THE NRL CYCLOTRON ION OPTICS SYSTEM

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Summary

A 9 foot radius, 135 degree, double focusing magnet, having a second order energy resolution less than 2×10^{-4} for 1 mm slits, is the principal element in the NRL Cyclotron Ion Optics System. A 21.5 inch wide pole face and 4.75 inch gap provide radial and axial acceptance angles of 3.0 degrees and 1.3 degrees respectively. This 80 ton magnet can be remotely traversed on a tracking system making it possible to bring the energy-analyzed beam into either of two experimental vaults for use along 8 different beam paths. Also described is the coupling of quadrupoles with the analyzing magnet in a manner that provides a minimum dispersion system when the full cyclotron beam intensity is desired on the experimental target. Ray-tracing calculations have been performed to determine the properties of the composite ion optics system, and the influence on image quality of power supply fluctuations and quadrupole misalignment. The design criteria for the system, the calculational methods used in the ray-tracing programs, and the results of ray-tracing along typical paths is discussed.

Introduction

The NRL Cyclotron Facility has been developed to meet the need of extending existing experimental programs in nuclear research to higher energies. In order to do this effectively, we have designed an ion optics system to deliver a beam at an experiment that approaches Van de Graaff quality in image size, angular divergence, and energy resolution. In order to achieve the required energy resolution, we direct the beam from the cyclotron first through a high resolution beam preparation magnet, and then switch it to one of eight beam paths in two experimental vaults. The analyzing magnet is coupled with the quadrupoles and switching magnets in a manner that assures approximately unit magnification and consequently meets the objectives of small spot size and small angular divergence at the targets. The full beam intensity can be delivered to a third experimental vault without energy analysis for experiments requiring high intensity. Also, the full beam intensity can be realized at the other eight experimental stations with a minimum of beam dispersion from the energy analyzing magnet in a manner to be described.

Computational Techniques

It is convenient to calculate the properties of ion optical systems to first order using the matrix techniques developed by S. Penner [1]. A program has been written in NELIAC-N (NELIAC-N is the compiler langauge for NAREC, the NRL digital computer) to carry out ray tracing calculations using the Penner [1] techniques. The quantities of interest in ray-tracing are the displacement from the optic axis, the angle with respect to the optic axis, and the deviation in momentum from the average momentum of the beam. These quantities are to be traced in two perpendicular planes—the horizontal plane (containing all of the dispersive elements in the present case), and the vertical plane. One can either ray trace along the horizontal and the vertical planes separately using 3 x 3 matrices, or one can ray trace in both planes simultaneously using 6 x 6 matrices. The NRL program has been written using 3 x 3 matrices.

Table I

MATRIX FUNCTION PACKAGE

(a) F SPACE (X, M1;)
(b) QUAD C (k, ℓ , M1;)
(c) QUAD D (k, ℓ , M1;)
(d) MAG R (n, ϕ, r, e1, e2, M1;)
(e) MAG Z (n, ϕ, r, e1, e2, M1;)
(f) C QUAD MAX (k, ℓ, M1; z)
(g) D QUAD MAX (k, ℓ , M1; z)
(h) R MAG MAX (n, ϕ, r, e1, M1; z)
(i) Z MAG MAX (n, ϕ, r, e1, M1; z)
(j) VECTOR (x, θ, Δ p/p, M1;)
(k) MX ZERO (M1;)
(l) PUT (M1, M2;)
(m) MX MULT (M1, M2, M3;)
(n) ACCUM MULT (M3, M8, M20;)
(o) DETR (M1; X)
(p) NVERS (M1, M2;)
(q) REVERS (M1;)
(r) FOCAL PLANE (M1; x)
(s) PT (M1;)

The matrix function package used in the NRL program is displayed in Table I. A set of forty matrices, labeled from M1 to M40, is dimensioned. Each matrix is a list of ten words: the first word, which is denoted by the label for the matrix, contains its own address; and the succeeding nine words contain the elements of the particular 3 x 3 matrix. This arrangement makes it possible to manipulate the matrices by reference only to the label for the matrix. The major properties of the function calls which are shown in Table I are as follows:

(a) This function call places the free space matrix for a field-free region of length X into a matrix (M1 in the illustration in Table I).

