

# Observations of Underdense Plasma Lens Focusing of Relativistic Electron Beams

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Particle Accelerator Conference 2007

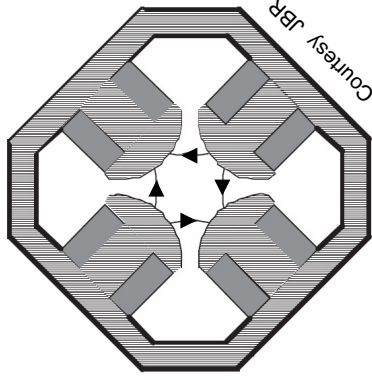
June 27, 2007 - Albuquerque, NM



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This work was performed under the auspices of the US Department of Energy under  
Contract No. DE-FG03-92ER40693 and W-7405-ENG-48.

# Advanced Electron Beam Lens

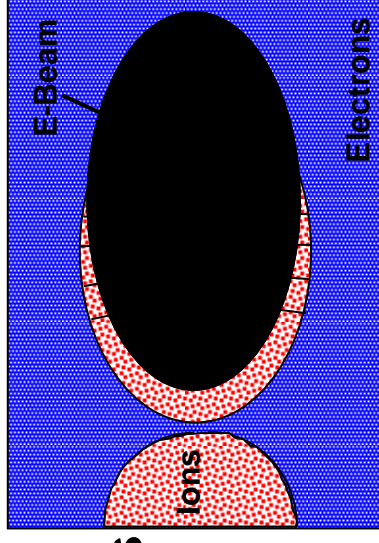


## Magnetic Quadrupoles

Uses magnetic forces to focus electron beam in one dimension at a time.

$$\vec{F}_{\perp} = q\vec{v} \times \vec{B}_{quad} = ecB'(y\hat{y} - x\hat{x})$$

$$B' \approx 250 \text{ T/m} \quad \text{State-of-the-Art Superconducting Quad}$$



## Underdense Plasma Lens

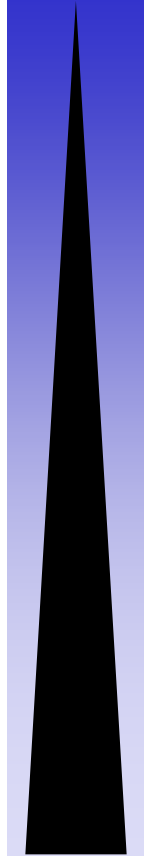
Uses electrostatic forces to focus electron beam in both dimensions.

$$\vec{F}_{\perp} = q\vec{E}_{ion} = 2\pi e^2 n_p (-y\hat{y} - x\hat{x})$$

$$B'_{equivalent} = 3 \times 10^{-11} n_p \text{ T/m}$$

## Adiabatic Focusing: Adiabatic increase in $B'$

- Circumvents limits on focusing due to synchrotron radiation induced chromatic aberrations.
- Plasma lens are ideally suited for adiabatic focusing.



Even “weak” plasma lens are immensely strong:

**150 T/m at  $5 \times 10^{12} \text{ cm}^{-3}$**

**1500 T/m at  $5 \times 10^{13} \text{ cm}^{-3}$**

**Head of the beam is not focused.**

# Plasma Lens Regimes

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## Overdense

$$n_b \ll n_p$$

Plasma cancels beam's space charge and remaining beam magnetic forces focus the beam

$$F_r \approx 2\pi n_b e^2 r$$

Since  $n_b$  is not generally uniform, overdense lens have significant aberrations.

### Spherical Aberrations:

(Gaussian Beam)

$$\frac{\Delta K}{K} \geq 0.22$$

Previous Experiments

## Underdense

$$n_b > \frac{n_p}{2}$$

Plasma electron ejected from beam entirely, uniform ion column focuses beam.

$$F_r = 2\pi n_p e^2 r$$

$n_p$  can easily be both uniform and adjustable.

