NON LINEAR SPACE CHARGE EFFECTS ON TRANSVERSE BEAM STABILITY*

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Abstract

Simbad code is used to study the combined effect of external non linearities and space charge non linearities on the beam stability using a simple FODO lattice. Gaussian and parabolic particle distribution are used for these simulations and results are compared with Möhl and Metral theoretical results.

INTRODUCTION

High intensity rings like FAIR and the LHC have motivated the study of the old problem of stability when non linear space charge and external non-linearities are present [1]. Recent studies [2] [3] has shown a discrepancy between simulations and theoretical stability diagrams. In this paper, simulations are performed with the recently developed code SIMBAD [4] and the results are compared with the stability diagrams for a Gaussian beam distribution.

The stability diagrams can be obtained from the dispersion relation [7]

$$\left(\int_{-\infty}^{\infty} \frac{\rho_x(\omega_{x,i})}{\omega_c - \omega_{x,i}} d\omega_{x,i}\right)^{-1} = (U_x - jV_x)_w\,, \quad (1)$$

where $\rho_x(\omega_{x,i})$ is the distribution function of horizontal betatron frequencies for the ith particle, ω_c is the coherent betatron angular frequency, $\omega_{x,i}$ is the horizontal betatron angular frequency of the ith particle.

Eq. 1 is a complex number. When ω_c is scanned from minus infinity to infinity, the left hand side of Eq. 1 draws a loop in the complex plane that correspond to the stability boundary diagram.

The right hand side of Eq. 1 is related with the space charge impedance through the following equation:

$$(U_x - jV_x)_w = \frac{je\beta I Z_x(w)}{2\omega_{xo} 2\pi R\gamma m_o} \quad (2)$$

where I is the beam current Z_x is the horizontal coupling impedance , and $\omega_{x,0}$ is the center of the distribution functions of the betatron frequencies of all particles. Similar expressions can be found for the vertical plane.

In order to find if the beam is stable the complex number given by the right hand side of Eq. 2 must be evaluated. If this number is within the stability boundary diagram, the beam is stable and if the number is outside the beam is unstable.

The impedance of the right hand side of Eq. 2 can be calculated for a Gaussian beam as [5]:

$$Z^{sc} = \frac{jZ_o R}{\beta^2\gamma^2\sigma^2}, \quad (3)$$

where $Z_o = \frac{4\pi}{c}$, c the speed of light, R the average radius of the accelerator, β and γ relativistic factor and σ the rms size of the beam.

BEAM SIMULATIONS IN SIMBAD

SIMBAD is an upgrade of the known code for space charge simulation ORBIT. SIMBAD uses the Particle in Cell (PIC) Algorithm to track particles in 3D through a sequence of transfer maps. Transfer maps are obtained from MAD, the software widely use for optics design in the accelerator community.

For the simulations of this paper, a continuous beam of $N = 1.79 * 10^9$ protons with Gaussian distributions in space as well as in energy was used in a simple array of 8 FODO cells. The Gaussian distribution in energy leads to Gaussian distribution of particle betatron angular frequencies through the equations:

$$\omega_{x,i} = Q_{x,i}\Omega_i, \quad (4)$$

where,

$$\Omega_i(p_i) = \Omega_o\left(1 - \eta\frac{\Delta p_i}{p_o}\right) \quad (5)$$

$$Q_{x,i}(p_i) = Q_{0,x}\left(1 + \xi_x\frac{\Delta p_i}{p_o}\right), \quad (6)$$

where η is the slippage factor and ξ is the chromaticity.

A preliminary test of the software was done by comparing the tune shift of the particles in the simulation and the corresponding tune shift predicted by [6]:

$$\Delta\nu = \frac{r_o}{(4\pi)^{3/2}}\frac{N}{\epsilon\beta^2\gamma^3 B_f} \quad (7)$$

with ro the classical radius of the particle,N the total number of particles ,ϵ the rms emittance, $B_f = I/I_{peak}$ the bunching factor, and β and γ the relativistic factors. From Eq. 7 a tune shift of 0.17 (dotted line in Fig. 1) is obtained which agrees with the center of the particles tune distribution. The histogram of Fig. 1 also shows how the particle tune distribution spreads as a consequence of the space charge.

Also, the oscillation amplitude of the beam was studied for a stable (Fig. 2) and unstable regimen (Fig. 3). These two plots are used as criterion of stability in the next section.

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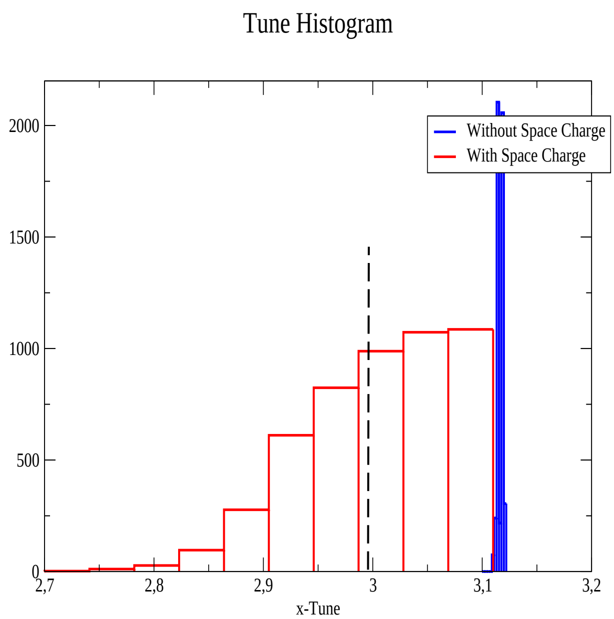