(b & c) These function calls are used to place matrices representing the converging plane or the diverging plane, of a quadrupole respectively, into a particular matrix. The arguments are $k = (B^{-1}\rho \cdot \nabla B)^{1/2}$, and ℓ, the effective length of the quadrupoles.

(d & e) These are matrices for the radial plane and the axial plane of a bending magnet, respectively. The arguments are: n, the field index, ϕ, the angle of deflection, r, the radius of curvature of the optic axis; e1, and e2, the angle the beam makes with the optic axis on entering and leaving the magnet, respectively.

(f, g, h & i) These function calls have similar names and the same notation as (b, c, d and e). They calculate the magnitude of the maximum deviation of a ray from the optic axis in the particular element, and put the result in a particular location, z. This information is important in the physical design of ion optical elements.

(j) This function puts a vector having the indicated arguments into a particular location.

(k) This function places zeroes in a matrix.

(l) This function puts the matrix contained in one location (M1, for example) into another location (M2, for example).

(m) This function multiplies two matrices and puts the results into a third location.

(n) This function performs a series of consecutive multiplications and stores the result in a particular location.

(o) This function takes the determinant of a matrix located in M1 and puts the result into X. This is useful in checking for computational errors as the determinant of any succession of transformations must be unity.

(p) This function takes the inverse of a matrix.

(q) This matrix is used in the manner discussed by Penner [1] when two succeeding bending magnets bend in opposite directions.

(r) This function calculates the focal plane for rays leaving a particular element of the system. The matrices representing the transformation from the object to the exit of the particular element in question are multiplied together, and placed in M1, for example. The location of the focal plane with respect to the element in question is then calculated using this function call. This function is useful in determining whether a selected set of parameters for the ion optical elements will result in a beam focussed at the required location.

(s) This function prints out the contents of a particular matrix in a standard format.

This program does not give any information about the second-order properties of a system. Any second-order aberrations in the system originate in the bending magnets as magnetic quadrupoles do not have any second-order aberrations. The aberrations in the bending magnets may be calculated from the work of Bretscher [2]. These second order effects may then be taken into account by using the degraded image as the object for subsequent first-order ray tracing.

Energy Analyzing Magnet

In order to establish the design criteria for the beam energy-analyzing magnet-system, we investigated the degradation in beam quality due to the various second-order aberrations. Using the notation of Bretscher [2], the energy resolution of an inhomogeneous-field magnet can be expressed as

$$\frac{\Delta E}{E} = \frac{2(1-N)}{R(1+M)}\Big(M\,\delta_r + A_{11}\,\alpha_r^2 + A_{44}\,\alpha_z^2 + A_{45}\,\alpha_z\,\delta_z + A_{55}\,\delta_z^2\Big)$$

in which R is the radius of curvature of the optic axis, M is the magnification, N is the field index, δ_r is the slit width, α_r and α_z are the beam divergences in the radial and axial planes, respectively, and δ_z is the slit height. The quantities A_{ij} were evaluated using a digital computer and except for the A_{11} term, the aberrations were found to be essentially independent of the magnet deflection angle. Since the A_{11} aberration can in principle be eliminated by suitable machining of the entrance and exit pole boundaries, the choice of magnet angle becomes independent of aberration considerations. However, the greater focal lengths associated with small deflection angles introduce other objections. Besides requiring an increase in vault space, these long focal lengths magnify the effects of magnet imperfections on resolution, and increase the effects of uncertainties in the analytic treatment of the magnet boundaries.

