TRANSVERSE BEAM BLOW-UP IN A STANDING WAVE LINAC CAVITY†

R. L. Gluckstern
University of Massachusetts, Amherst, Massachusetts

and

H. S. Butler
University of California, Los Alamos Scientific Laboratory
Los Alamos, New Mexico

Summary

The theory of the interaction of a beam with transverse modes in a standing wave linear accelerator is summarized and applied to the design parameters for recently proposed proton linacs. Approximate results are derived for the limiting current and these are checked and extended by detailed numerical calculation of the build-up of transverse fields. For the contemplated linacs, the current limit is of the order of 10 amperes, safely above the design values.

Introduction

High-current proton linacs are currently being designed for use both as injectors for high-energy synchrotrons and as facilities for direct experiments with mesons and nucleons. Since high-current electron traveling wave linacs are known to exhibit the phenomenon of beam blow-up, we have tried to evaluate the interaction of high-current proton beams with transverse modes in a standing wave linac in order to assess the seriousness of this phenomenon.

Theory

The theory of the interaction of a bunched beam with transverse modes in a cavity has been developed[1,2] in analogy with Wilson's description[3] of beam blow-up in a traveling wave electron linac. The modes of the first transverse band of the cavity are assumed to be oscillating with given amplitudes. The m^{th} (narrow) beam bunch is then assumed to enter the cavity with a certain initial displacement x_m and angle x_m'. This bunch interacts with the existing transverse modes which change its trajectory. The currents generated by this moving bunch will then feed energy into the transverse modes leading to changes in the amplitudes of these modes. If these amplitudes are able to build up sufficiently, the beam will be deflected into the structure and will be lost.

If $H_j^{(m)}$ is the appropriately normalized (complex) amplitude of the magnetic field in the j^{th} mode as the m^{th} beam bunch enters, one can write for the change in amplitude per beam pulse

$$H_j^{(m+1)} e^{-i\omega_j \Delta t} - H_j^{(m)} = \sum_k S_k\left[W_{jk}H_k^{(m)} - \overline{W}_{jk}H_k^{(m)*}\right] - \epsilon_j H_j^{(m)} - i(x_m - iK_j x_m')e^{i\alpha_j/2} \quad (1)$$

Here $\epsilon_j = \dfrac{\pi\omega_j}{\omega_o Q_j}$, $S_k = \dfrac{eI_o gL^2}{\pi^2 pc}\left(\dfrac{c^2}{\omega_o\omega_k}\right)(Z_o LR_k^2)$,

$$\frac{4W_{jk}}{\pi^3} = \frac{e^{i(\alpha_j-\alpha_k)} - 1}{\alpha_k^2(\alpha_j - \alpha_k)} - \frac{(e^{i\alpha_j}-1)(\alpha_j+\alpha_k)}{\alpha_j^2\alpha_k^2} + \frac{ie^{i\alpha_j}}{\alpha_j\alpha_k}. \quad (2)$$

The average beam current is I_o, the beam-bunch separation is $\Delta t = 2\pi/\omega_o$,† the quantity $Z_o LR_k^2$ is directly related to the r/Q for the cavity, and

$$\alpha_j = (k_j - \frac{\omega_j}{v})L \quad (3)$$

is the slip of the bunch relative to the traveling wave component of the j^{th} mode. The quantity $\overline{W}_{jk}$ is defined as

$$\overline{W}_{jk}(\alpha_j,\alpha_k) = W_{jk}(\alpha_j,-\alpha_k)^*, \quad (4)$$

and K_j is related to the relative amplitude of the transverse and axial components of the electric field in the j^{th} mode. All other symbols are either obvious or are defined in Refs. 1 and 2. The assumptions made in the derivation of Eq. (1) are:

1) The frequency separation of adjacent modes is larger than the natural width (related to the Q) of each mode.

2) Only those components of the mode traveling with approximately the same velocity as the beam are important.

3) Only those effects which are linear in the displacement and angle of the beam relative to the axis are retained.

4) The beam bunches are narrow and are equally spaced.

