

STABLE SUPERCONDUCTING COILS

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Introduction

The use of superconducting coils in large systems has been retarded by the fact that the design of superconducting magnets has been more of an art rather than a straightforward application of physics and engineering principles with the subsequent evolution of a predictable coil design. Superconducting coils have been designed up to the present using mainly data obtained from similar coils. The problem of scaling data to coils of different sizes and shapes was a formidable one and when building a coil of a type that had not previously been built, its performance could not be predicted---sometimes within 50%. This situation is tenable when dealing with small coils, however it becomes intolerable in large coil systems where the investment in materials and labor can be considerable.

This paper presents an analysis as well as a summary of tests of stabilized superconducting coils. Stabilization achieves two results. First, the short sample current carrying capacity is consistently achieved in coils with no size or shape effects. Second, the uncontrolled transition to the normal state with the resultant loss of liquid helium and magnetic field is eliminated. These two results combine to allow superconducting coils to be designed and built with the assurance that their performance will have no more uncertainty in it than the usual engineering tolerances.

Stabilized Conductor

The principle of stabilizing a superconductor is discussed qualitatively in Ref. 1. This paper presents an analysis of the behavior of a short piece of stabilized superconductor in an externally applied field. The results are later expanded to arrive at coil behavior. By a "stable superconductor" is meant, one that returns to the superconducting state following a disturbance, either self-generated (such as a flux jump) or externally generated (vibration, rapid external field changes, temporary excess in current, etc....).

Let us now derive the requirements for such a conductor. Consider a superconductor in good electrical and thermal contact with a normally conducting substrate exposed to the liquid helium bath and in an externally applied magnetic field. Consider what must occur in a well cooled conductor as current is increased. At low currents all the current flows in the superconductor until it reaches its maximum current carrying capacity. If the conductor is well cooled so that the conductor temperature rise can be neglected, then a further increase in current will result in a "spilling over" of the excess current from the superconductor to the normal substrate. The net result is a sharing of the total current between the superconductor and the substrate. The superconductor must develop

some resistance to limit its current, otherwise it would take on more current. Obviously, if not enough cooling is provided, the temperature rise due to current in the substrate will result in a lowering of the current carrying capacity of the superconductor resulting in more current in the substrate and a complete transfer of current to the substrate may occur.

To determine the stability of the conductor we must analyze the behavior of the conductor when current is shared between the substrate and the superconductor.

If we define $0 \leq f \leq 1$ as being the fraction of the total current I which flows in the substrate, then the voltage per unit length of the conductor is:

$$v = \rho I f / A \quad (1)$$

where ρ and A are the resistivity and cross sectional area of the substrate. The conductor temperature T is given by:

$$T - T_b = v I / hP = \rho I^2 f / hPA \quad (2)$$

where h is the heat transfer per unit surface area per unit temperature rise from the conductor to the cooling bath at a temperature T_b and is assumed to be a constant. The perimeter of the conductor that is exposed to the cooling bath is P .

Under conditions of current sharing between superconductor and substrate, the temperature T , and the externally applied magnetic field determine the current in the superconductor.

Let us define $I_{ch}(T_b)$ as the maximum current a superconductor can carry at the bath temperature T_b and in an externally applied magnetic field H --this is the so-called short sample or H - I curve of the superconductor. Since the current carrying capacity of superconductor decreases with temperature, the current in the superconductor I_s is given by:

$$I_s / I_{ch} = g [(T - T_b) / (T_{ch} - T_b)] \quad (3)$$

$$g(0) = 1 \quad g(1) = 0.$$

Here T_{ch} is the superconductor critical temperature in a field H and g represents the functional relationship between the superconductor current and temperature. For most superconductors (2) the functional relationship is well approximated by a straight line:

$$I_s / I_{ch} = [1 - (T - T_b) / (T_{ch} - T_b)] \quad (4)$$

It is useful to introduce a stability parameter.

$$a \equiv \frac{\rho I_{ch}^2}{hPA (T_b - T_{ch})}$$

Use of equations (1) and (2) and the definition of a result in the following after some simplification:

$$f = \frac{(I/I_{ch}) - 1}{(I/I_{ch}) [1 - a (I/I_{ch})]} \quad (5)$$

$$vA / \rho I_{ch} = \frac{(I/I_{ch}) - 1}{1 - a (I/I_{ch})} \quad (6)$$

$$(T - T_b) / (T_{ch} - T_b) = \frac{a I/I_{ch} (I/I_{ch} - 1)}{1 - a (I/I_{ch})} \quad (7)$$

Equations 5, 6 and 7 are shown plotted in Fig. 1, 2 and 3 respectively for several values of a .