By using one large magnet to obtain the desired energy resolution rather than a number of smaller ones, the magnet can be rather easily traversed to provide high resolution beams for experimental areas on opposite sides of the cyclotron vault. Other factors favoring the selection of one large magnet are: (a) the return yoke requires approximately the same quantity of iron as does a magnet of smaller radius for the same degree of iron saturation; (b) a single regulated power supply is required, resulting in lower power supply costs and smaller image deterioration due to power supply fluctuations; (c) it has fewer entrance and exit boundaries to distort the beam; (d) less vault space is required for beam preparation.

A summary of the analyzing magnet characteristics is shown in Table II.

Table II

Summary of Magnet Characteristics

Deflection Angle	135 degrees
Radius of Curvature	9 feet
Field Index	0.5
Energy Resolution for 1 mm Slit: (Full Width at Half-Maximum)	
First Order	1.82×10^{-4}
Second Order	$< 2.00 \times 10^{-4}$
Radial Acceptance Angle	± 1.5 degrees
Axial Acceptance Angle	± 0.65 degrees
Gap at Optic Axis	4.75 inches
Pole Face Width	22.5 inches
Maximum Field Strength	5.63 kilogauss
Power Supply Rating	34.3 KW
Weight of Iron	153,800 lbs.
Weight of Copper (Hollow Conductor Coils)	11,560 lbs.

General Quadrupole Arrangement

The virtual source for the ion optics system is assumed to be similar to that of ORIC. A quadrupole doublet located in the magnet yoke images the virtual source at infinity. Then another quadrupole doublet, located near the entrance slit of the analyzing magnet, images the source onto the entrance slit of the analyzing magnet. We are using 4 inch I.D. beam pipes and a standard magnetic quadrupole with 4 inch aperture, 8 inch pole length, and maximum gradient of 4 kilogauss/inch. A ray tracing calculation was performed to be sure that the beam will not strike the beam pipe or the quadrupoles in going from the cyclotron to the entrance slit of the analyzing magnet. The assumed properties of the virtual source were:

	Radial Source	Vertical Source
Location:	10' upstream from 1st quadrupole	Infinity
Size:	0.070"	0.275"
Divergence:	0.010 rad.	0.0

For this calculation the first quadrupole doublet was oriented so that the converging plane of the first quadrupole was in the horizontal plane. This arrangement makes a larger angular acceptance available where it is required the most. The second quadrupole doublet is oriented with the diverging plane upstream in the horizontal plane. The exit of the second quadrupole doublet is 5' from the entrance slit of the analyzing magnet. With this arrangement, the beam is easily confined within the required apertures. The source is imaged onto the 1 mm entrance slit of the analyzing magnet with a magnification of approximately one-half in the horizontal plane, thus making it possible for all of the beam to go through the entrance slit of the analyzing magnet. Of course the angular divergence in the horizontal plane is increased from 0.010 rad to 0.0228 rad (1.3°). The image of the virtual source in the vertical plane will be very small, since the source is located at infinity. The results of these calculations were used in selecting the design criteria for the beam analyzing magnet.

If the properties of the radial virtual source turn out to be substantially different from those used in the calculations, the location of the second quadrupole doublet can be changed to get the best compromise between high transmission through the entrance slit of the analyzing magnet, and limitations imposed by the maximum angular acceptance of the beam analyzing magnet. It had been thought that another way to vary the size of the image at the entrance slit would be to vary the spacing between the individual elements of each quadrupole doublet. Calculations indicate that small changes in this spacing have a negligible effect on the image size. Hence, this extra degree of freedom is not worth the mechanical difficulties involved and will not be used.

Calculations for a typical pair of quadrupole doublets arranged as above were performed to determine the degree of regulation required for the quadrupole power supplies. A coherent variation of 0.06% in all four power supplies was found to produce a 5% deterioration in image size. Power supplies whose regulation is not poorer than 0.05% throughout the current range will be used.