5) The energy gain in a tank can be neglected.

6) External transverse focusing is not included.

Equation (1) can be solved only after further assumptions. However, the question of stability of the solution is related to the homogeneous part

†The beam bunch frequency $\omega_o/2\pi$ is usually a submultiple of the frequency of the accelerating mode.

of Eq. (1) which in principle can be solved. Specifically, the solution will stay within bounds if all of the eigenvalues of the homogeneous equation have a magnitude less than unity. This leads to a starting current, I_S, below which the solutions will be bounded.

Approximate Solution for Blow-Up

In order to obtain the character of the solution of Eq. (1) we have assumed that only a single mode is important. This leads, for the homogeneous equation, to the solution

$$H_j^{(m)} \sim \lambda^m \tag{5}$$

with

$$\lambda = \mathrm{Re}\left[e^{i\theta}(1-\epsilon+SW)\right] \pm \sqrt{S^2|\overline{W}|^2 - \left\{\mathrm{Im}\left[e^{i\theta}(1-\epsilon+SW)\right]\right\}^2} \tag{6}$$

Here $\theta_j = \omega_j\Delta t$ to within a multiple of 2π, and all subscripts have been dropped.

In the cases of interest ϵ is of the order 10^{-3}. Since

$$W = U + iV \tag{7}$$

is of order 1, the values of S which are of interest are also of order ϵ or lower. If $|\theta|$ is much larger than ϵ, the solution for λ is

$$|\lambda| \simeq |e^{i\theta}(1 - \epsilon + SW)| \simeq 1 - \epsilon + SU \tag{8}$$

and stability simply requires $|\lambda| < 1$, or

$$S < \epsilon/U, \text{ for } |\theta| \gg \epsilon. \tag{9}$$

If on the other hand $\theta = 0$ (resonance between the transverse mode and the beam frequency) one finds

$$\lambda = 1 - \epsilon + SU \pm S\sqrt{\overline{W}^2 - V^2}. \tag{10}$$

Stability then requires

$$S < \epsilon/U, \quad \begin{cases}\text{for } |\theta| \gg \epsilon \text{ and} \\ \text{for } \theta = 0, \ |\overline{W}| < V,\end{cases} \tag{11}$$

and

$$S < \epsilon/\left[U + \sqrt{\overline{W}^2 - V^2}\right], \text{ for } \theta = 0,$$

$$|\overline{W}| > V, \tag{12}$$

where we have combined the cases $\theta = 0$ and $|\theta| \gg \epsilon$ in Eq. (11).

Amplitude Growth

In order to obtain a simple guide for the amplitude growth we will neglect all modes other than $k = j$ in Eq. (1) as well as neglect the term $\overline{W}$ compared with W. (This is not always valid, but leads to qualitatively correct results as can be seen from Eqs. (11) and (12).) In this case one has

$$H^{(m+1)}e^{-i\theta} = H^{(m)}(1-\epsilon+SW) - i(x_m - iK\, x_m')e^{i\alpha/2}. \tag{13}$$

We will further assume that (at least for the first tank) all values of x_m and x_m' are identical, so that one has

$$H^{(m)} = -i(x - iK\, x')\frac{(1 - \lambda^m)}{(1 - \lambda)}\, e^{i\alpha/2 + i\theta}, \tag{14}$$

where

$$\lambda = e^{i\theta}(1 - \epsilon + SW). \tag{15}$$

The angular deflection a proton experiences in traversing the cavity can be shown to be given in general by

$$L\Delta x_m' = \frac{\pi^3}{2}\,\mathrm{Im}\left\{\sum_j S_j H_j^{(m)} e^{-i\alpha_j/2}\,\frac{\sin^2(\alpha_j/2)}{(\alpha_j/2)^2}\left(1 - \frac{K_j\alpha_j}{L}\right)\right\}. \tag{16}$$

