A WAVEGUIDE RESONANT RING ELECTRON ACCELERATOR*

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ABSTRACT

This paper analyses a proposed accelerator, less than 1 meter long, capable of accelerating bunched electrons with a peak current of 100 A for a period of 50 rf cycles (20 nsec). The accelerator consists of an S-band disk-loaded circular waveguide, similar in design to the SLAC machine, incorporated into a waveguide resonant ring. Analysis of this system indicates that significant power buildup can occur if the length of the accelerator part of the ring is less than 1 meter. Further analysis of the power buildup and energy storage is made for a variety of rings having an accelerator section which is three, four, and five wavelengths long. Following the initial buildup of ring energy a beam of current bunched into 1/6th wavelength (60°), having a peak value of 100 A and an initial energy of 100 kV, is injected into the first cavity of the accelerator. Such a beam would extract half of the energy within the ring in approximately 20 nsec and have a nominal output energy of 10 MeV. By operating at 100°K, five such accelerators could be pumped by the same power source, thus increasing the output energy level to 50 MeV.

The problem of handling a 100-A beam at low velocities is also investigated and a method utilizing space-charge neutralization by ionized gas is discussed.

INTRODUCTION

The accelerator is shown pictorially in Fig. 1b. It incorporates a section of circular waveguide accelerator as one leg of a waveguide ring. The waveguide ring is shown schematically in Fig. 1a Waveguide rings of this sort possess many interesting properties which have been investigated by a number of people.[1-5] A brief review of the operation of the ring using the notation of S. J. Miller will be included for completeness. The coupler, if lossless and matched at all four ports, can be represented by the scattering matrix S,

$$S = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{bmatrix} \qquad (1)$$

The matrix elements s_{1m} are called the scattering or coupling coefficients and give the contribution from a wave incident at port m to the outport wave at port 1. For example, if the wave a_1 is incident at port 1 then $s_{31} \times a_1$ will appear at port 3. The complete output wave at port 3, b_3, is given by,

$$b_3 = \sum_{m=1}^{n} S_{3m} a_m \qquad (2)$$

For a four-port device, such as a coupler, n is 4. We may choose the reference planes of the coupler as we see fit and if the choice is made so that the output wave at port 4 is in phase with the input at port 1 we may rewrite S as,

$$S = \begin{bmatrix} 0 & 0 & jC & \sqrt{1-C^2} \\ 0 & 0 & \sqrt{1-C^2} & jC \\ jC & \sqrt{1-C^2} & 0 & 0 \\ \sqrt{1-C^2} & jC & 0 & 0 \end{bmatrix} \qquad (3)$$

where C is the voltage coupling coefficient of the directional coupler. Build up of the wave in the ring occurs in the following manner. A wave is incident at port 1. Some of this wave is coupled to port 3. It then propagates around the ring to port 2, being attenuated in the process. Some of this wave incident at port 2 is now coupled to port 4, but the rest goes across to port 3, and if the path length around the ring is an integral number of wavelengths then the waves incident at ports 1 and 2 will add in phase at port 3 and add 180° out of phase at port 4. In this case the wave leaving port 3 after one complete trip around the ring has been made is,

$$b_3 = jC + jCT\sqrt{1-C^2} \qquad (4)$$

The process repeats again and again and after N trips around have been made the wave out of port 3 is,

$$b_{3_N} = jC\sum_{n=0}^{N}\left(T\sqrt{1-C^2}\right)^n = jC\,\frac{1-\left(T\sqrt{1-C^2}\right)^N}{1-T\sqrt{1-C^2}} \qquad (5)$$

Similarly the wave out of port 4 after N trips around the ring have been made is,

$$b_{4_N} = \sqrt{1-C^2} + (jC)^2\,T\sum_{n=0}^{N-1}\left(T\sqrt{1-C^2}\right)^n \qquad (6a)$$

and by performing the summation we obtain,

$$b_{4_N} = \sqrt{1-C^2} - \frac{C^2T\left(1-\left[T\sqrt{1-C^2}\right]^{N-1}\right)}{1-T\sqrt{1-C^2}} \qquad (6b)$$

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In the limiting case, as N goes to infinity, b_{3_N} and b_{4_N} become,

$$\lim_{N\to\infty} b_{3_N} = b_{3_\infty} = j\frac{C}{1-T\sqrt{1-C^2}} \qquad (7)$$

$$\lim_{N\to\infty} b_{4_N} = b_{4_\infty} = \sqrt{1-C^2} - \frac{C^2T}{1-T\sqrt{1-C^2}} \qquad (8)$$

