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LABORATORY FOR ELEMENTARY-PARTICLE PHYSICS

Cornell University

Spin Transport in the International Linear Collider

Jeffrey C. Smith

CLASSE, Cornell University, Ithaca, NY, USA

and

SLAC, Menlo Park, CA, USA

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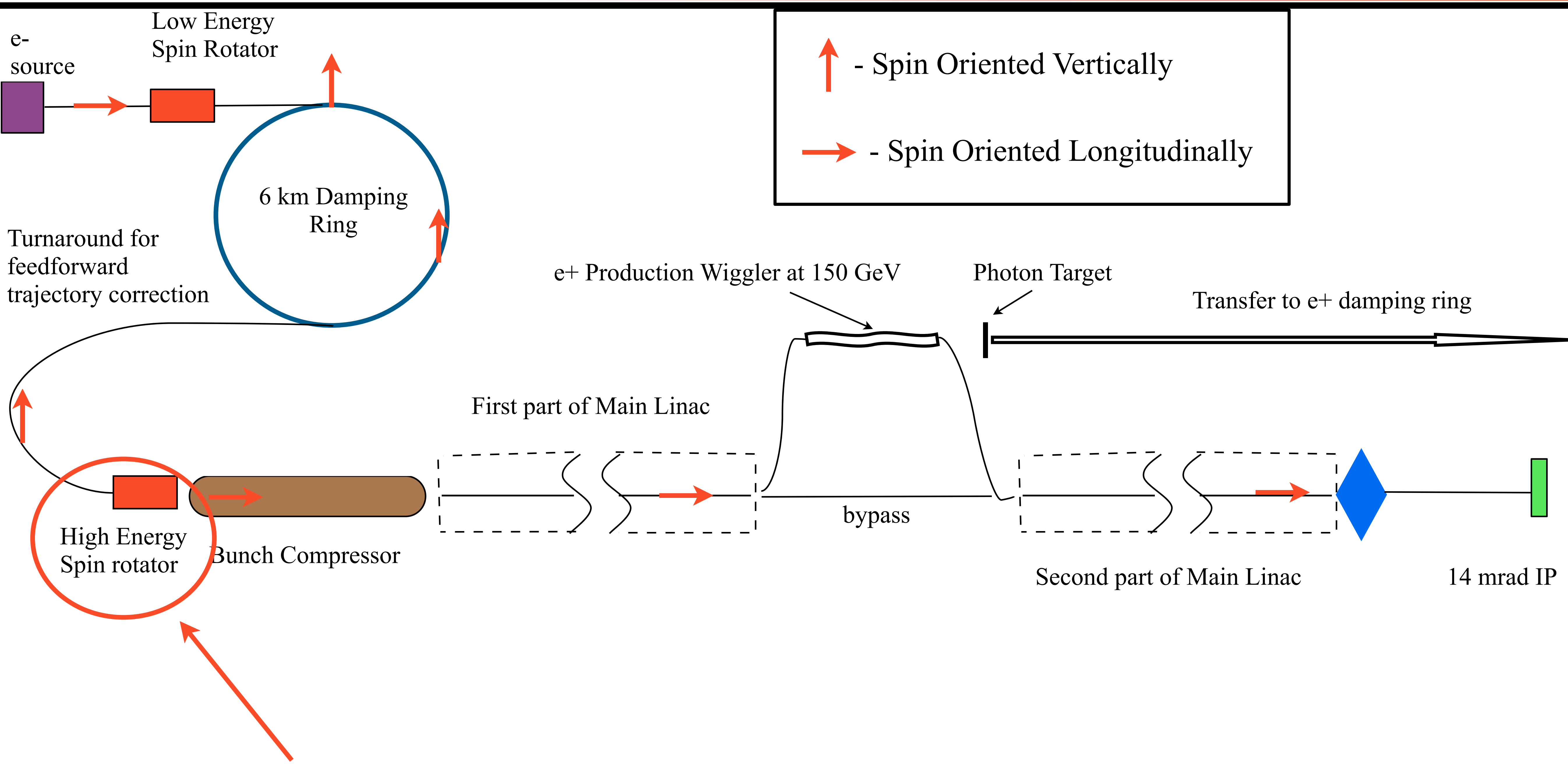
Contributed Oral Presentation