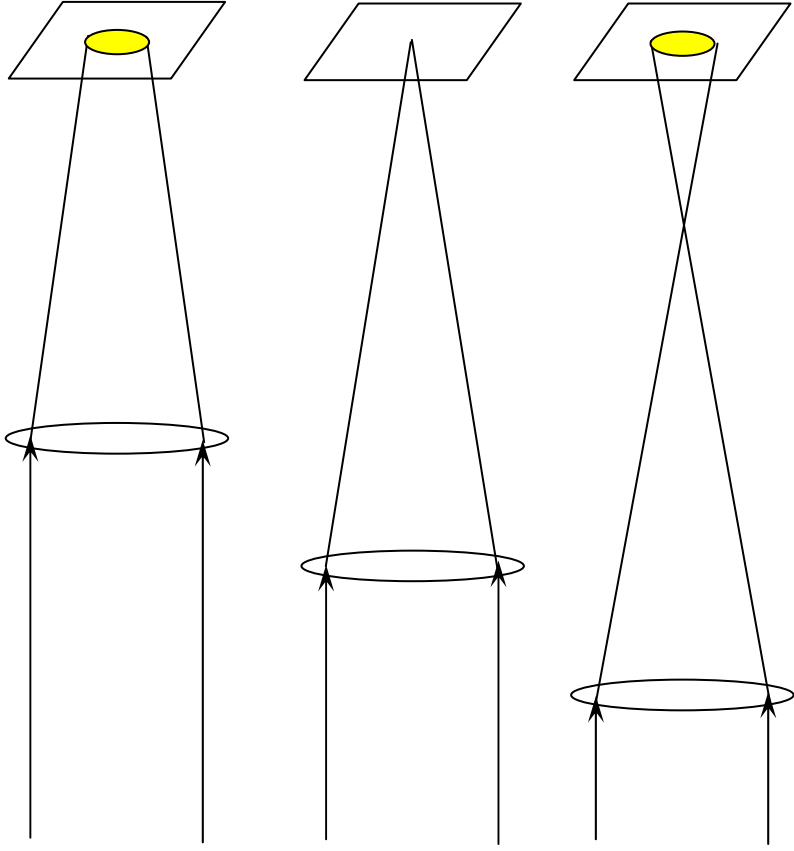
### Spherical Aberrations:

$$\frac{\Delta K}{K} \approx 0$$

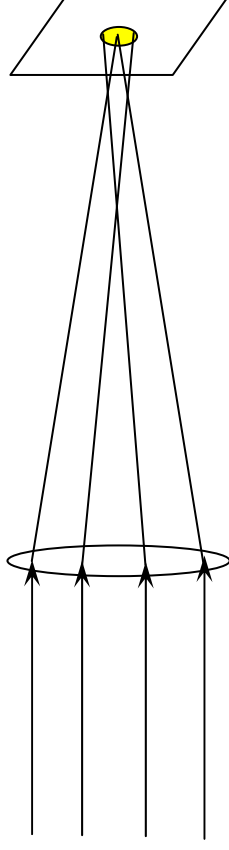
Our New Results

# A Simple Lens Experiment

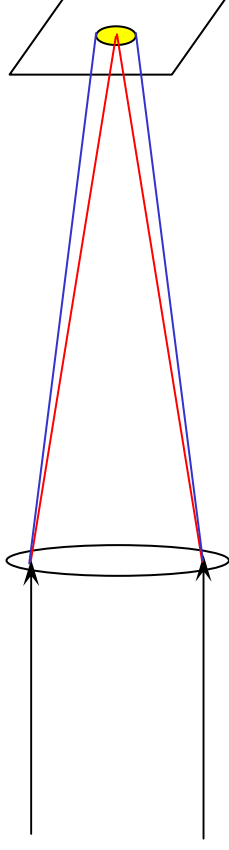
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Focusing a Perfect Beam Through  
a Waist with a Perfect Lens



