\vec{s}}{dt'} = \frac{ge}{2m} \vec{s} \times \mathbf{v} \times \langle \vec{B}_y \rangle, \tag{1}$$

where

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}; \quad g = 5.58536; \quad t' = \text{proper time}; \quad \text{and}$$

$$\langle \vec{B}_y \rangle = \frac{1}{\oint dz} \int \vec{B}_y dz$$

is the average guide field in the laboratory frame. Clearly, the precession caused by the vertical field does not affect the axial polarization. Since the octant field gradient is very small, the only depolarization effects come from the horizontal field components at the magnet edges. Since the particles have a vertical betatron motion, they will experience rotating horizontal field components in crossing the magnet edges. These can contribute large depolarizing effects if their angular frequency is equal to the precession frequency.

In the laboratory frame, the horizontal components at the location of the particle are the x (radial) and z (longitudinal) components which can be written in the form

$$B_x = \frac{\partial B}{\partial y} y + \frac{\partial^2 B}{\partial_x \partial_y} xy + \dots \tag{2}$$

$$B_z = \frac{\partial B}{\partial y} z + \frac{\partial^2 B}{\partial_x \partial_y} xz + \dots$$

For the ZGS it is a good approximation to assume that the betatron oscillation is sinusoidal

$$y = A \cos(v_y \theta + \phi)$$

where

$$\theta = \frac{z}{R} \quad \text{with} \quad R = \frac{1}{2\pi} \oint dz \tag{3}$$

Substituting (3) into (2) and making use of the fact that curl B=0, neglecting higher order terms, and expanding

$$\frac{\partial B}{\partial x} \quad \text{and} \quad \frac{\partial B}{\partial z}$$

in Fourier series, we obtain

$$B_x = \frac{1}{2} A \sum_{k=0}^{\infty} a_k \cos \{ (k \pm v_y) \theta \pm \phi \}$$

$$B_z = \frac{1}{2} A \sum_{k=0}^{\infty} b_k \sin \{ (k \pm v_y) \theta \pm \phi \}$$

In the laboratory frame, the average angular velocity of the particle is given by the cyclotron frequency

$$\frac{d\theta}{dt} = \frac{e\langle B_y \rangle}{m\gamma}$$

Using this and going to the particle rest frame and adding the Thomas precession term, we obtain

$$b_x = \frac{\gamma A}{2} \sum_{k=0}^{\infty} a_k \cos \omega_k t'$$

$$b_z = \frac{A}{2} \sum_{k=0}^{\infty} b_k \sin \omega_k t'$$
(4)

where

$$\omega_k = (k \pm \nu_y + \gamma) \frac{e\langle B_y \rangle}{m}$$

and  $t'$  is the proper time. From (1), the precession frequency is

$$\omega_p = \frac{g}{2} \nu \frac{e\langle B_y \rangle}{m}$$

and the resonance condition is given by  $\omega_p = \omega_k$ , so that

$$\frac{g}{2} \gamma = k \pm \nu_y + \gamma$$

or

$$\left(\frac{g}{2} - 1\right)\gamma = k \pm \nu_y$$

Equation (4) can be written in terms of rotating fields

$$b_x = (b_+ + b_-) \cos \omega_k t'$$

$$b_z = (b_+ - b_-) \sin \omega_k t'$$

where

$$b_+ = \frac{A}{4} (\nu a_k + b_k)$$

$$b_- = \frac{A}{4} (\nu a_k - b_k)$$

The field  $b_+$  rotates clockwise and  $b_-$  counterclockwise. Since the spin precesses clockwise, only the  $b_+$  term has an effect. The  $b_-$  term gives  $\omega_k = -2\nu_p$  so that in every spin rotation the net depolarizing impulse will be zero.