Figure 2 shows the voltage-current characteristics of unit length of the conductor. Since we are interested in the behavior of this conductor in a coil which is inductive, we should examine the stability at constant current.

Two distinct types of operation are possible depending on the value of a . For $a < 1.0$ no voltage appears until $I = I_{ch}$ (the superconductor current carrying capacity.) For $I > I_{ch}$ the voltage increases gradually with current. The characteristic is everywhere single valued. For $a > 1.0$ the operation is more complicated:

$$\text{For: } 0 < I \leq \frac{I_{ch}}{\sqrt{a}}$$

Single valued operation with all the current in the superconductor ($v = 0$).

$$\text{For: } \frac{I_{ch}}{\sqrt{a}} \leq I \leq I_{ch}$$

Double valued operation with either all the current in the superconductor or all the current in the substrate.*

$$\text{For: } I_{ch} < I$$

Single valued operation with all the current in the substrate.

Completely stable operation results for $a \leq 1.0$. For $a > 1.0$ stable operation is limited to currents up to I_{ch}/\sqrt{a} . In the region of current above I_{ch}/\sqrt{a} steady operation with all the current is still possible up to I_{ch} , however, should the conductor suffer a disturbance then all the current will immediately switch to the substrate.

Coil Performance

Now that we have examined the operation of a conductor in an external magnetic field, we can *Operation on the negative resistance part of the curve is unstable for a constant current type of operation.

relate its performance to that of a coil in which the local magnetic field varies from point to point so that I_{ch} and T_{ch} vary, and the substrate resistivity ρ may exhibit magneto-resistive effects.

For simplicity let us restrict ourselves to a coil energized so that the same current flows through all the conductors in series. We must first obtain the length of conductor at a given field within a coil. We may define a function q :

$$q (H/I) = q (G) = \lim_{\Delta G \rightarrow 0} \frac{\Delta L}{\Delta G} \quad (8)$$

such that the length of conductor L_{12} between points having a ratio of magnetic field to current of G_1 and G_2 is:

$$L_{12} = \int_{G_1}^{G_2} q (G) dG \quad (9)$$

This functional relationship is obtained from a field plot for the particular coil in question and, at a given current, represents the distribution of conductor versus the magnetic field locally with the coil.

In the previous section we have derived the voltage per unit length as a function of conductor geometry, how well it is cooled, the current I flowing through it and the following field dependent quantities: ρ , I_{ch} , T_{ch} . In a coil the local magnetic field H is a function of the current I so that the incremental coil voltage for an element at a location within the coil with a ratio of magnetic field to current G is:

$$\begin{aligned} dv (I) &= v \left[I, I_{ch} (H), T_{ch} (H), \rho (H) \right] q (G) dG \\ &= v \left[I, I_{ch} (G), T_{ch} (G), \rho (G) \right] q (G) dG \end{aligned} \quad (10)$$

The coil terminal voltage is obtained by integrating over the full range of G existing in the coil

$$v (I) = \int_{G=0}^{G_0} v \left[I, I_{ch} (G), T_{ch} (G), \rho (G) \right] q (G) dG \quad (11)$$

As an illustration let us consider a superconductor that can be characterized by a current carrying capacity that drops linearly with increasing magnetic field, and whose critical temperature T_{ch} also drops linearly with magnetic field. These assumptions can be regarded either as close approximations to the actual superconductor behavior or as a linearization of the actual superconductor characteristics in the vicinity of the operating condition. Also, for purpose of simplification let us neglect the variation of the substrate resistivity with magnetic field.

With these assumptions, equation (6) can be rewritten as:

$$vA / \rho I_d = \frac{1 - (I/I_d) [1 - (1 - G/G_0) \beta]}{(1 - \beta) [a_d (I/I_d) - 1]} \quad (12)$$

The coil stability parameter a_d is now based on the design value of current I_d and the critical temperature T_{cd} at the maximum magnetic field within the windings at the design current $H_d = G_0 I_d$.

$$a_d = \rho I_d^2 / hPA (T_{cd} - T_b) \tag{13}$$

I_d is determined by the intersection of the superconductor current carrying capacity versus magnetic field at the bath temperature and a line of slope G_0 corresponding to the maximum ratio of magnetic field to current within the windings.