Figure 1: Particle tune distribution without space charge (narrow histogram) and with space charge (wide histogram). The dotted line represent the tune predicted by Eq. 7 considering the space charge effect

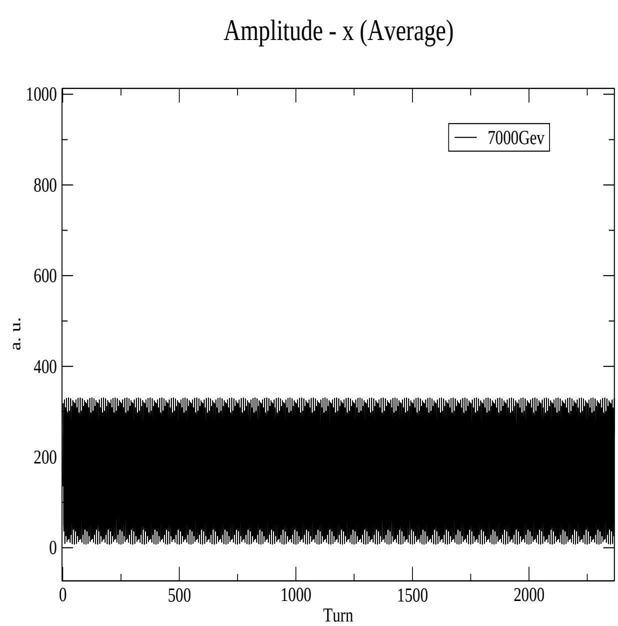

Figure 2: The amplitude of the oscillations of the beam is roughly constant. The beam should be inside the stability boundary diagram

STABILITY DIAGRAMS VS SIMULATIONS

Stability boundary diagram Fig. 4 is obtained for the distribution of particles of our simulation using Eq. 1. Several simulations are performed in different points of the UV plane by changing the impedance (Eq. 3). The points

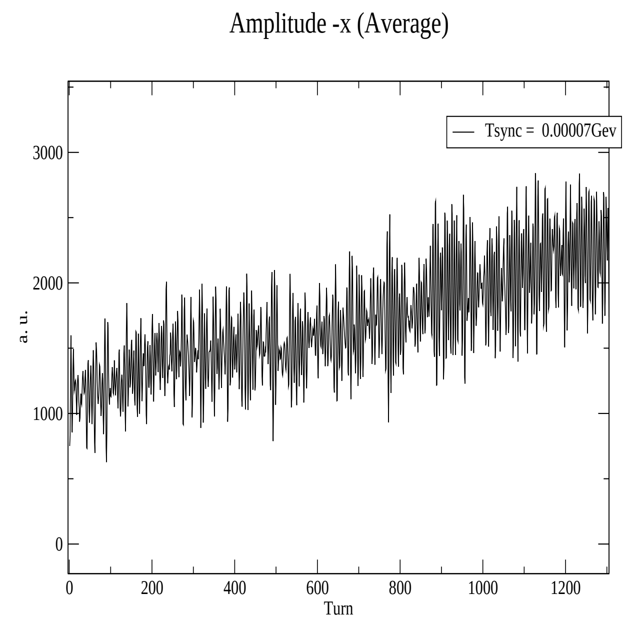

Figure 3: The amplitude of the oscillations of the beam get significantly bigger with turn number. The beam should be outside the stability boundary diagram

found by Eq. 2 are well inside the stability boundaries, even though some simulations are clearly unstable.

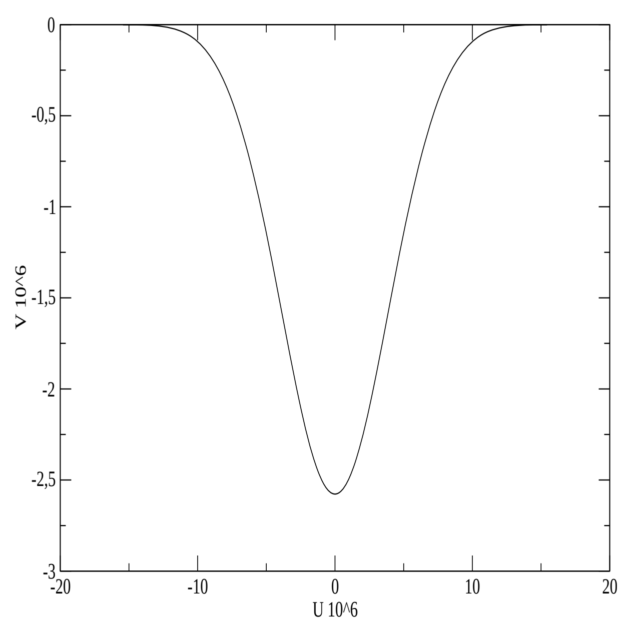

Figure 4: Stability boundary diagram vs simulations

Discrepancies might be due to the simplified model used for comparison. Future works will use dispersion relations that will take into account non linear space charge and external non linear forces as proposed in reference [7].

CONCLUSIONS

Simulations performed with SIMBAD have reproduced the tune shift expected from basic theoretical formulations. However, the stability limits of the beam are not consistent with the Möhl relations either because a very simplified model was used or a major drawback of the simulation. Work will continue in two directions, improving the simulation and using a more sophisticated model.

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