RESONANT FREQUENCY CONTROL OF SUPERCONDUCTING RF CAVITIES*

Peyton Z. Peebles, Jr.
University of Tennessee
and
Oak Ridge National Laboratory

Abstract

Resonant frequency of superconducting radio frequency (RF) cavities, which are used in linear accelerator research, may rapidly shift over many bandwidths because of vibration. In this paper an all electrical method is analyzed which can correct for these frequency shifts. The method depends upon the use of a voltage-variable reactance to control frequency. The general characteristics of control are first determined. Power distribution in the system is then found.

1. Introduction

In much of the present research, concerned with resonant radio frequency (RF) cavities for heavy ion linear accelerators, the cavity must have a small phase velocity relative to the speed of light. These velocities may be realized using cylindrical structures loaded by a helix.[1] However, a problem in helically-loaded cavities results from vibration of the helix. Resonant frequency may shift plus-and-minus several hundred hertz which ordinarily might be no problem. On the other hand, if the cavity is superconducting with a quality factor, Q_o, on the order of 10^8, these shifts represent several hundred bandwidths. Such shifts cannot be tolerated in linear, heavy ion accelerators and must be compensated. An all-electrical means for vibration compensation is the subject of this paper.

2. The General Control Method

A resonant RF cavity behaves quite similar to a single-tuned resonant circuit for frequencies near a cavity mode frequency. The effect of coupling to the cavity at any point can be modeled by a transformer.[2] The cavity and its coupled ports may then be represented as shown in Fig. 1(a). Transmission lines connected to ports 1 and 3 are assumed to be terminated in line-matched impedances R_{C1} and R_{C3} respectively. These ports in a system would carry excitation and response signals respectively of the cavity.

Port 2 is used to control frequency. The general method involves changing the resonant frequency ω_o by reflecting a reactance into the cavity. The reactance at port 2 is derived from a voltage-variable impedance Z_o which terminates an assumed lossless transmission line of length L. The impedance Z_o, called the line-load, is of the form

$$Z_o = R_o + jX_o, \tag{1}$$

where only the reactance X_o is assumed here to be a function of a control voltage and $j = \sqrt{-1}$ as usual. We shall assume that for small changes ΔX_o in X_o about a nominal value $\bar{X}_o$ that

$$X_o = \bar{X}_o + \Delta X_o = \bar{X}_o\,(1 + \delta) \tag{2}$$

where

$$\delta = \Delta X_o / \bar{X}_o \tag{3}$$

is the fractional change in line-load reactance.

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Cavity Load Admittance

Let Z_L be the impedance terminating port 2. Then for a lossless transmission line of length L having a phase constant β and characteristic impedance Z_{C2} the admittance is

$$\frac{1}{Z_L} = \frac{1}{R_L} - j\,\frac{1}{X_L} \tag{4}$$

where we find

$$\frac{R_{C2}}{X_L} = \frac{(R_{C2} - X_o y)(X_o + R_{C2} y) - R_o^2 y}{R_o^2 + (X_o + R_{C2} y)^2} \tag{5}$$

$$\frac{R_{C2}}{R_L} = \frac{R_{C2} R_o (1 + y)^2}{R_o^2 + (X_o + R_{C2} y)^2} \tag{6}$$

$$y = \tan(\beta L). \tag{7}$$

Circuit Solutions

We shall need the solution for the current i_2 of Fig. 1(a). All currents are easily found if certain assumptions are made which make interpretations easier. The primary effect of port 3 is to reflect a resistance into the cavity loop. If ωL_6 is small in relation to R_{C3} this resistance is $\omega^2 M_3^2 / R_{C3}$. Since frequency ω will never be far from its resonant value ω_o this resistance is approximately constant and equal to $\beta_3 R_s$ where β_3 is a coupling coefficient. In a similar manner we neglect all port-side inductances and define coupling coefficients for all ports by

$$\beta_k = \frac{\omega_o^2 M_k^2}{R_s R_{Ck}}, \quad k = 1,2,3. \tag{8}$$

Current solutions become

$$i_1 = \frac{(e_g/R_{C1})\left[R_s(1+\beta_3) + R_{con} + j\omega L - j\frac{1}{\omega C_s} + jX_{con}\right]}{R_s(1+\beta_1+\beta_3) + R_{con} + j\omega L - j\frac{1}{\omega C_s} + jX_{con}} \tag{9}$$

$$i_2 = \frac{-je_g\sqrt{\beta_1 R_s / R_{C1}}}{R_s(1+\beta_1+\beta_3) + R_{con} + j\omega L - j\frac{1}{\omega C_s} + jX_{con}} \tag{10}$$

$$i_3 = \frac{-e_g\sqrt{\beta_1/(\beta_2 R_{C1} R_{C2})}\,(R_{con} + jX_{con})}{R_s(1+\beta_1+\beta_3) + R_{con} + j\omega L - j\frac{1}{\omega C_s} + jX_{con}} \tag{11}$$

where $L = L_2 + L_3 + L_5 + L_s$ and

$$R_{con} = \frac{\beta_2 R_s R_{C2}}{R_L}, \quad X_{con} = \frac{-\beta_2 R_s R_{C2}}{X_L} \tag{12}$$

are control resistance and reactance as seen by the cavity loop due to port 2.