The parameter β is the ratio of H_d to H_m . H_m is the magnetic field obtained by the intersection of the linearized curve of superconductor current carrying capacity versus magnetic field with the zero current axis

$$\beta = \frac{G_0 I_d}{H_m} \tag{14}$$

Equation (12) represents the characteristic of the conductor between the $v = 0$ and the $v = \rho I/A$ lines.

To perform the integration in Eq. (11) we must have a conductor distribution function $q(G)$. A simple function $q(G)$ results for a long solenoid of winding thickness large compared with its diameter and end effects neglected. In this case $q(G)$ is simply the total conductor length L divided by G_0 and is the same for all values of $0 \leq G \leq G_0$.

Let us first assume stable single valued operation to obtain the coil terminal characteristics. These will then be generalized to include unstable characteristics. The first integration of equation (14) then goes from a value of G where $v = 0$ to G_0 where current is still shared between the superconductor and substrate:

$$\begin{aligned} \frac{VA}{\rho I_d L} &= \int_{1 - \frac{(I/I_d) - 1}{\beta I/I_d}}^1 \frac{1 - (I/I_d) \left[1 - (1 - G/G_0) \beta \right]}{(1-\beta) \left[a_d (I/I_d) - 1 \right]} d(G/G_0) \\ &= - \frac{\left[(I/I_d) - 1 \right]^2}{2\beta (1-\beta) (I/I_d) \left[a_d (I/I_d) - 1 \right]} \end{aligned} \tag{15}$$

The above equation holds until either all the current is expelled from the superconductor at $G = G_0$ or the local characteristic becomes vertical at the point in the coil where $v = 0$. The first condition is:

$$\frac{I}{I_d} = \frac{\sqrt{\beta^2 + 4 a_d (1-\beta)} - \beta}{2 a_d (1-\beta)} \tag{16}$$

The second condition is:

$$I/I_d = 1/a_d \tag{17}$$

for $a_d < 1.0$ the first condition always occurs before the second one.

Under this condition, in the inner layers of the coil, which are exposed to the highest field, the current is all in the substrate. As one goes

out radially the current is shared between the superconductor and the substrate, the fraction of current in the substrate decreasing to zero as one goes outward radially.

In this region the voltage is given by:

$$\begin{aligned} \frac{VA}{\rho I_d L} &= \int \frac{1 - a_d (I/I_d)^2 (1-\beta)}{\beta (I/I_d)} \frac{1 - (I/I_d) \left[1 - (1 - G/G_0) \beta \right]}{(1-\beta) \left[a_d (I/I_d) - 1 \right]} d(G/G_0) \\ &+ \int \frac{(I/I_d) d(G/G_0)}{1 - a_d (I/I_d)^2 (1-\beta)} \\ &= \left\{ (I/I_d) \left[a_d (I/I_d) - 1 \right] (1-\beta) + 2 (I/I_d) - 1 \right\} / 2\beta \end{aligned} \tag{18}$$

$$\frac{\beta^2 + 4 a_d (1-\beta) - \beta}{2 a_d (1-\beta)} \leq I/I_d \leq 1/a_d$$

At $I/I_d = 1/a_d$ the region of sharing of current between superconductor and substrate shrinks to zero, so that for $I/I_d > 1/a_d$ the current is carried either by the superconductor or the substrate, and the terminal voltage is given by:

$$\begin{aligned} \frac{VA}{\rho I_d L} &= \int \frac{(I/I_d) d(G/G_0)}{1 - a_d (I/I_d)^2 (1-\beta)} \\ &= \left[a_d (I/I_d)^2 (1-\beta) + \beta (I/I_d) - 1 \right] / \beta \end{aligned} \tag{19}$$

$$I/I_d > 1/a_d$$

We have up to now restricted ourselves to $a_d < 1.0$, which results in single valued terminal characteristics. For $a_d > 1.0$ no sharing of current can result and the expression, Eq. (19), derived for $I/I_d > 1/a_d$ holds for this case. We must remember however that $V = 0$ is also a possible operating line up to $I = I_d$.