For a fixed value to T there is an optimum value of C which permits a maximum buildup to occur. This value is,

$$C = \sqrt{1-T^2} \qquad (9)$$

By substituting this value of C into the previous equations, b_3 and b_4 simplify to,

$$b_{3_N} = j\frac{1-T^{2N}}{\sqrt{1-T^2}} \quad ; \quad b_{3_\infty} = j\frac{1}{\sqrt{1-T^2}} \qquad (10)$$

and

$$b_{4_N} = T^{2(N-1)} \quad ; \quad b_{4_\infty} = 0 \qquad (11)$$

Since we are primarily interested with the buildup of the wave, we may drop the j and so consider only the amplitude of the wave in the ring. We are also concerned with the power gain G in the ring which is equal to the square of the wave amplitude,

$$G_N = \frac{(1-T^{2N})^2}{1-T^2} \quad ; \quad G_\infty = \frac{1}{1-T^2} \qquad (12)$$

From the last equation it is apparent that if the value of T is close to 1, then the power gain in the ring can become quite large.

An accelerator utilizing feedback, similar in concept to this resonant ring, was built by Messrs. Mullard Ltd. in collaboration with A. E. R. E.[6] By using a feedback they were able to increase their input power from 2 Mw to an effective 4 Mw and so obtained an increased energy output from their linear accelerator. Our purpose in investigating a resonant ring is directed toward achieving a design having a large amount of energy stored per unit length to facilitate accelerating beams having a peak current of 100 amps. It is further desired that the accelerator have an output energy of at least 10 MeV, and be as compact as possible.

ACCELERATOR PARAMATERS

The constant gradient $2\pi/3$ mode has been selected for the accelerator since this mode suppresses the adverse radial forces of the TM_{11} mode, maintains a constant (hence maximum) gradient throughout the accelerator, and a large amount of technology pertaining to it already exists. The parameters used in the analysis of the performance of the accelerator include: the shunt resistance, r; the Q; the attenuation constant, α; and the group velocity, v_g. In the Stanford linear accelerator reports, SLAC-7 and SLAC-8, curves and tables for these parameters are given for eighty-five cavities. Polynominals were determined which fit the data given. To accommodate a 100-amp beam it is desired that the irises be as large as possible, and so the polynominals were used to extrapolate the values for the cavity dimensions and the properties of the cavities back from the fifteenth cavity of the SLAC section through the first and on for another 185 cavities. The "a" dimension of the irises for these 200 cavities ranges from 0.479 inch at the first cavity to 0.678 inch at the last.

Groups of 9, 12, 15, and 18 of these cavities were then combined to permit analysis of accelerators of 3, 4, 5, and 6 wavelengths respectively. After determining the essential properties of a ring having a given accelerator section, a new section, formed by dropping a cavity from one end and adding the next cavity to the other end, was incorporated into the ring. This new resonant-ring accelerator was then analyzed. The process of deletion of a cavity from one end and adding a cavity to the other was continued until each of the 200 cavities had been included in at least one accelerator section.

Plots of the power gain, the energy stored, and the electric field are shown in Fig. 2, Fig. 3, and Fig. 4 respectively. In Figs. 3 and 4 the values for the ordinates correspond to an input power of 6 Mw. From Fig. 3 it is seen that for each length of accelerator section, the energy stored decreases as the diameter of the iris increases. The opening of the iris is accompanied by a reduction in the attenuation constant of the cavities which allows greater buildup of power, but at the same time the velocity of propagation is increasing which results in a reduced ratio of stored energy to input power. For the values used, the increase in group velocity is more significant than the increased power buildup. It can also be seen from Fig. 3 that although the longer sections have more attenuation that the shorter section and so less power buildup, the energy stored in the sections is almost independent of length. Figure 4 indicates that variation of electric field strength as a function of accelerator length and iris opening. It is seen that as the irises become larger, which is desirable for transporting a high-current beam, both the stored energy and accelerating gradient decrease. A compromise must therefore be sought which will optimize the output energy and pulse length for a given beam current and application.