In order to obtain estimates, one must assign a value to K. In the approximation of small coupling along the axis between adjacent cells it can be shown that

$$K\alpha \ll L. \tag{17}$$

Since typical values of x' are of the order $2\pi x/\lambda_t$ where λ_t, the transverse oscillation wavelength, is much larger than L, all terms in K may be dropped, leading to

$$L\Delta x' \simeq -\frac{\pi^3}{2}\, xS\, \mathrm{Re}\left\{e^{i\theta}\,\frac{(1-\lambda^m)}{1-\lambda}\right\}\frac{\sin^2(\alpha/2)}{(\alpha/2)^2}. \tag{18}$$

The most serious amplitudes are reached in the steady case for $\theta = 0$, but these are quickly reduced once $|\theta| > \epsilon$. Since ϵ is very small and since even a 1% variation in ω_j will lead to variations in θ of the order of 0.4 rad, at most one or two tanks will be near enough to resonance to make the amplitude growth serious. In this case

$$L\Delta x_\infty' \sim -\frac{\pi^3}{2}\, x\,\frac{S(\epsilon-SU)}{(\epsilon-SU)^2 + S^2V^2} \sim -\frac{\pi^3}{2}\, x\,\frac{S}{\epsilon-SU}, \tag{19}$$

which implies that the fractional increase in amplitude is of the order of the ratio of the actual current to the starting current. Moreover the sign of the deflection is such as to increase rather than decrease the transverse focusing.

Numerical Calculations

The rough guides in Eqs. (11), (12), and (19) have been obtained by assuming that only one mode contributes to the beam growth. In order to see the effect of taking into account several modes, a series of numerical evaluations of Eqs. (1) and (16) have been performed to investigate the following effects:

1) Dependence of the starting current on ϵ.

2) Difference between narrow band and wide band structures.

3) Effect of resonance ($\theta = 0$) taking into account several modes, and determination of the width of the resonance.

Figures 1 and 2 show the growth of the deflection $(L\Delta x'/x)$ for a single mode, below and above the starting current defined in Eqs. (11) and (12). In this case the transverse band was taken to run from 1190 to 1230 Mc/sec which yields a value of $\theta = 0.363$. These results show the instability associated with currents above the starting value. (For all cases depicted by the figures in this paper the following parameter values were assumed: frequency of the π-mode = 800 Mc/sec, beam frequence = 200 Mc/sec, $\epsilon = 0.01$, and number of cells per section = 40.) Figures 3 and 4 show a corresponding calculation taking into account 3 modes. In this case the starting current has been raised by about 18%. Further investigation showed that in all cases tried the starting current for more than one mode was higher than that for a single mode, implying some draining of the field build-up into adjacent modes. Figure 5 gives the explicit results for 1, 3, 5, 7, and 9 modes. The variation due to the presence of other modes is not great, amounting to 10 ± 8% for those cases tried.

Figures 6 and 7 are appropriate to the resonant case, $\theta = 0$, with a single mode and show a decrease in the starting current of 44% compared with the non-resonant cases. This is consistent with Eq. (12). The width of the resonance is shown in Fig. 8 where the starting current is plotted against θ in the region near $\theta = 0$. The width is clearly given by $\theta \sim \epsilon$, implying

$$\frac{\delta f_j}{f_j} \sim \frac{1}{Q_j}. \tag{20}$$

The circles show computer runs which were made for various values of S/S_o and θ/ϵ. The arrows show whether the value of S/S_o was above or below the starting current and in addition indicate how close the run was to blow-up. The solid curve was derived from theory.

Figures 9 and 10 depict the resonant case, $\theta = 0$, for three modes. As with the non-resonant case the inclusion of more modes raised the value of the starting current. The width of the resonance, shown in Fig. 11, is again given approximately by $\theta \sim \epsilon$, but here the pattern is unclear and further studies are being made of the dependence of starting current on θ for more than one mode.

Several runs were made with different values of ϵ. In each case the starting current was found to be proportional to ϵ. Numerical values were then obtained primarily for $\epsilon = 10^{-2}$ rather than for the more appropriate $\epsilon = 10^{-3}$ in the interest of calculation time.

Several runs were made with different locations of the transverse band and for different bandwidths. The starting currents obtained were insensitive to these changes, except for the resonant cases described above.