Figure 4 shows typical voltage current characteristics for $\beta = 0.5$ and for several values a_d .

The coil characteristics can be summarized in a manner similar to the single conductor characteristics. For $a_d < 1.0$ single valued terminal characteristics result, with no voltage appearing until the full current carrying capacity is reached in the inner layers of the coil. As the current is increased further a terminal voltage appears which increases with current and the current is gradually driven out of the conductor. A decrease in current follows the same characteristic and

upon reaching the current carrying capacity of the superconductor the voltage disappears. For $\alpha_d > 1.0$ the operation is different. For:

$$0 \leq I/I_d \leq \frac{\sqrt{\beta^2 + 4\alpha_d(1-\beta)} - \beta}{2\alpha_d(1-\beta)}$$

all superconducting operation results. For

$$\frac{\sqrt{\beta^2 + 4\alpha_d(1-\beta)} - \beta}{2\alpha_d(1-\beta)} \leq I/I_d \leq 1.0$$

double valued operation results with either completely superconducting or current completely in the substrate in part of the coil.

For: $I/I_d > 1.0$

the operation is single valued with current completely in the substrate in part of the coil and rest of coil superconducting.

Results

We are now in a position to interpret superconducting coil current carrying capacity as measured on short samples of superconductor. If we have a coil with a high value of α_d , then, provided a disturbance is large enough, operation switches from a fully superconducting state to an operation with part of the coil fully normal. Once this has occurred the current must be reduced to the single valued region of coil operation--the higher the value of α_d , the lower must the current be reduced before fully superconducting operation can be restored. This is the case with most superconducting coils built to date--the coil is subjected to disturbances (usually flux jumps), which usually trigger the coil into a partly normal type of operation. Once voltage appears the current must be reduced almost to zero before the coil can be re-energized. While this type of operation is tolerable for laboratory coils, it becomes more and more of a problem the larger the coil, not only because of the uncertainty in the current at which the coil will go normal, but also because of the magnetic energy released upon transition into the normal state. Operation of a coil at $\alpha_d \leq 1.0$ results in a coil which not only is consistently operable to the full current carrying capacity of short samples of superconductor, but also does away with the uncontrolled transition to the normal state characteristic of conventional superconducting coils.

Experimental Results

A superconducting coil using the principles outlined above was constructed using heat treated Nb-25% Zr .010" dia. wire. This wire was chosen specifically because it had exhibited very high short sample current carrying capacity, but its current carrying capacity in coils was very much less than Nb-25% Zr that was not heat treated and which, in short samples, had a much lower current carrying capability. For example, a .010" dia. wire of heat treated Nb-25% Zr wire has an I_{ch} of 85 to 110 amps at 40 kgauss. The coil's internal

diameter was 5 inches and it was 13 inches long. The conductor consisted of copper strip .040" by 0.5 inches in cross-section with nine longitudinal grooves into which the nine strands of .010" dia. wire were inserted.

This conductor was then wound into pancake-type coils which were stacked with a clearance so that both sides of each pancake coil were exposed to helium. It was later determined that the fabricating procedure used produced many breaks in the superconducting wire. Inspection revealed that breaks occurred in a single wire locally, and no places were found where more than one wire was broken at the same point. Therefore one would expect only 8/9ths of the total short sample current carrying capacity for $V=0$. (Note that the resistance encountered in transferring current laterally between superconductors is negligibly small when the distance between breaks is large).

The simple analysis presented in this paper is not adequate for exact comparison to the terminal characteristics of the experimental coil, since the effect of substrate magnetoresistance, the variability of the heat transfer coefficient with temperature, the breaks in the superconductor and the actual distribution of the conductor length versus magnetic field must be taken into account.

Nevertheless the coil stability parameter α_d and β are measures of the expected performance.

The experimental coil was designed with $\alpha_d = 0.35$ and $\beta = 0.7$, thus the terminal characteristics were predicted to be everywhere single valued, and fully superconducting operation was predicted up to the full current carrying capacity of the superconductors used.

The measured steady state voltage-current characteristics for the coil as a whole is shown in Fig. 5 (the current is the total current in nine parallel superconducting wires). The circles show the voltage with increasing current, the squares with decreasing current. The current at which voltage first appeared was 710 amps. The central field was 38 kgauss and the calculated field at the wire was 42 kgauss. The terminal characteristics were reproducible and independent of the previous history of current within the range of current shown in Fig. 5.