To get an estimate of the minimum energy output of the accelerator, calculations will be made based on an accelerator section having the maximum size irises. It is also assumed that a klystron of the type being developed for SLAC which has a peak output power of 24 Mw will be available for the input power to the ring. From Fig. 2, it is seen that the power in a ring whose accelerator section is three wavelengths long would buildup to 360 Mw, have a gradient of 48 Mw/m, and the amount of energy stored would be 9.7 joules. The electrons coming out would initially have a maximum energy of 14.4 MeV and would extract 0.1 joule per cycle for a bunched beam having a 60° phase spread and having a peak current of 100 amps. This rate of extraction corresponds to a power flow out of the ring of 286 Mw which is over ten times the rate that energy is being fed into the ring. At this rate the stored energy will decrease to half its original value after approximately 50 bunches have been accelerated. This would take 17.5 ns. During this time, the output energy of the beam will have decreased to 10 MeV, $1/\sqrt{2}$ of its original value. The curves shown in Fig. 2 indicate the power which the ring will buildup to as the number of trips goes to infinity. However, the pulse length from the klystron is 2.5 μs and so the fill-time and discharge-time combined must not exceed 2.5 μs. In addition to this limitation it is desirable to discharge the ring as soon as possible to keep the amount of energy converted to heat down to a minimum. From the relations in Eq. (12), it is seen that the ratio of the power buildup after N trips to the total buildup possible is,

$$\frac{G_N}{G_\infty} = (1 - T^{2N})^2 \tag{13}$$

From this equation, we can solve for N, the number of trips required to reach a certain percentage of the final value, that is

$$N = \frac{\log\left(1 - \sqrt{G_N/G_\infty}\right)}{2 \log T} \tag{14}$$

If we wish to let the power buildup to 95 percent of its final value, then for the 3-wavelength accelerator under consideration the number of trips required is 51. Each trip takes 24 ns and so the time required to buildup to the 95 percent level is 1.62 μs; well below the pulse length limitation.

Similar calculations for a 6-wavelength section having the maximum diameter irises and for 3- and 6-wavelength sections having the minimum diameter irises are included in Table 1.

Table 1
Summary of Calculations
for Various Accelerator Sections

I	II	III	IV	V	VI	VII	VIII	IX
3	10	195	52.8	12.0	16.2	11.5	18.9	1.25
3	200	350	42.4	9.2	13.3	9.4	18.2	1.62
6	19	106	37.8	12.2	23.8	16.8	12.8	1.44
6	200	195	31.7	9.5	20.0	14.1	11.9	1.73

Where the columns are:

- I Number of wavelengths in the accelerator section
- II Input cavity number
- III 0.95 x G (Mw)
- IV E_z, Maximum electric field on axis (Mv/m)
- V Energy stored in ring (Joules)
- VI Initial output energy of electrons (MeV)
- VII Output energy of electrons when half the energy has been depleted (MeV)
- VIII Time required to extract half of the stored energy (ns)
- IX Time required to buildup ring power to 95 percent of G (μs)

The values shown in columns V and VI correspond to the energy an electron would have if it was always on the peak of the wave. Since there is a 60° phase spread to the electrons, all of these electrons cannot have this peak value. Also, the electrons slip in phase until they achieve their terminal phase. Analysis of the dynamics involved results in the following equations of motion in the axial direction of the accelerator,[7]

$$\frac{d\gamma}{d\xi} = \alpha \cos 2\pi\Delta \tag{15}$$

$$\frac{d\Delta}{d\xi} = \frac{1}{\beta} - \frac{\gamma}{\sqrt{\gamma^2-1}} \tag{16}$$

where,

- γ = mass of electron in units of electron rest mass
- ξ = axial distance in units of free space wavelength
- α = energy an electron on the peak of the wave can gain in one wavelength in units of electron rest energy
- Δ = phase of electron in number of cycles with respect to the crest of the travelling wave
- β = ratio of phase velocity of accelerating wave to the velocity of light

Equations (15) and (16) were numerically integrated on a computer using the Runge-Kutta 2^{nd}-order technique for various values of α. The computer program was run for a number of input velocities. For each input velocity, a number of variations of β as a function of ξ were run. And for each of these configurations results were computed for a set of values of input phase beginning at +80° and going to -70°, computations being made at increments of 10°. The results of these computations indicate that the variation in output energy due to phase spread, phase slippage, and input velocities corresponding to 100 kV ±50 kV can be kept within an energy band of 15 percent. By using a value of $\beta < 1$ in the first two cavities, it is possible to bunch the electrons tightly at some phase angle in front of the crest. Doing this results in a more homogeneous energy output, but at a reduced level.