WEOAAB01

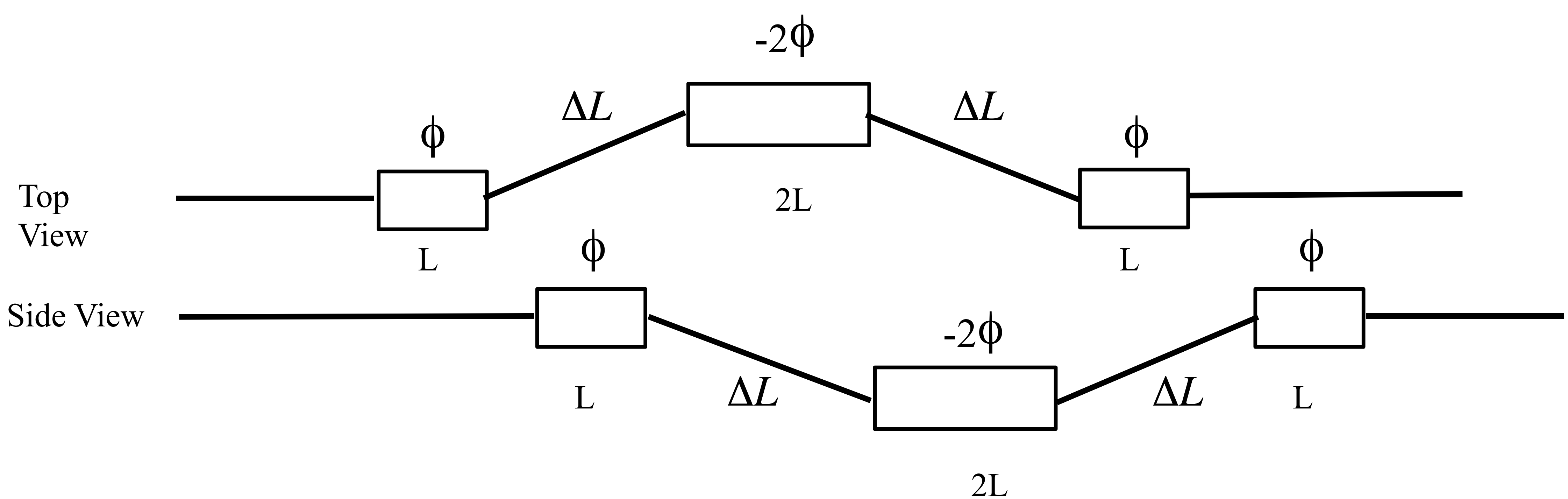
ILC Layout with Spin Rotators



- The High Energy Spin Rotator after Damping Ring (at 5 Gev) will be discussed here.

Use a Half Serpent?

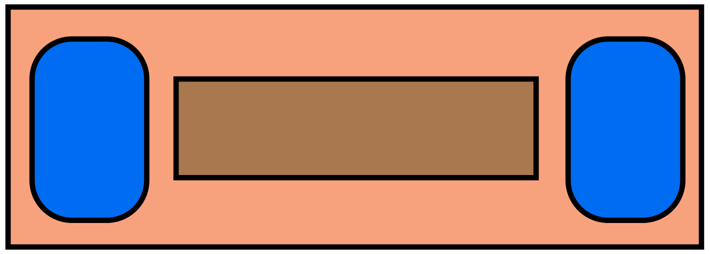
- Could use nested horizontal and vertical chicanes to manipulate spin
  - Simple design
  - But must be careful about synchrotron radiation emittance growth and R_56 term...
    - Each vertical bend would have to be about 1000 meters long to keep vertical emittance from growing even 2%
    - R_56 ~800 meters in such a setup -- totally unacceptable
    - Spin rotation is fixed, we want full variability in exiting polarization



The Solenoid Solution

- A Solenoid can be used instead to perform the spin manipulation
  - However, solenoids also roll the beam introducing x-y coupling
  - The key is rotating the spin and decoupling the beam.
    - This can be done by spitting the solenoid in half and introducing a canceling symmetry between the two halves.