Due to the breaks the actual current per wire should be based on only eight wires, yielding 88.8 amps per wire (average), a value in the middle of the range of measured short sample characteristics of the material used at 42 kgauss.

Conclusions

With stabilization the usual argument against the use of superconducting coil systems that "they are very promising, however, achievement of the design performance is uncertain", can be laid to rest. Stabilized superconductors completely eliminate all sorts of coil effects which have been observed, such as large coil effects, split coil effects, etc. These effects are no more than

situations in which one would normally expect disturbances such as flux jumps, vibration, and conductor movement to be more severe. Stabilization not only eliminates these effects but allows the design of superconducting magnets using only data generated on short samples, requiring only that in long lengths of the same material any short sample will perform within a specified deviation.

The operation of a stabilized coil is a much simpler matter than a conventionally wound superconducting coil. Effects such as training which require that the coil be driven normal several times before achieving required coil performance are completely eliminated. Should the current in a stabilized coil be increased accidentally or intentionally above the full current carrying capacity of the superconductor, a small resistance will appear. This results in increased refrigeration requirements, but no loss of magnetic field--in fact continuous operation is possible under this condition up to the capacity of the refrigeration system. If the current is reduced, fully superconducting operation is restored at the full superconductor current carrying capacity. This performance is to be contrasted to the uncontrolled transition to the normal state of conventional superconducting coils which result in complete loss of magnetic field at coil currents which usually fall far short of the current carrying capacity of short samples of the superconductor.

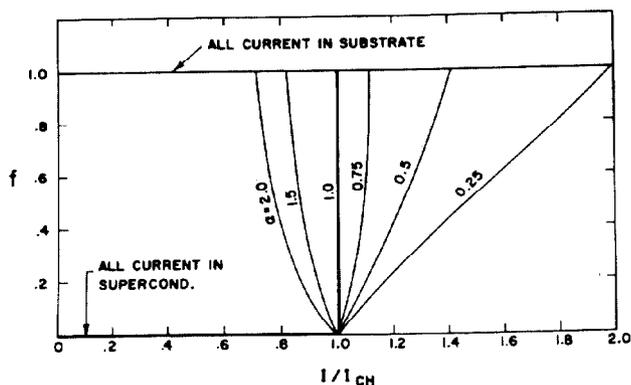


Fig. 1. Current sharing between a stabilized superconductor and a cooled substrate. f is the fraction of total current conducted by substrate.

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References

1. A. R. Kantrowitz and Z. J. J. Stekly, "A New Principle for the Construction of Stabilized Superconducting Coils", *Appl. Phys. Lett.* **6**, 56, (1965).
2. R. D. Cummings and W. N. Latham, "An Experimental Measurement of the Dependence of the H-I Curve of Nb-Zr Wire on Temperature. Avco-Everett Research Laboratory, Report No. AMP-141, May, 1964.

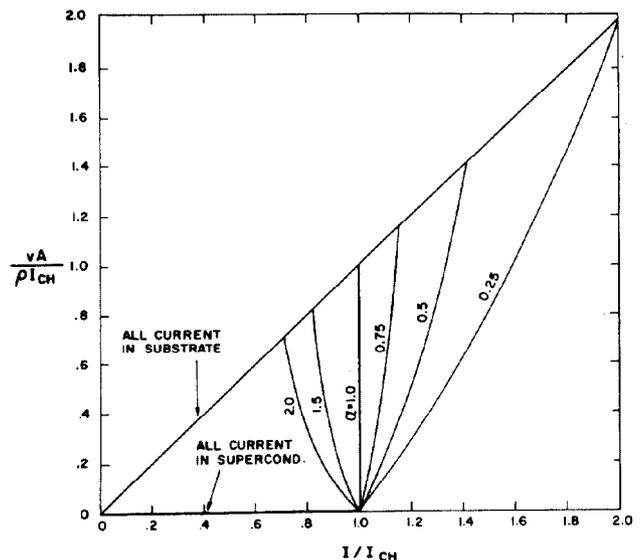


Fig. 2. Voltage-current characteristic for a stabilized superconductor-substrate combination.