Using either the classical mechanics of tops or quantum mechanics, we can solve for the probability of a transition from spin 1/2 to spin -1/2. This is given by

$$P_{\frac{1}{2}, -\frac{1}{2}} = 1 - \exp - \frac{(\nu_p r)^2}{2} [f^2(t) + g^2(t)]$$

where

$$f(t) \approx \int_{-t_0}^t \cos \frac{\lambda t^2}{2} dt, \quad g(t) \approx \int_{-t_0}^t \sin \frac{\lambda t^2}{2} dt$$

and

$$\lambda = \frac{d}{dt} (\omega_p - \omega_k) \quad \text{at} \quad \omega_p = \omega_k$$

The quantity  $r$  is given by

$$r = \frac{b_+}{\gamma \langle B_y \rangle} = \frac{A}{4 \langle B_y \rangle} \left( a_k + \frac{b_k}{\gamma} \right)$$
(5)

We can now show how the values of these machine parameters lead to a small transition probability.

The ZGS has four long straight sections and four short straight sections. Since the fields in the octants are uniform, we have only the horizontal components of the magnetic field at the edges. The edge angles are different for the long and short straight sections and for the short straight sections have a small energy dependence. Similarly, the length of the short straight section is slightly energy dependent. We characterize the edge angles by  $\delta_L$  and  $\delta_S$  and the straight section lengths by the  $\frac{1}{2}$  angles  $\theta_L$  and  $\theta_S$ . Let  $\vec{n}$  be a vector normal to the edge and we then have

$$\frac{\partial B_y}{\partial x} = \frac{\partial B_y}{\partial n} \sin \delta$$

$$\frac{\partial B_y}{\partial z} = \frac{\partial B_y}{\partial n} \cos \delta$$

Then the coefficients  $a_k$  and  $b_k$  are determined from

$$a_k = \frac{2}{\pi} \int_0^{\pi} \frac{\partial B_y}{\partial n} \sin \delta \cos k\theta d\theta$$

$$b_k = \frac{2}{\pi} \int_0^{\pi} \frac{\partial B_y}{\partial n} \cos \delta \sin k\theta d\theta$$

Let  $L$  = the effective length of the edge field and note that  $k$  is a multiple of four. For the ZGS, we then get

$$a_k = \frac{8}{\pi} \frac{\partial B_y}{\partial n} \frac{2 \sin \frac{kL}{2R}}{k} \left( \sin^2 \delta_L \cos k\theta_L + \sin^2 \delta_S \cos k\theta_S \right)$$

$$b_k = \frac{8}{\pi} \frac{\partial B_y}{\partial n} \frac{2 \sin \frac{kL}{2R}}{k} \left( -\cos^2 \delta_L \sin k\theta_L + \cos^2 \delta_S \sin k\theta_S \right)$$

Determining  $\frac{\partial B_y}{\partial n}$  from the equation of motion in the  $y$  direction, we find that

$$\frac{\partial B_y}{\partial n} = \frac{\pi \nu^2 \langle B_y \rangle}{4L(\sin \delta_L + \sin \delta_S)}$$

This then gives

$$a_k = \frac{2v^2 \langle B \rangle}{R} \frac{\sin \delta_L \cos k\theta_L + \sin \delta_S \cos k\theta_S}{\sin \delta_L + \sin \delta_S}$$

$$b_k = \frac{2v^2 \langle B \rangle}{R} \frac{-\cos \delta_L \sin k\theta_L + \cos \delta_S \sin k\theta_S}{\sin \delta_L + \sin \delta_S}$$

From Eq. (5) we thus see that  $r$  is proportional to  $v^2/y$ .

In the following table we then show the values of the above terms for the ZGS. In most cases we note that the fact that the straight sections have different lengths leads to smaller values of  $a_k$  and  $b_k$  and hence of  $r$  and  $P_{\frac{1}{2}, -\frac{1}{2}}$ .