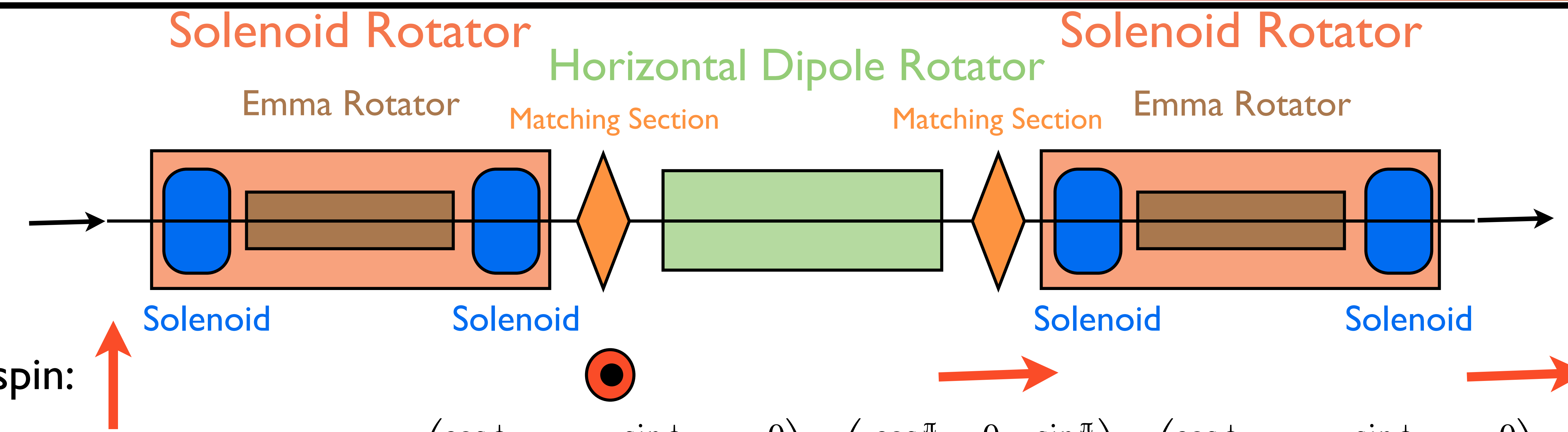
Emma Rotator



Solenoid Solenoid

    - First solenoid rotates spin by half the desired total
    - Then a transfer line which is +1 in x and -1 in y will reflect the beam about the y-axis
    - Finally, the second solenoid (of equal strength) rotates the spin the rest of the way as it rotates the beam back to a flat state.
  - Changing the spin rotation angle is simply done by changing the strength of the two solenoids.

Fully Flexible System



$$\Omega_{tot} = \Omega_{sol34} \cdot \Omega_{bend} \cdot \Omega_{sol12} = \begin{pmatrix} \cos\phi_{sol34} & -\sin\phi_{sol34} & 0 \\ \sin\phi_{sol34} & \cos\phi_{sol34} & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \cos\frac{\pi}{2} & 0 & \sin\frac{\pi}{2} \\ 0 & 1 & 0 \\ -\sin\frac{\pi}{2} & 0 & \cos\frac{\pi}{2} \end{pmatrix} \cdot \begin{pmatrix} \cos\phi_{sol12} & -\sin\phi_{sol12} & 0 \\ \sin\phi_{sol12} & \cos\phi_{sol12} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -\sin\phi_{sol34}\sin\phi_{sol12} & -\sin\phi_{34}\cos\phi_{12} & \cos\phi_{sol34} \\ \cos\phi_{sol34}\sin\phi_{sol12} & -\cos\phi_{34}\cos\phi_{12} & \cos\phi_{sol34} \\ -\sin\phi_{sol12} & \sin\phi_{sol12} & 0 \end{pmatrix}$$

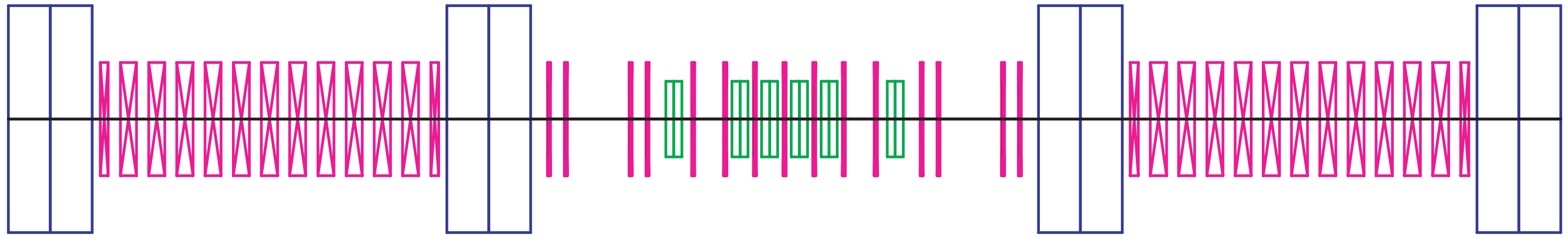
The incoming orientatiin is vertical, so