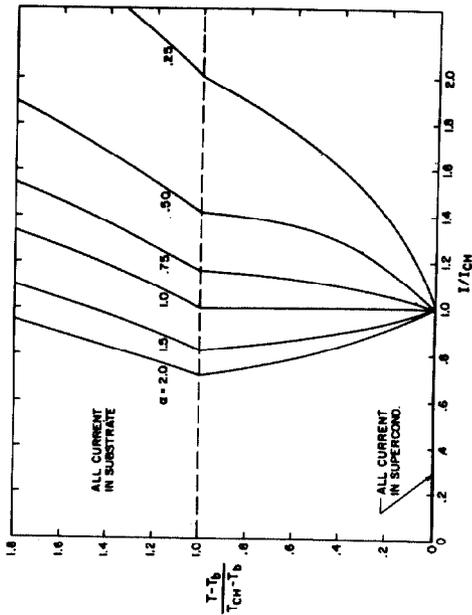


Fig. 3. Temperature rise of a stabilized superconductor as a function of current and cooling parameter.

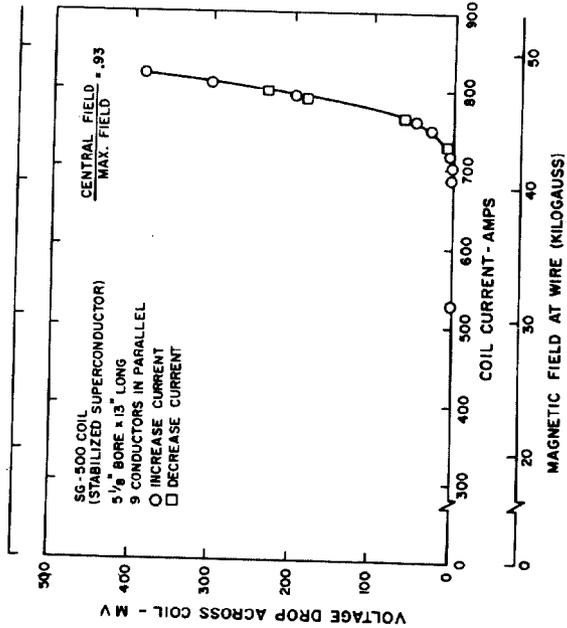


Fig. 5. Experimentally determined characteristic for an SG-500 coil wound with stabilized superconductor. The limit of superconductivity corresponded to 710 amps which was consistent with the full current carrying capacity of the superconductors used. The excursion beyond this limit was reversible.

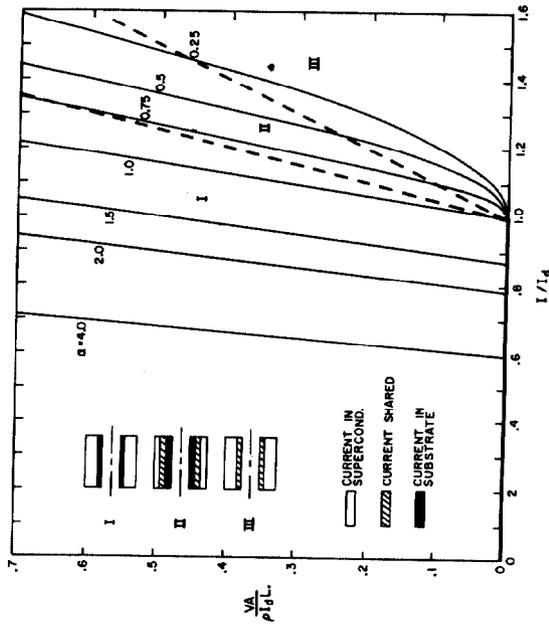


Fig. 4. Voltage-current characteristic for a long solenoid wound with a stabilized conductor plotted for a value of $\beta = 0.5$. In region I, to the left of the dashed line, the resistive portion of the coil has all the current in the substrate. In region III, in all portions of the coil the current is either all in the superconductor or is shared between the superconductor and the substrate. In region II, the current is in the substrate at the inner layers of the coil, is shared between substrate and superconductor in the intermediate layers and is all in the superconductor in the outer layers. For $\alpha > 1.0$ and $I/I_d < 1.0$, operation may be either on the $V = 0$ line up to $I/I_d = 1.0$ or on the corresponding curve of α . This results in double valued operation in a certain current range.