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A COMPUTER PROGRAM FOR OPTICAL MATCHING SYSTEMS*

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Summary A computer program has been developed for finding an optical matching system of particle beams in the transverse (x-x', y-y') phase space. It is well-known that the two sets of symmetric quadrupole triplets could be easily obtained for this purpose starting from a thin lens approximation. However, a system composed of only four lenses (with a fixed geometry) is often desirable in order to save the available space. Four simultaneous algebraic equations for four unknowns (magnet strength) are solved numerically starting from a thin lens approximation and the solutions are improved successively. In general, the program finds the strength of each magnet in less than one minute. (IBM 7094.) Maximum beam excursion is also computed to find the best solution when there are several possible combinations.

Introduction

The purpose of the computer program described here is to find the strengths of quadrupole lenses that can transform the emittance ellipse in the transverse (x-x', y-y') phase space into the acceptance ellipse of the next focusing system. Since the area of these ellipses do not change in going through the matching system, there are, in general, four parameters to be matched. Although it is possible to use distances between magnets of the matching system as free variables to be adjusted, there are many cases for which the geometrical set-up has to be fixed. For example, there may be other devices (bending magnets, slit boxes, current transformers, etc.) that cannot be moved easily. At least four lenses are then required to match two arbitrary ellipses.

A system composed of two sets of symmetric triplets (excited symmetrically) are often used. It is well-known that, starting from a thin lens approximation, one can easily find such a system for given emittance and acceptance ellipses.[1] Programs for digital as well as analog computers have been used in many places for finding a matching system between the injector and the synchrotron. One disadvantage of such systems with two triplets is the requirement of a relatively long space which may cause a longitudinal phase spread. For example, in a typical proton linac[2] for meson factories, the emittance of the Alvarez section has to be matched to the acceptance of the iris-loaded waveguide section. The size of the additional phase spread due to a drift space between two sections restricts its length to less than $\sim$ 4m.[3] If the linac is to be used as an injector of a synchrotron, this restriction is more stringent. Also, two separate Alvarez sections with different focusing systems might have to be matched when the particle velocity is still very small ($v/c \sim .2$). The effect of the drift space on phase spread is then quite serious. It is therefore desirable to have a computer program for four lenses (minimum number required) which, together with measurements of the beam shape in (x-x', y-y') phase spaces, would make it possible to adjust the strength and, if necessary, polarity of each lens. Preferably, this should be done by on-line computers as a part of the over-all computer control of the accelerator.

Program

The matching efficiency E_x or E_y of a system in one direction is defined here as the percentage fraction of the overlapping area of two ellipses (with the same area) to be matched. The overall efficiency E is then the product of E_x and E_y. Two methods have been used in solving four simultaneous equations with four unknown quantities (i.e., strengths of four lenses), the gradient-search (or the steepest-descent) method and the Newton's method.[4,5]

Gradient-Search Method

Since the geometry of the system is fixed, the overall efficiency E can be expressed as a function of fixed parameters (distances between lenses, lengths of lenses) and four unknown lens strengths (g_i; $i = 1,2,3,4$). To make the program simple, the polarity of each lens is set to (+)(-)(+)(-) in x (or y) direction. Other arrangements can of course be investigated by slightly modifying the program. The range of the value of (g_i) is limited by the range of lens strengths and the solution for (g_i) should not lie outside of this range.

The procedure used is as follows: Assign a random value (not exceeding the maximum value) to each g_i and compute E. Take four partial derivatives $\partial E/\partial g_i$, either numerically or using analytical expressions. Change g_i to $g_i + \Delta g_i$ where Δg_i is proportional to $\partial E/\partial g_i$. Repeat this until E reaches a maximum point. If this maximum value is less than an efficiency desired, repeat the entire process starting from a new random set of (g_i). When a satisfactory value of E is obtained, calculate the largest beam excursion in x and y directions (for unit phase space areas) as well as their locations so that, if several different sets of (g_i) are found, the final choice can be made from this beam quality.

Newton's Method

Instead of solving exact equations containing exponential and trigonometric functions, one can start from the thin lens approximation and improve the solution by a successive iteration. A very convenient formalism, based on an electrical analogue of a ladder network, is developed by Hereward.[1] This method is especially suited to a computer which has built-in facilities for complex numbers.

For the focusing direction of a lens with the strength g and the length s, the unknown parameter is $C = g \cdot \sin(gs)$. For the defocusing direction of the same lens, one starts with the approximation $-C = -g \cdot \sinh(gs)$, the parameter C being common to both directions. Another approximation is that the effect of a finite length of a lens with the length s is simply to add a free space with the distance s/2 on both sides of the lens position. This corresponds to

$$\frac{1-\cos(gs)}{g\sin(gs)} = (s/2), \quad \frac{\cosh(gs)-1}{g\sinh(gs)} = (s/2) \quad (1)$$

The relations between two ellipses then reduce to four algebraic equations for four unknown C's. Starting again from a random set of (C_i), one tries Newton's method until a convergence is obtained. The strength (g_i) can be calculated from $C_i = g_i \cdot \sin(g_i s)$. If the resulting value of the overlapping efficiency E is less than, say, 50%, a new random set of (C_i) will be used. For E larger than 50%, two equations for the defocusing direction is modified by replacing $-C_i$ by $-C_i + \Delta_i$ where

$$\Delta_i = g_i \cdot \sin(g_i s) - g_i \cdot \sinh(g_i s), \quad (2)$$

(g_i) being the solution of the first approximation. Equations for the focusing direction are unchanged. The entire procedure is repeated until (Δ_i) is essentially zero. Finally, (s/2) is replaced by the proper form in (1) but, in most cases, this last refinement is not necessary. The calculation of the beam quality (the maximum excursion and its position) is the same as in the gradient-search method.

One advantage of this method is that the polarity of each lens does not have to be assigned at the beginning of the calculation. A random value for C_i could be negative as well as positive and the final arrangement is not restricted to (+)(-)(+)(-).

Discussions

Many cases for which at least one set of (g_i) is known to give 100% efficiency have been tried by both methods. When the gradient-search is employed, it is difficult to reach the maximum point even with a very small step. Presumably, utilization of second derivatives ($\partial^2 E/\partial g_i^2$) could remove part of this difficulty but this would increase the computing time. A fixed polarity for each lens is another disadvantage compared to the method which is based on a successive iteration for algebraic equations.

It should be emphasized here that there are many cases for which no practical solutions exist or several solutions are equally satisfactory. If the value of the admittance is much larger than the emittance value and the final beam quality is relatively unimportant, a perfect matching is not necessarily required. On the other hand, if a matching system is to be used for a large number of different emittance-admittance combinations with a close to 100% efficiency, four lenses with a fixed geometry may be inadequate without auxiliary adjustment lenses.

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