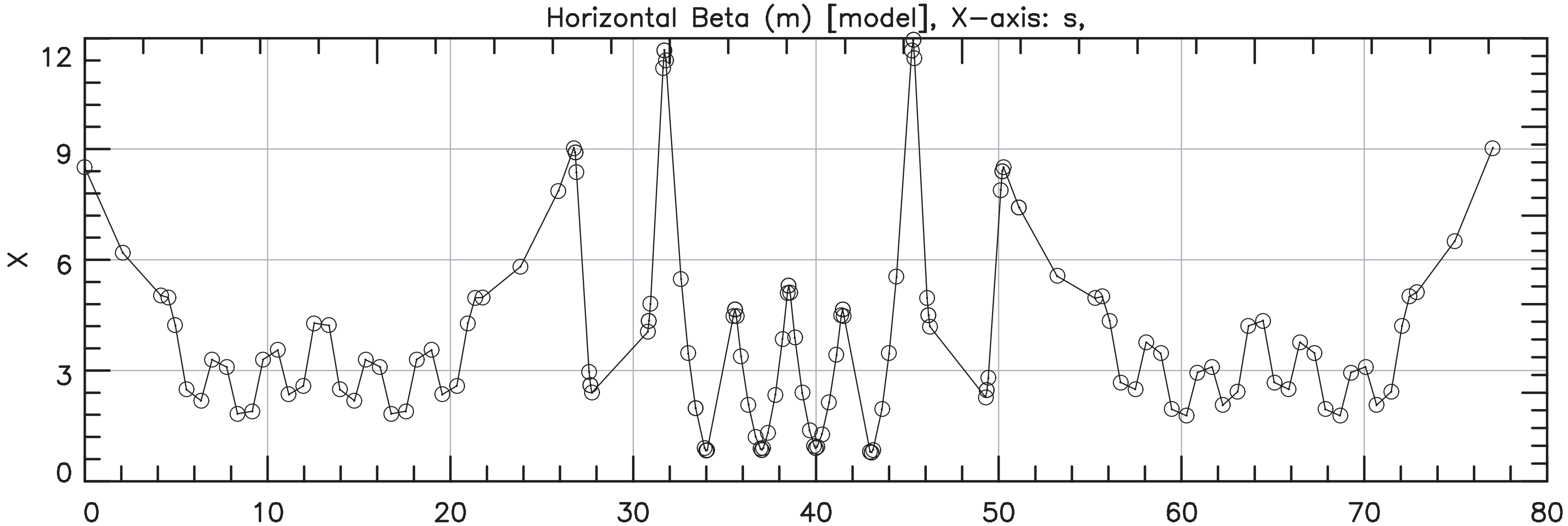
$$\bar{S} = \Omega_{tot} \cdot \begin{pmatrix} 0 \\ \pm 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \mp\sin\phi_{sol34}\cos\phi_{sol12} \\ \pm\cos\phi_{sol34}\cos\phi_{sol12} \\ \pm\sin\phi_{sol12} \end{pmatrix}$$