$k$	$v$	$\sin \delta_L \cos k\theta_L$	$\sin \delta_S \cos k\theta_S$	$-\cos \delta_L \sin k\theta_L$	$\cos \delta_S \sin k\theta_S$	$\frac{a_k}{C}$	$\frac{b_k}{CY}$	$\frac{1}{C} \left( \frac{b_k}{a_k + \frac{b_k}{Y}} \right)$	$r$	$P_{\frac{1}{2}, -\frac{1}{2}}$
4	1.785	.20690	-.14266	-.41875	.33158	.06424	-.04883	+.01541	$4.1 \times 10^{-6}$	.00023
4	2.678	.20690	-.14623	-.41875	.31971	.06067	-.03698	.02369	5.1	.0008
8	4.016	.14436	.12486	-.75603	-.59388	.26922	-.33613	-.06691	17.9	.02
8	4.909	.14436	.12567	-.75603	-.59050	.27003	-.27430	-.00427	.9	.00008
12	6.248	.05373	-.09085	-.94624	.80618	-.03712	-.02242	-.05954	16.0	.042
12	7.140	.05373	-.09159	-.94624	.80374	-.03786	-.01996	-.05782	14.2	.042
16	8.479	-.04734	.04751	-.95239	-.94166	.00017	-.22338	-.22321	38.4	.366
16	9.371	-.04734	.04798	-.95239	-.94080	.00064	-.20203	-.20139	31.5	.312
20	10.710	-.13921	.00055	-.77326	.98740	-.13866	.01999	-.11867	23.3	.235
20	11.603	-.13921	.00055	-.77326	.98740	-.13866	.01846	-.12020	22.7	.258

Table I

As we can see from the table, the transition probability is reasonably small up to  $\gamma = 7.1$ . In order to go to higher energies, we must still reduce this depolarization. Using two quadrupoles, spaced  $180^\circ$  apart, each with a 25-in effective length and a gradient of 50 G/in for a total  $B'L = 2500$  G-in/in, we can achieve the following tune changes and transition probabilities. These quadrupoles are pulsed with a 10- $\mu$ s rise time and have a 2-ms flattop.

$k$	$d\nu_y/dn$	$P_{\frac{1}{2}, -\frac{1}{2}}$
4	.0071	$\sim 0$
4	.0042	$\sim 0$
8	.0027	$\sim 0$
8	.0022	$\sim 0$
12	.0017	.0002
12	.0015	.0002
16	.0012	.003
16	.0011	.003
20	.0009	.003
20	.0009	.003

Table II

There is thus a depolarization of about 2% from the resonances. There is also a small probability of spin flip when off of resonance, and calculations show this to be about 4% during the acceleration cycle. We additionally expect about 3% depolarization between the source and the ZGS and about 1% from the ZGS to the experimenters. We anticipate then a net polarization of 65% in our extracted beam of about  $10^9$  protons per pulse. We feel that the effects of magnet errors and/or misalignments will be smaller than the major contributions that we have considered.

## Section II:

### Extraction and Measurement of Polarization

In order to tune through the depolarizing resonances, we must be able to extract beam before and after the resonance and to measure the amount of depolarization. When a measurable depolarization occurs,

we will turn on the pulsed quadrupoles just before and move the pulse in time until we minimize the depolarization. This will be done on "front-porch" flattops set just before resonance and just after. Beam will be extracted using an energy loss target and with slow spill. It has been predicted theoretically by Wolfenstein<sup>7</sup> and shown experimentally in cyclotron experiments<sup>8</sup> that depolarization does not occur in any energy loss target provided the scattering angle is small. Extraction on these "front-porch" flattops will also be necessary for the high energy physics experiments since some will be done as a function of energy from about 3.0 GeV/c to 12.0 GeV/c.

To measure the polarization of the extracted beam, we are assembling a pair of double arm spectrometers which will measure the elastic proton-proton scattering of the polarized beam from a liquid hydrogen target (Fig. 1). We will measure the left-right asymmetry ( $A$ ) between the two spectrometers and use the  $p$ - $p$  asymmetry parameter ( $A_{pp}$ ) measured in other experiments to give the beam polarization ( $P$ ).

$$A = P A_{pp}$$

At each energy we will use a scattering angle where  $A_{pp}$  is measured and large. As can be seen in Fig. 2,

appreciable polarizations exist at all measured energies up to 17.5 GeV.