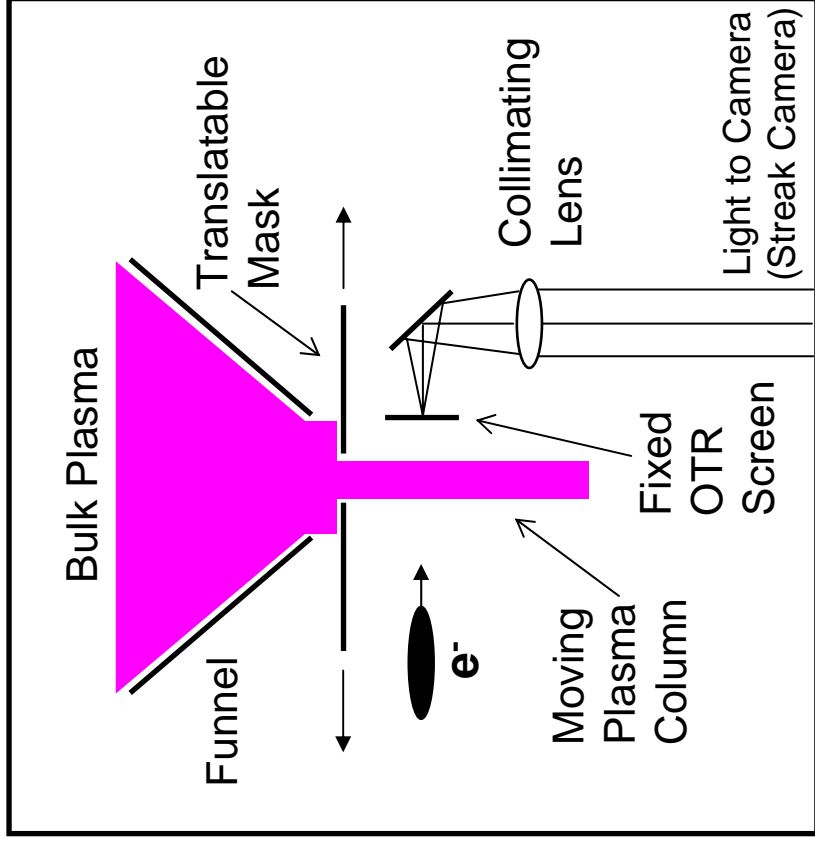
The Waist with Spherical Aberration



The Waist with Chromatic Aberration

# Underdense Plasma Lens Exp Parameters

A 1.25 cm wide moving window is placed in front of the fixed 5 cm plasma column. The resulting translatable plasma column is used to focus the beam onto a fixed OTR screen.



Schematic of the Experiment

## Experimental Parameters

Peak Plasma Density	$4.9 \times 10^{12} \text{ cm}^{-3}$
Plasma Width (FWHM)	19.3 mm
Beam Energy	14.8 MeV
Beam Charge	18.8 nC
Beam Duration ( $\sigma_t$ )	22 psec
Initial Beam ( $\sigma_x$ )	692 $\mu\text{m}$
Beam Emittance ( $\epsilon_{x,n}$ )	87 mm-mrad
Peak Beam Density	$2.5 \times 10^{12} \text{ cm}^{-3}$
Focal Length $f = 1/Kl$	1.9 cm

$$n_b \approx \frac{n_p}{2}$$

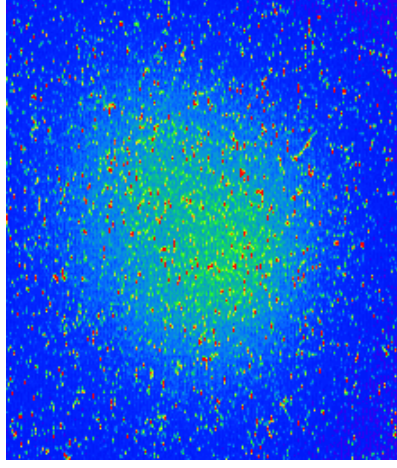
**Just on the boundary of the underdense regime on average.**