Fig. 1(b) illustrates the various impedances reflected into the cavity loop. In these figures

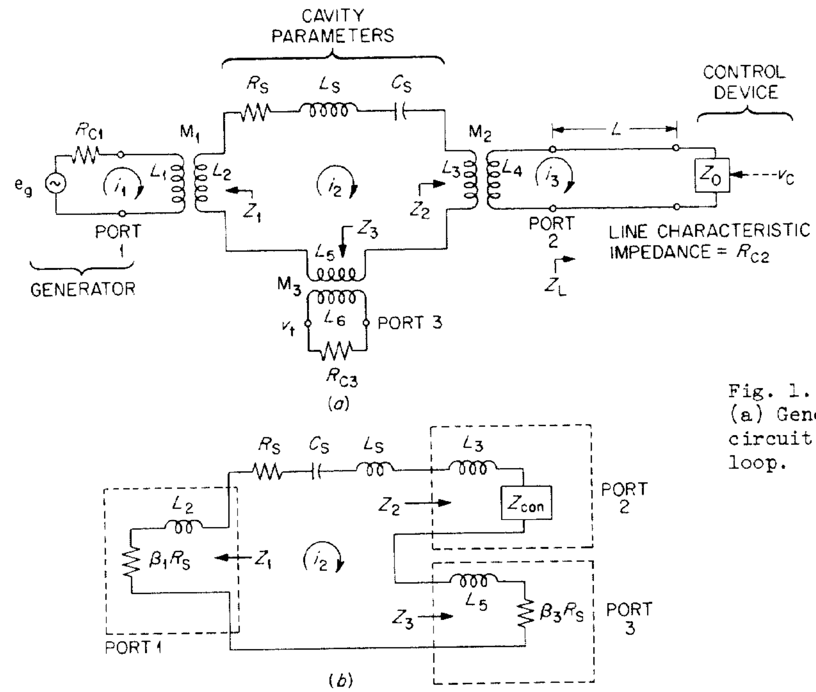

Fig. 1. Equivalent circuits. (a) General circuit and (b) circuit referred to cavity loop.

$$Z_{con} = R_{con} + jX_{con}. \qquad (13)$$

Frequency Control Equation

From Fig. 1(b) the undisturbed cavity resonant frequency ω_o occurs when $Z_{con} = 0$ and is $\omega_o = (LC_s)^{-1/2}$. When $Z_{con} \neq 0$, due to the control circuit, the resonant frequency in system operation shifts to a new value $\omega = \omega_o + \Delta\omega_o$ such that the loop reactance is zero. This result occurs when the denominator reactance equals zero in (10). Assuming $\Delta\omega_o \ll \omega_o$ and equating the reactance to zero gives

$$\frac{\Delta\omega_o}{\omega_o} = \frac{\beta_2}{2Q_o}\frac{R_{C2}}{X_L} \qquad (14)$$

where Q_o is the "unloaded" Q of the cavity $Q_o = \omega_o L/R_s$.

The result (14) is fundamental to all following results on frequency control. The fractional frequency control is related to the behavior of R_{C2}/X_L which is determined by the line-load behavior and line length L through (5) and (7).

3. Frequency Control Characteristics

Using (5) and (14) we obtain the general frequency control result

$$\frac{\Delta\omega_o}{\omega_o/Q_o} = \frac{\beta_2}{2}\,\frac{[1 - 2w(1+\delta)y][2w(1+\delta)+y] - z^2y}{z^2 + [2w(1+\delta)+y]^2} \qquad (15)$$

where for convenience we have defined

$$w = \overline{X}_o/(2R_{C2}), \quad z = R_o/R_{C2}. \qquad (16)$$

The important information to be gained from (15) is the behavior of $\Delta\omega_o Q_o/\omega_o$ as a function of δ for various w and z. Such results are possible if a reasonable method is found for selecting y, which amounts to choosing the line length L in Fig. 1(a).

The choice of y should lead to large values of control range (extremes of $\Delta\omega_o Q_o/\omega_o$) and large slopes of $\Delta\omega_o Q_o/\omega_o$ at the origin (where $\delta = 0$). To find reasonable values for y a number of special cases of (15) were examined depending upon whether $R_o = R_{C2}$, $R_o^2 \ll R_{C2}^2$ or $R_o = 0$. In every case evaluations indicated that a reasonable choice of y was one near

$$y = -2w. \qquad (17)$$

We now use (17) in (15) and note that the nominal frequency offset $\overline{\Delta\omega_o}$ given by

$$\frac{\overline{\Delta\omega_o}}{\omega_o/Q_o} = \beta_2 w \qquad (18)$$

(when $\delta = 0$) is of little importance since the cavity can be designed to give a desired resonant frequency after (18) is included. Subtraction of the offset gives the important relative control function:

$$\frac{\Delta\omega_o - \overline{\Delta\omega_o}}{\omega_o/Q_o} = \frac{\beta_2(1+4w^2)}{4z}\,\frac{2\left(\frac{2w\delta}{z}\right)}{1+\left(\frac{2w\delta}{z}\right)^2}. \qquad (19)$$