Conclusions

1) A reliable guide to the current limit for transverse beam blow-up is given by

$$S_o = \epsilon/U \simeq \epsilon = (\pi\omega_j/\omega_o Q_j) \tag{21}$$

where we have taken U to be of order unity. The corresponding current is

$$eI_s \sim \frac{\pi^3 pc(\omega_j/c)^2}{L^2(Z_o LR_j{}^2)Q_j}. \tag{22}$$

If one defines a shunt impedance per unit length for a transverse mode as

$$r_\ell \equiv \frac{\left[(c/\omega_j)\int_o^L dz(\partial E_z{}^j/\partial x)\cos k_j z\right]^2}{L \times \text{Power Loss}}, \tag{23}$$

Eq. (22) can eventually be written as

$$eI_s = \frac{\pi^3\, pc(c/\omega_j)}{2L^2 r_\ell}. \tag{24}$$

2) Measurement of the fields in the transverse mode for the LASL cloverleaf structure leads to an estimate

$$I_s \sim 10 \text{ amp}, \tag{25}$$

safely above the design value of 20 ma.

3) The estimate in Eq. (24) was made assuming a single deflecting mode and is increased when neighboring modes are included. The results appear to be approximately independent of bandwidth as long as it is wide enough for the modes to be clearly distinct.

4) In the unlikely event that there is a resonance between the deflecting mode frequency and a multiple of the beam frequency, the starting current is lowered by less than a factor 2. The relative "width" of this resonance is of the order Q^{-1}.

References

†Work performed under the auspices of the U. S. Atomic Energy Commission.

1. R. L. Gluckstern, AADD-38, Brookhaven National Laboratory Report, July 1964; Proceedings of the MURA Conference on Linear Accelerators, Stoughton, Wisconsin, July 1964 (to be published).

2. R. L. Gluckstern, P-11/RLG-1, LASL internal report, August 1964 (unpublished).

3. P. B. Wilson, HEPL Report No. 297, Stanford, June 1963.

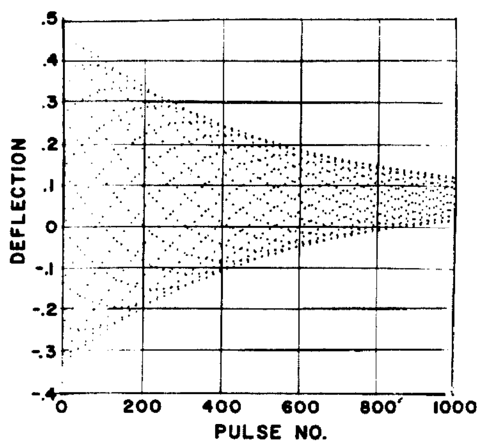

Fig. 1. Deflection after m pulses for a single mode; non-resonant case, below starting current. $f_{d\pi}$ = 1190 Mc/sec, f_{do} = 1230 Mc/sec, and S = 0.01053 = 0.8 S_o.

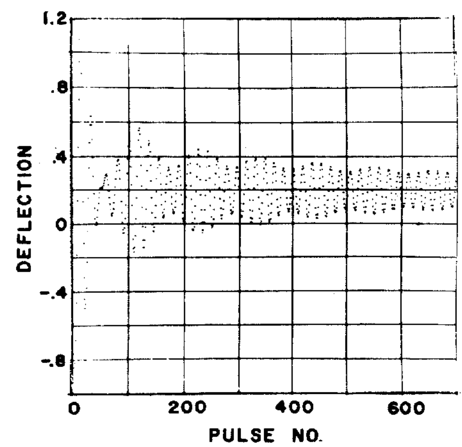

Fig. 3. Deflection after m pulses for three modes; non-resonant case, below starting current. $f_{d\pi}$ = 1190 Mc/sec, f_{do} = 1230 Mc/sec, S = 0.01184 = 0.9 S_o.