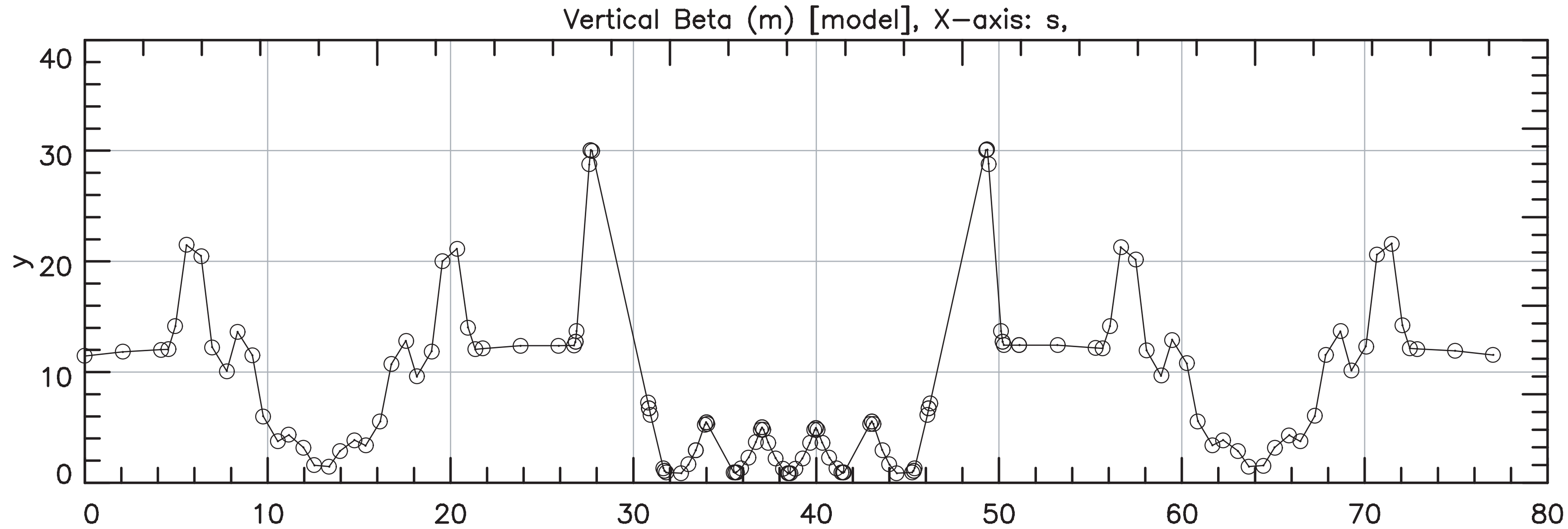
If the solenoidal fields are reversible then any arbitrary spin orientation can be achieved.

Current Design

- System works over entire range of exit polarization
  - Design a hybrid of Emma/Walker/Schmid/ Smith







Current Design Characteristics

- Full flexibility on outgoing spin by manipulating solenoid strengths.
- Emittance tends to blow up in Emma Rotators due to chromaticity
  - decreasing phase advance per Emma FODO cell and lengthening the quads lowers emittance growth
  - Spin Rotator must be before bunch compressor or large energy spread will blow up emittance
  - current design limits the growth to 0.01 nm in ideal case (no misalignments)
- R_56 = -6 mm which is small compared to the -800 mm in bunch compressor
- Changing solenoid strengths changes solenoid focusing so matching sections do need to be tweaked to maintain beta functions.
- Optimizing the matching sections also improves emittance dilution.

Spin Dynamics

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- Relativistic spin motion in an electromagnetic field is governed by the T-BMT equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{s} = \Omega_{BMT}(\mathbf{r},\mathbf{p},t)\times\mathbf{s}$$

$$\Omega_{BMT}(\mathbf{r},\mathbf{p},t) = -\frac{q}{m\gamma}\left[(1+G\gamma)\mathbf{B} - \frac{G\mathbf{p}\cdot\mathbf{B}}{(\gamma+1)m^2c^2}\mathbf{p} - \frac{1}{mc^2}\left(G+\frac{1}{1+\gamma}\right)\mathbf{p}\times\mathbf{E}\right]$$

- For no electric fields or longitudinal magnetic fields, T-BMT takes on a simpler form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{p} = -\frac{q}{m\gamma}\{\mathbf{B}_\perp\}\times\mathbf{p} \qquad \text{Lorenz}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{s} = -\frac{q}{m\gamma}\{(G\gamma+1)\mathbf{B}_\perp + (1+G)\mathbf{B}_\parallel\}\times\mathbf{s} \qquad \text{T-BMT}$$

For fixed orbit deflections (fixed ratio $\frac{\mathbf{B}_\perp}{\gamma}$), spin precession increases with energy

- In other words, if the orbit is deflected by an angle ϕ then the spin is rotated by an angle $G\gamma\phi$ relative to the orbit.
- G = 0.00116 for electron, so with $\gamma = 4.9\times 10^5$, spin precession can be quite large

BMAD Spin Tracking

- Spin Tracking has been implemented in BMAD using a spinor-quaternion transfer map method:

Spinor $= \Psi = (\psi_1, \psi_2{}^T)$, ψ_1 and ψ_2 are complex numbers

$$s = \Psi^\dagger \sigma \Psi \Longleftrightarrow \Psi = \frac{1}{\sqrt{2(s_3+1)}} \begin{pmatrix} 1+s_3 \\ s_1 + is_2 \end{pmatrix}$$

Spin tracking via:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Psi = -\frac{i}{2}(\sigma \cdot \Omega)\Psi \qquad \text{T-BMT}$$

$$\Psi = e^{-i\frac{\alpha}{2}e\cdot\sigma}\Psi_i$$

$$\Psi(z,\theta) = (a_0 \mathbf{1}_2 - \mathbf{i}\mathbf{a}\cdot\sigma)\Psi(z_i,\theta_i)$$

And the Four-Vector, $\mathbf{A} = (a_0, \mathbf{a})$ describes the transfer map for each element. Tracking through any element is simply achieved via the application of these quaternions in sequence. This results in very fast tracking times.

Spin Transfer Maps

- The quaternion Four-Vector is expressed as a Taylor Series in particle orbit.
- Maps have been found to second order for bends, quadrupoles and sextupoles.
- Solenoids and RF cavities currently have a first order map.
  - Fringe fields in solenoids insignificant in spin motion
  - RF cavity fields also insignificant, even for 7000 of them in the main linacs

Numerical Spin Tracking

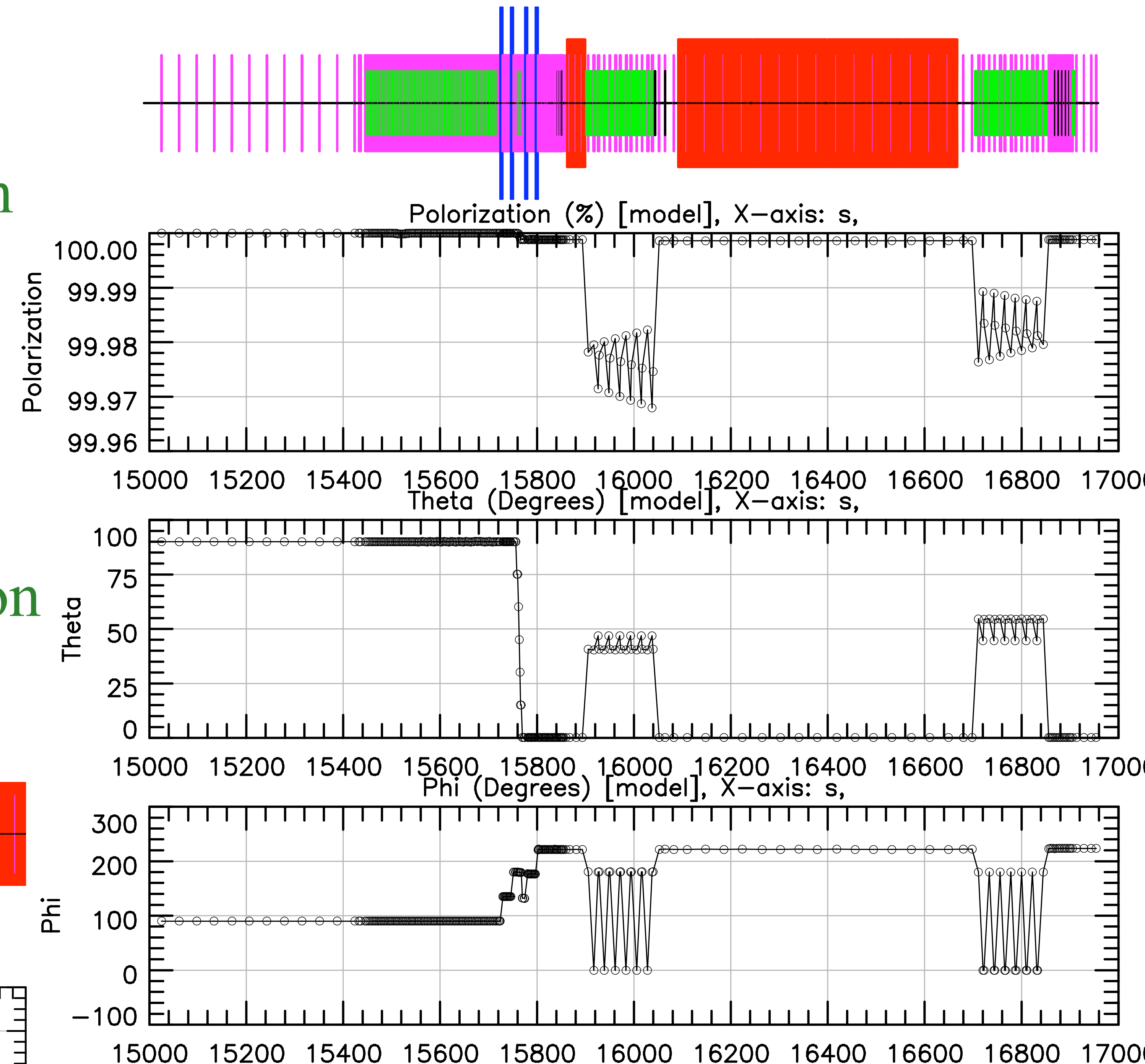
- A numerical spin tracker has also been implemented. It is an extension of the Boris-like numerical integrator of Stolz, Cary, Penn and Wurtele
- The Boris Method is second order accurate, requires only one force calculation per particle per step and preserves conserved quantities more accurately over long distances than a Runge-Kutta integration scheme.
- In many cases, it is also more efficient than Runge-Kutta.