Minimum Dispersion Beam Transport

The dispersion experienced by the cyclotron beam in traversing the energy analyzing magnet (11 cm for a one percent momentum spread) would prevent the steering of an intense, unresolved beam to an experiment's target station if the analyzing magnet were used in its normal operating mode. The following arrangement of quadrupole lenses overcomes this difficulty. The ion optic system preceeding the analyzing magnet is focused to form a virtual image 37 cm beyond the entrance of the analyzing magnet. This results in an image along the magnet's optic axis that is 127 degrees from the emergent boundary. In traversing this angle the ions undergo half a betatron oscillation and emerge parallel to the optic axis. Quadrupole lenses then condense this parallel beam to a focal point at the experiment station. Ions of slightly different momenta will emerge in nearly parallel beam bundles and will be focused at different planes along the optic axis. The net result at the target, however, is a small increase in the beam spot size.

The ion-optics computer program was used to determine the beam properties at the emergent boundary of the analyzing magnet as well as at the target. The largest radial excursion of the beam at the analyzer exit was within the 4 inch acceptance aperture of the succeeding quadrupole elements and the beam divergence from the optic axis was less than 0.007 radians for a 1 percent momentum change. At the target the dispersion is reduced by a factor of 8 from that of normal operation allowing a 5 mm diameter beam spot to be obtained from an extracted beam with the same emittance properties as ORIC ($\Delta E/E \cong 0.36\%$). If necessary, the size of the beam on target can be further reduced by optically coupling the homogeneous field switching magnets so that their inherent dispersion is algebraically opposite to that of the energy analyzing magnet when used in the above manner.

Switching Magnets

The three switching magnets shown in Figure I in paper EE-14 will not be installed when the cyclotron is brought into initial operation. However, sufficient design effort has been put into the system to be sure that the cyclotron building is laid out in accordance with the desired ion optical objectives. That is, the system consisting of a switching magnet and quadrupoles is able to deflect the beam into any of a number of desired beam paths and image the virtual source at desired target locations with approximately unit magnification. To check the feasibility of the system a study was made of the switching magnet into Room 3, with deflection angles of 0, $\pm$ 22-1/2°, $\pm$ 45°. The switching magnets for Rooms 1 and 2 are covered by this study since the deflection angles are the same or essentially the same, except that these switching magnets deflect in only one direction.

The switching magnets achieve double focussing by means of non-normal entrance and exit of the beam to the magnet. The switching magnets will be designed with variable entrance angle of the beam with respect to the field boundary, and fixed exit angle for each of the fixed deflection angles. This gives the desired flexibility in the optical properties of the switching magnets, and is absolutely necessary for the Room 3 switching magnet where deflection on either side of the beam is required.

The double focussing conditions for uniform-field bending magnets given by Cross [3] have been evaluated on a digital computer [4]. This study furnishes accurate input information for ray tracing calculations.

A quadrupole doublet is placed between the exit slit of the analyzing magnet and the entrance to the switching magnet. This quadrupole doublet images the exit slit at infinity. This image at infinity is then used as an object for the switching magnet. For 45° deflection, the switching magnet produces an image 8-1/2 feet from the exit of the switching magnet with approximately unit magnification. This image can easily be transferred to the required experimental location by means of a pair of quadrupole doublets. The position of these quadrupole doublets can be adjusted to give the magnification desired in combination with the previously discussed switching magnet and quadrupole combination. For 22-1/2° deflection, the radius of curvature of the optic axis is larger, and it is impossible to use the switching magnet to focus the beam at a near distance to the switching magnet. Hence, magnification of approximately unity would not be preserved. However, in this case, the image located at a large distance can be used as a virtual object for a quadrupole doublet and it can be refocussed to a shorter distance resulting in the desired magnification of unity.

This arrangement of quadrupoles, switching magnets and the remotely traversable energy analyzing magnet, provides a flexible facility for precision nuclear physics research.

References

1. S. Penner, Rev. Sci. Instr. 32, 150 (1961).

2. M. M. Bretscher, ORNL-2884, Focusing Properties of Inhomogeneous Magnetic Section Fields, April 20, 1960.

3. W. G. Cross, Rev. Sci. Insti. 22, 717 (1951).

4. These results are contained in a forthcoming NRL Report, P. Shapiro, S. Podgor, and R. B. Theus, NRL Report 6248.