FURTHER RESEARCH AREAS

The values determined for the power and electric field in the accelerator are much greater than what is currently found in accelerators. Tests at Stanford using standing wave resonators have indicated that the proposed level of operation of their accelerator is considerably below the breakdown level.[8] If it should be found that the values determined above exceed the allowable levels of operation then the power from the klystron can be split to feed two rings operating in tandem. The increased complexity and cost of such an arrangement would at least be partially compensated for by an increased energy inherent in this arrangement. If the accelerator is cooled, the resistance and so the attenuation constant decrease. At -200°C approximately the temperature of liquid nitrogen, the dc resistance of copper is one-tenth that at room temperature. If a similar reduction occurs for the attenuation constant at radio frequencies, then the power gain in a ring could be increased by a factor of five over that obtainable at room temperature. Or, the input power could be split to feed five rings. Such a technique would enable one to obtain electrons of considerable energy using a single r-f source. Whether such a scheme is feasible will depend to a large degree on the stability of operation of the resonant rings. One further point which requires mentioning is the technique to be used in preventing a space-charge-caused radial blow up of the beam. Currently under consideration is the utilization of positive ions along the beam path to neutralize the charge of the electrons. For a peak current of 100 amps in a beam having a radius of 1 cm, the required ion pressure for neutralization would be 10^{-8} Torr. The scarcity of research in this field and the lack of agreement about the results of experimenters indicates the need for further experimentation before any conclusions can be drawn.

REFERENCES

1. Miller, S. J., "Travelling Wave Resonator and High Power Testing," Microwave Journal, Vol. 3, No. 9, p 50, 1960.
2. Tischer, R. J., "Resonance Properties of Ring Circuits," IRE Transactions PGMTT, Vol. MTT 5, No. 1, p 51, 1957
3. Tomiyasu, K., "Effect of a Mismatched Ring in a Travelling Wave Resonator," IRE Transactions PGMTT, Vol. MTT 5, No. 4, p 267, 1957.
4. Tomiyasu, K., "Attenuation in a Resonant Ring Circuit," IRE Transactions PGMTT, Vol. MTT 8, No. 2, p 253, 1960.
5. Twisleton, J. R. G., "Some Properties of Travelling-Wave Resonance," Proceedings IEE, Oct. 1958, p 108.
6. "Engineering," Vol. 174, p 161, 1952.
7. Chu, E. L., "The Theory of Linear Electron Accelerators," Stanford University ML Report 140, 1951.
8. Borghi, R., Stanford Linear Accelerator Center, Private communication, 1965.

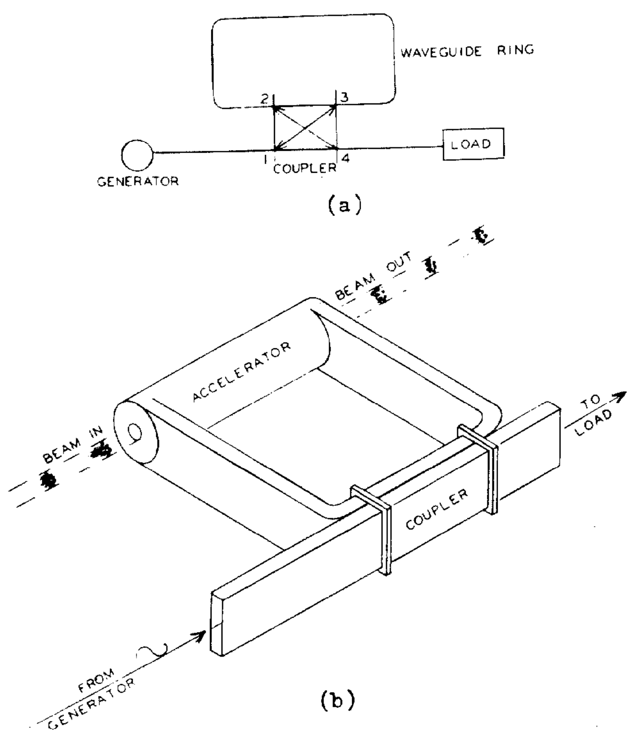

Fig. 1. (a) Resonant ring circuit. (b) Resonant ring accelerator.