We should point out another interesting possibility. The source can also produce polarized deuterons. These can also be accelerated and extracted. The deuterons can then be passed through a stripper. The proton could then go through the polarimeter while the polarized neutrons, whose energy is well determined, would then provide a monoenergetic polarized neutron beam for high energy experimentation.

At the present time, there are four approved experiments and three proposals or letters of intent to use these facilities. These experiments propose investigating the following topics:

1. Measurement of total and differential elastic cross sections with the initial spin states parallel and antiparallel. This uses the polarized beam and a polarized target.
2. Measurement of the polarization of  $\Lambda$ 's produced by a polarized proton beam. This could lead to high intensity polarized hyperon beams.
3. Measurement of inclusive cross sections for both charged and neutral final states.
4. Study of resonance production in PP and PN interactions.
5. Study of parity violations in proton scattering processes.
6. Study of interactions of polarized protons in the 12-ft bubble chamber.
7. Polarization in proton neutron elastic scattering.

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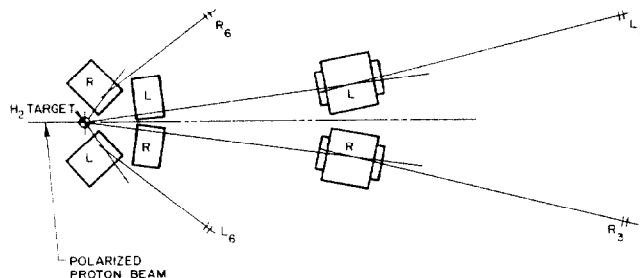


Fig. 1 High Energy Polarimeter Showing Right and Left Arms of Spectrometer for Detecting Asymmetry in Elastic Scattering of Polarized Protons

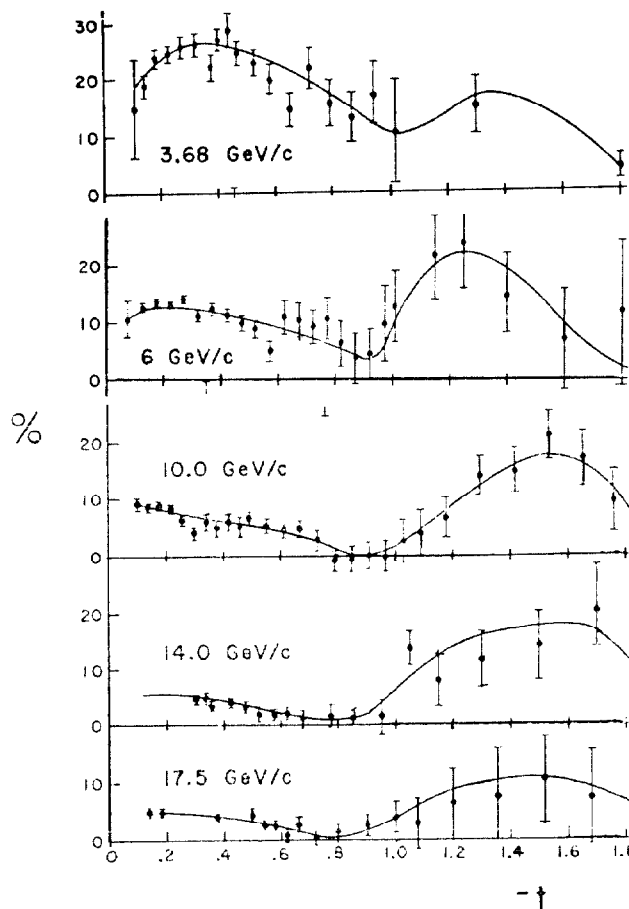


Fig. 2 Polarization Measurements in Elastic Proton-Proton Scattering