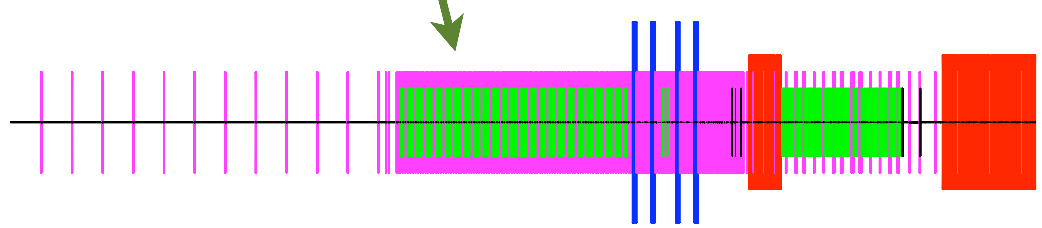
Damping Ring

- Damping Ring designed to preserve polarization
  - Spin tune (for 5 GeV) = $G\gamma = 11.35$
  - No fractional ratios between spin tune and betatron tune so that no spin-orbit coupling occurs
  - Even if on spin-tune resonance at 4.8 GeV, after only 8000 turns beam not in DR long enough for large depolarization to occur.
  - Beam not in DR long enough for spin-flip radiation to be a concern.
  - Misalignments not a concern, but extraction polarization direction is sensitive to injection polarization direction.
- But what about after the damping ring?
  - Gamma function grows large.
  - Perhaps some depolarization occurs.

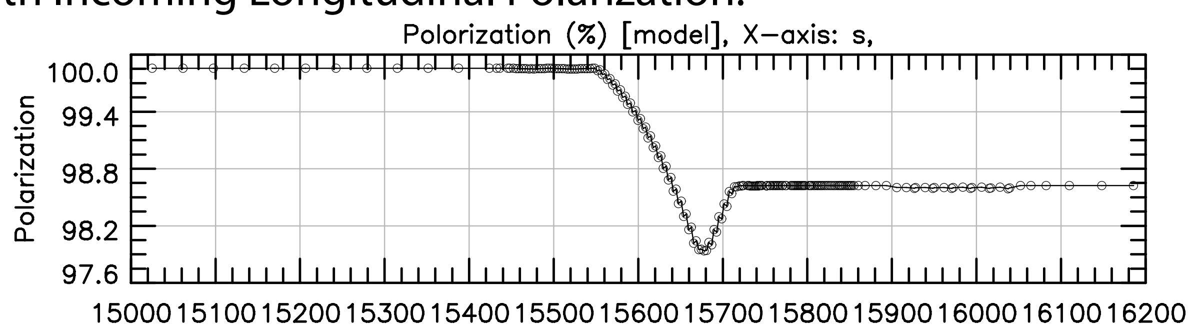
Ring To Main Linac

- At 5 Gev gamma function still relatively small
- Some measurable effect in bunch compressor wigglers but no net depolarization
- Misalignments have little effect
- Spin rotator should be after turnaround, or else, depolarization in turnaround