# Plasma Focusing Results

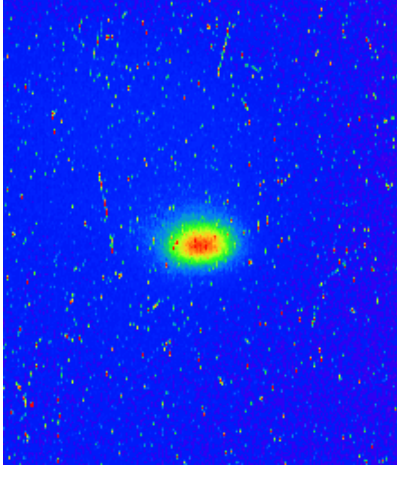
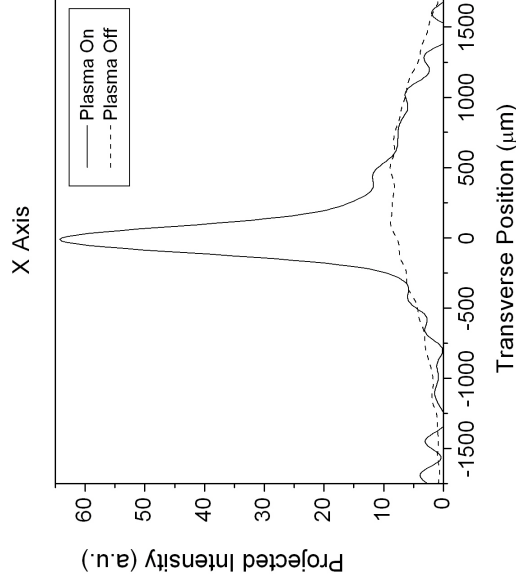
Beam Spot Before:  
x FWHM = 1630  $\mu\text{m}$   
y FWHM = 1540  $\mu\text{m}$   
 $n_b = 2.5 \times 10^{12} \text{ cm}^{-3}$

Beam Spot After (Ave.):  
x FWHM = 260  $\mu\text{m}$   
y FWHM = 420  $\mu\text{m}$   
 $n_{b,\text{core}} \approx 5 \times 10^{13} \text{ cm}^{-3}$

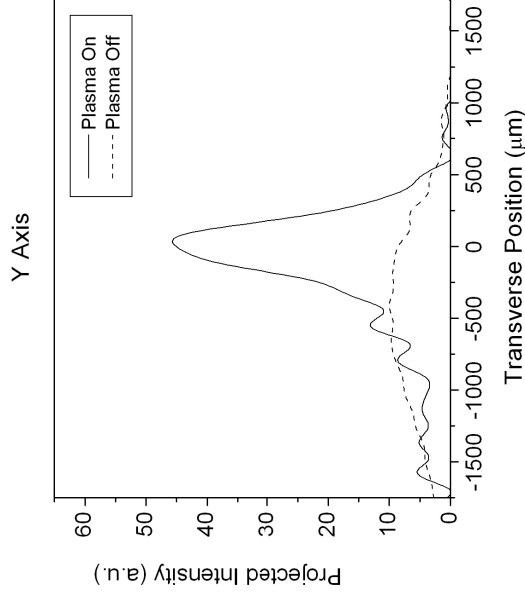
The transverse area of the beam core is reduced by a factor of 23.



Unfocused – 5 electron pulses



Focused – 1 electron pulse



Plots of the image intensity of the above photographs  
(normalized to 1 electron pulse).

# Examining the Beam Envelope Near the Waist

The transverse evolution of an electron beam can be described by (neglecting space charge effects) the beam envelope equation:

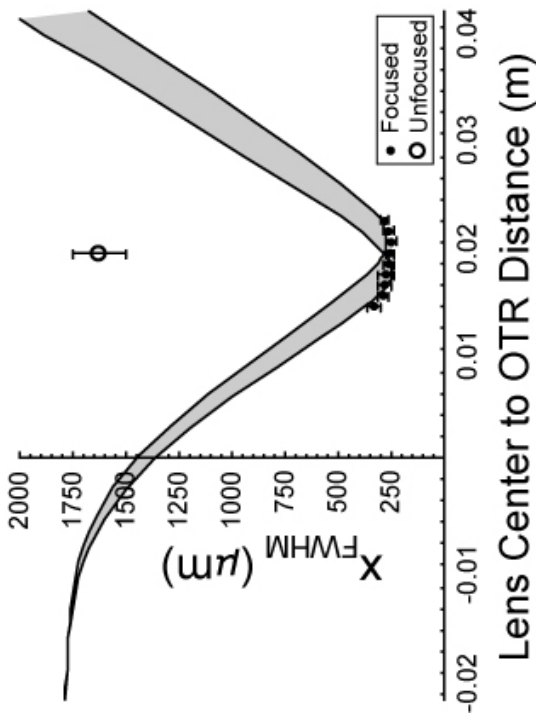
$$\frac{d^2 \sigma_x}{dz^2} + K \sigma_x = \frac{\mathcal{E}_{eff}^2}{\sigma_x^3}$$