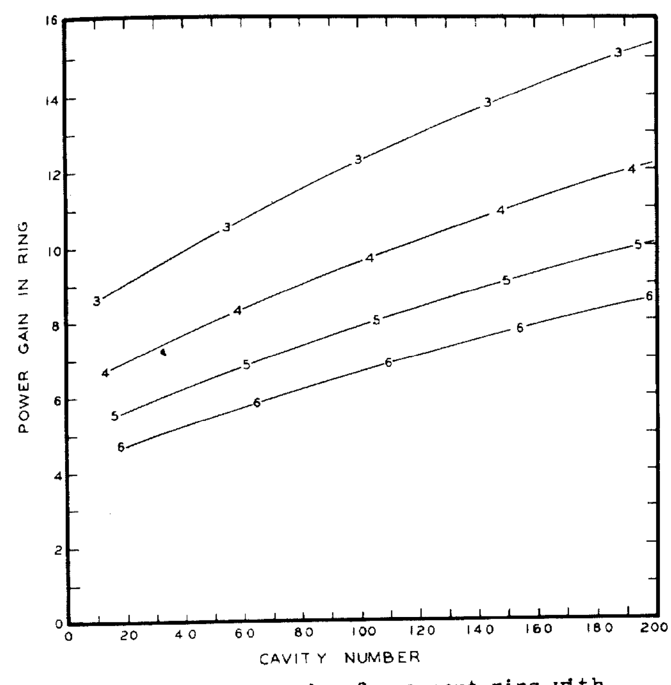

Fig. 2. Power gain of resonant ring with 3λ to 6λ accelerator sections. Abscissa indicates cavity number at entrance end.

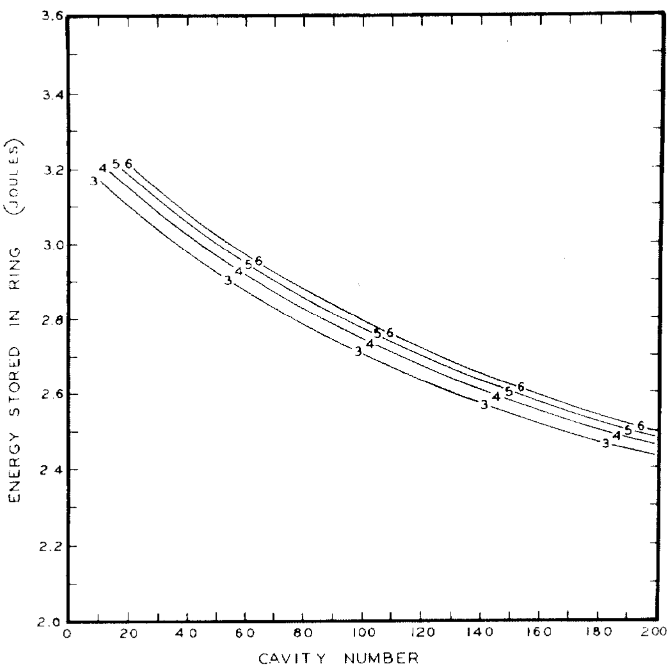

Fig. 3. Energy stored in resonant ring with 3λ to 6λ accelerator sections for 6 Mw source. Abscissa indicates cavity number at entrance end.

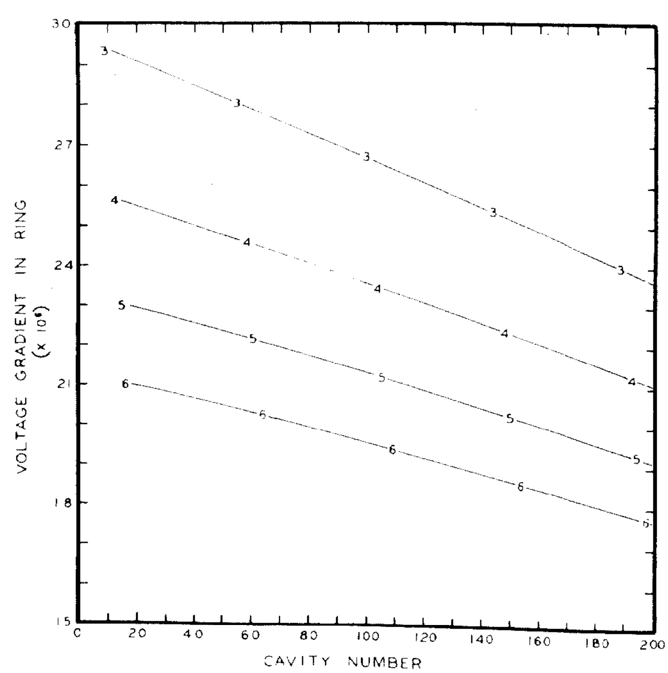

Fig. 4. Maximum electric field on axis with 3λ to 6λ accelerator sections for 6 Mw source. Abscissa indicates cavity number at entrance end.