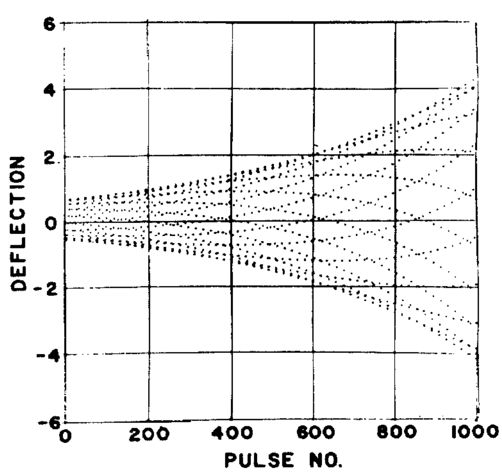

Fig. 2. Deflection after m pulses for a single mode; non-resonant case, above starting current. $f_{d\pi}$ = 1190 Mc/sec, f_{do} = 1230 Mc/sec, S = 0.01579 = 1.2 S_o.

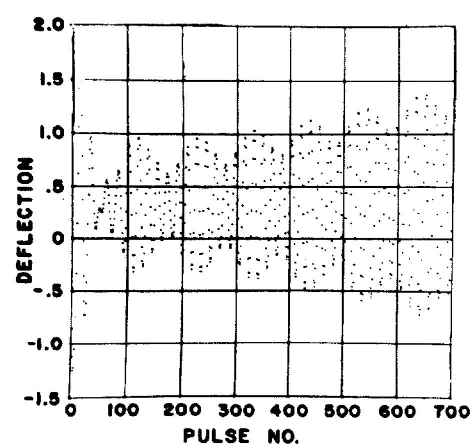

Fig. 4. Deflection after m pulses for three modes; non-resonant case, above starting current. $f_{d\pi}$ = 1190 Mc/sec, f_{do} = 1230 Mc/sec, S = 0.01711 = 1.3 S_o.

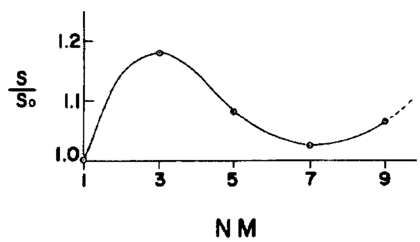

Fig. 5. Normalized starting current, S/S_o versus number, NM, of modes permitted to take part in the interaction.

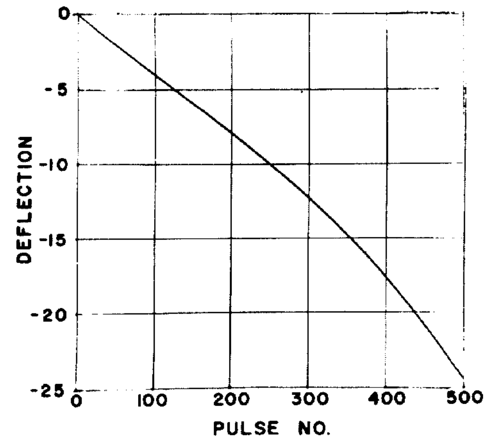

Fig. 7. Deflection after m pulses for one mode; resonant case, above starting current. $f_{d\pi}$ = 1178.4308 Mc/sec, f_{do} = 1218.4308 Mc/sec, S = 0.007 = 0.7 S_o.

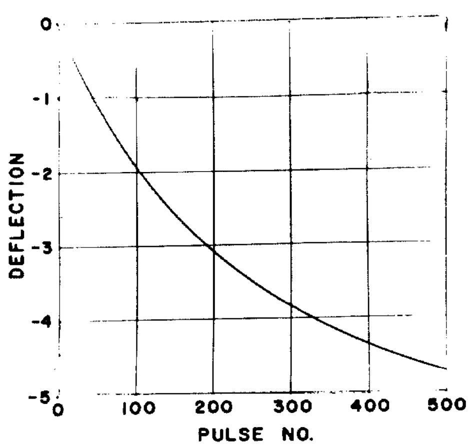

Fig. 6. Deflection after m pulses for one mode; resonant case, below starting current. $f_{d\pi}$ = 1178.4308 Mc/sec, f_{do} = 1218.4308 Mc/sec, S = 0.004 = 0.4 S_o.