For our underdense plasma lens the focusing strength  $K$  is a function of  $z$ :

$$K = \frac{2\pi r_e n_p(z)}{\gamma}$$

Using the known plasma lens parameters, and leaving the effective emittance as the free parameter, the beam envelope equation was fit to the data.

The focused electron beam spot was measured at several different lens/screen spacing near the waist.



**Behavior of the Beam Spot Size Near the Waist**

**Effective Emittance Value Extracted from Best Fit:**

$$\mathcal{E}_{eff,x,n} = 110 \text{ mm-mrad}$$

# Focusing: Aberrations and Saturation

From the envelope data:

$$\mathcal{E}_{eff, x, n} = 110 \text{ mm-mrad}$$

From quadrupole scans:

$$\mathcal{E}_{x, n, 0} = 87 \text{ mm-mrad}$$

$$\mathcal{E}_{eff}^2 = \mathcal{E}_0^2 + \frac{\sigma_0^4}{f^2} \left( \frac{\Delta K}{K} \right)^2$$



**Total Lens Aberrations:**

$$\frac{\Delta K}{K} = 0.076 \pm 0.006$$

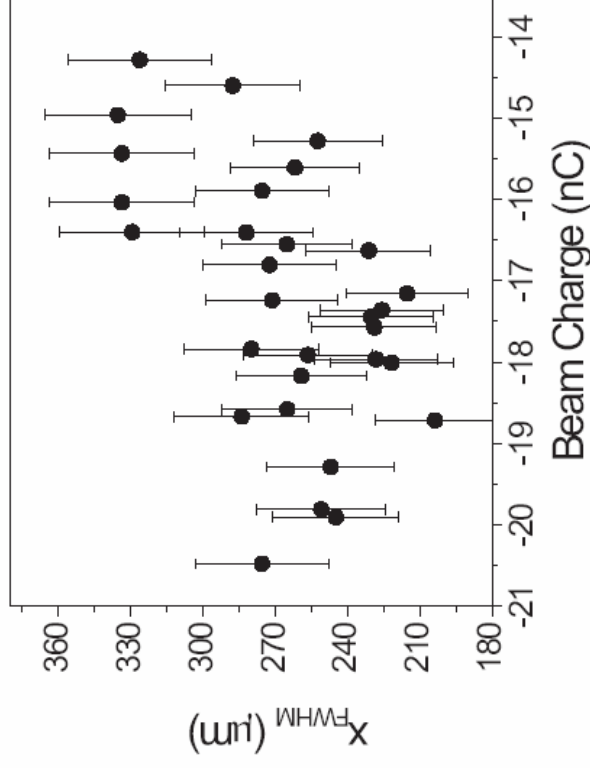
Overdense

Minimum:

$$< 0.22$$

**Chromatic Contribution:**

$$\left( \frac{\Delta K}{K} \right)_{chromatic} \leq 0.025$$



Dependence of the focused beam spot size on the beam charge under otherwise similar conditions. The saturation of the focusing effect is consistent with a transition to underdense operation.

**Strong evidence that we have low-aberration underdense behavior even at the regime threshold.**



# Conclusions

- We have demonstrated a compact, high demagnification plasma lens for relativistic electron beams operating at the threshold of the underdense regime.
- The aberrations of the underdense lens are much lower than in the overdense case, as predicted by theory, even near the threshold.
- Recent work pointing out the problem of **ion collapse** in the ILC afterburner scenario also has implications for final focus plasma lens, especially those of the adiabatic variety. The **near threshold underdense regime** may prove optimal.

## Further Analysis

Efforts are underway to match the time-integrated and time-resolved measurements made in this experiment to simulation.