Fig. 2 shows a normalized plot of (19). The peak-to-peak frequency control range is

$$\left|\text{peak-to-peak}\left(\frac{\Delta\omega_o - \overline{\Delta\omega_o}}{\omega_o/Q_o}\right)\right| = \frac{\beta_2(1+4w^2)}{2z} = \frac{\beta_2(R_{C2}^2 + \overline{X}_o^2)}{2R_oR_{C2}}. \qquad (20)$$

From (20) we see that to achieve a large frequency control range one should select β_2 large, X_o large in relation to R_{C2}, R_o small, and R_{C2} small (making β_2 large).

4. Circuit Power Distribution

The currents found earlier may be used to determine the various average powers in the circuit. The usual powers of interest are: transmitted power P_{RC3}, cavity power P_{cavity} and control circuit power P_{con}.

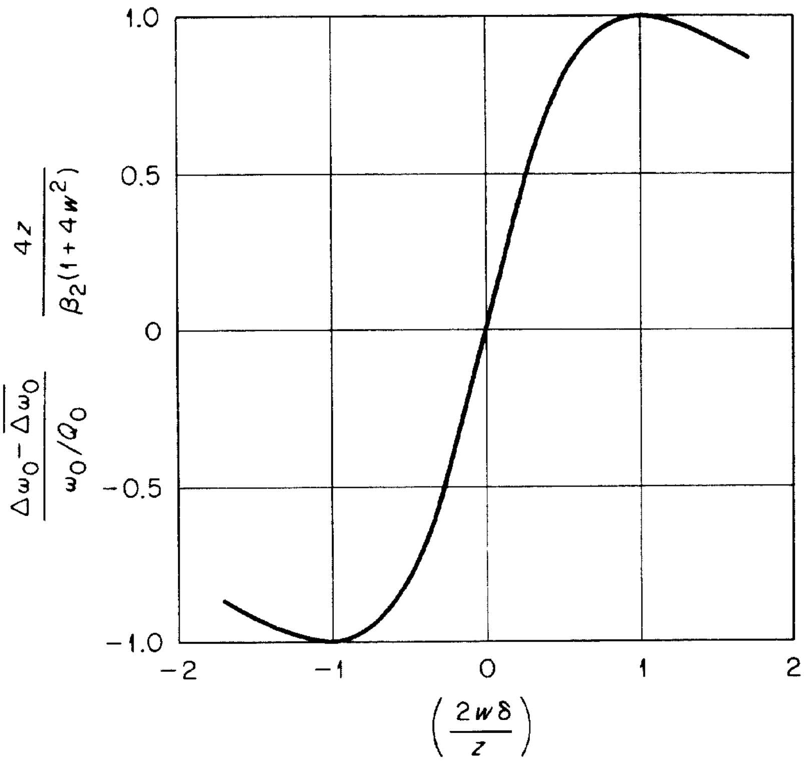

Fig. 2. Normalized frequency control function.

These are given by

$$P_{RC3} = \frac{|i_2|^2}{2}\beta_3 R_s \qquad (21)$$

$$P_{cavity} = \frac{|i_2|^2}{2} R_s \qquad (22)$$

$$P_{con} = \frac{|i_2|^2}{2} R_{con} \qquad (23)$$

using (10). Useful explicit expressions follow by noting that operating frequency will usually be such that $j\omega L - (j/\omega C_s) + jX_{con} \approx 0$ in (10) and the incident power P_{inc} to the cavity is

$$P_{inc} = \frac{|e_g|^2}{8\,R_{C1}}\,. \qquad (24)$$

From the explicit expressions it can be shown that for $\delta = 0$

$$\frac{P_{con}}{P_{cavity}} = 2\left|\text{peak-to-peak}\left(\frac{\Delta\omega_o - \overline{\Delta\omega_o}}{\omega_o/Q_o}\right)\right|. \qquad (25)$$

Thus, for large frequency control capability the control power $P_{con} \gg P_{cavity}$.

5. Discussion and Summary

An all-electrical means of controlling the resonant frequency of a superconducting cavity has been analyzed in this paper. Of special interest has been frequency control over many unloaded cavity bandwidths to compensate for vibration effects in helix loaded cavities.

The applicable circuit is given in Fig. 1(a). Analysis leads to the frequency control function of (19) which is plotted in Fig. 2. The origin of the plotted function corresponds to a fixed cavity offset frequency $\Delta\omega_o$ given by (18). The control curve applies to either inductive or capacitive line-loads. For inductive loads w is positive and for capacitive loads w is negative. The proper transmission line length L is found from (7) using (17).

Power distribution in the circuit was found and the individual powers are given by (21) through (24). The power required by the control device may be much larger than the power of the cavity, depending upon the amount of frequency control required, as given by (25).

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References

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