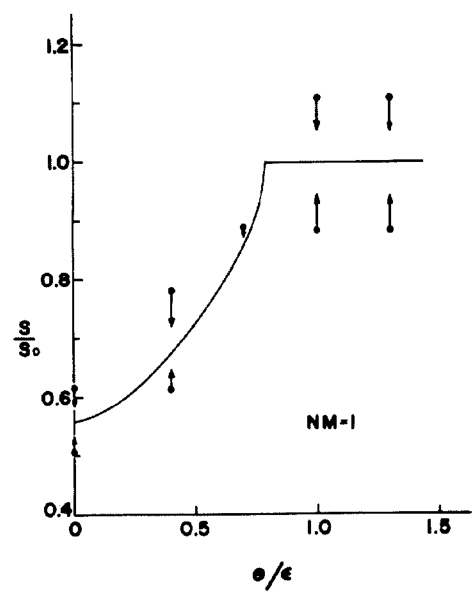

Fig. 8. Normalized starting current, S/S_o, versus resonance parameter, θ, for one mode.

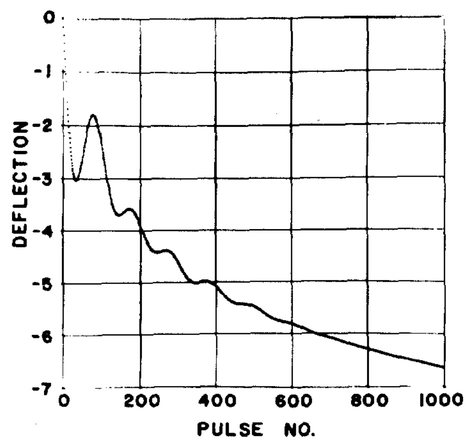

Fig. 9. Deflection after m pulses for three modes; resonant case, below starting current. $f_{d\pi}$ = 1178.4308 Mc/sec, f_{do} = 1218.4308 Mc/sec, S = 0.007 = 0.7 S_o.

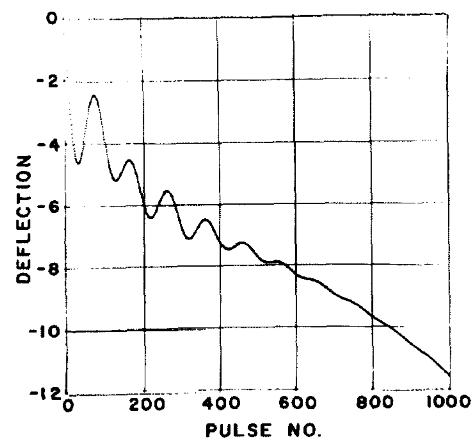

Fig. 10. Deflection after m pulses for three modes; resonant case, above starting current. $f_{d\pi}$ = 1178.4308 Mc/sec, f_{do} = 1218.4308 Mc/sec, S = 0.011 = 1.1 S_o.

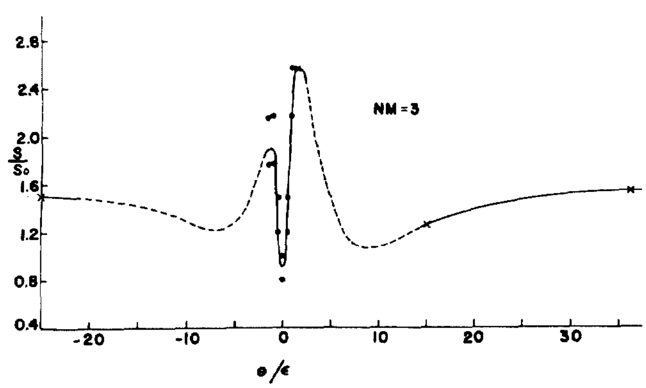

Fig. 11. Normalized starting current, S/S_o, versus resonance parameter, Θ, for three modes.