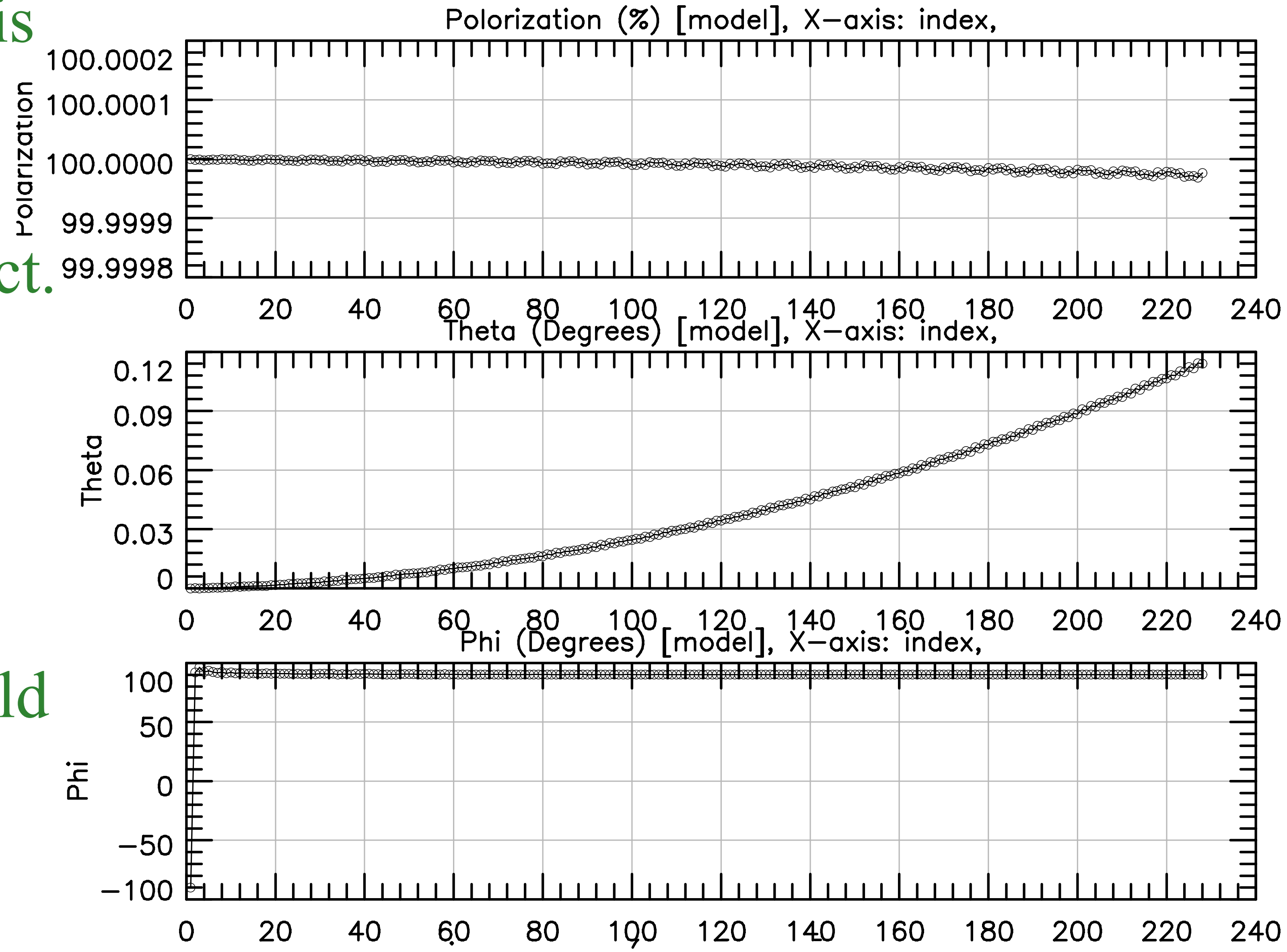


With Incoming Longitudinal Polarization:



Main Linac

- Lots of RF cavities but effect is just too weak for any depolarization too occur
- Misalignments have little effect.
- Tunnel bending to follow Earth's curvature causes polarization vector to curve
- This isn't including Earth magnetic field which has a field integral of 0.54 T.m along the ML which will also effect the spin
  - Spin rotator can be adjusted to take out effect on spin precession.

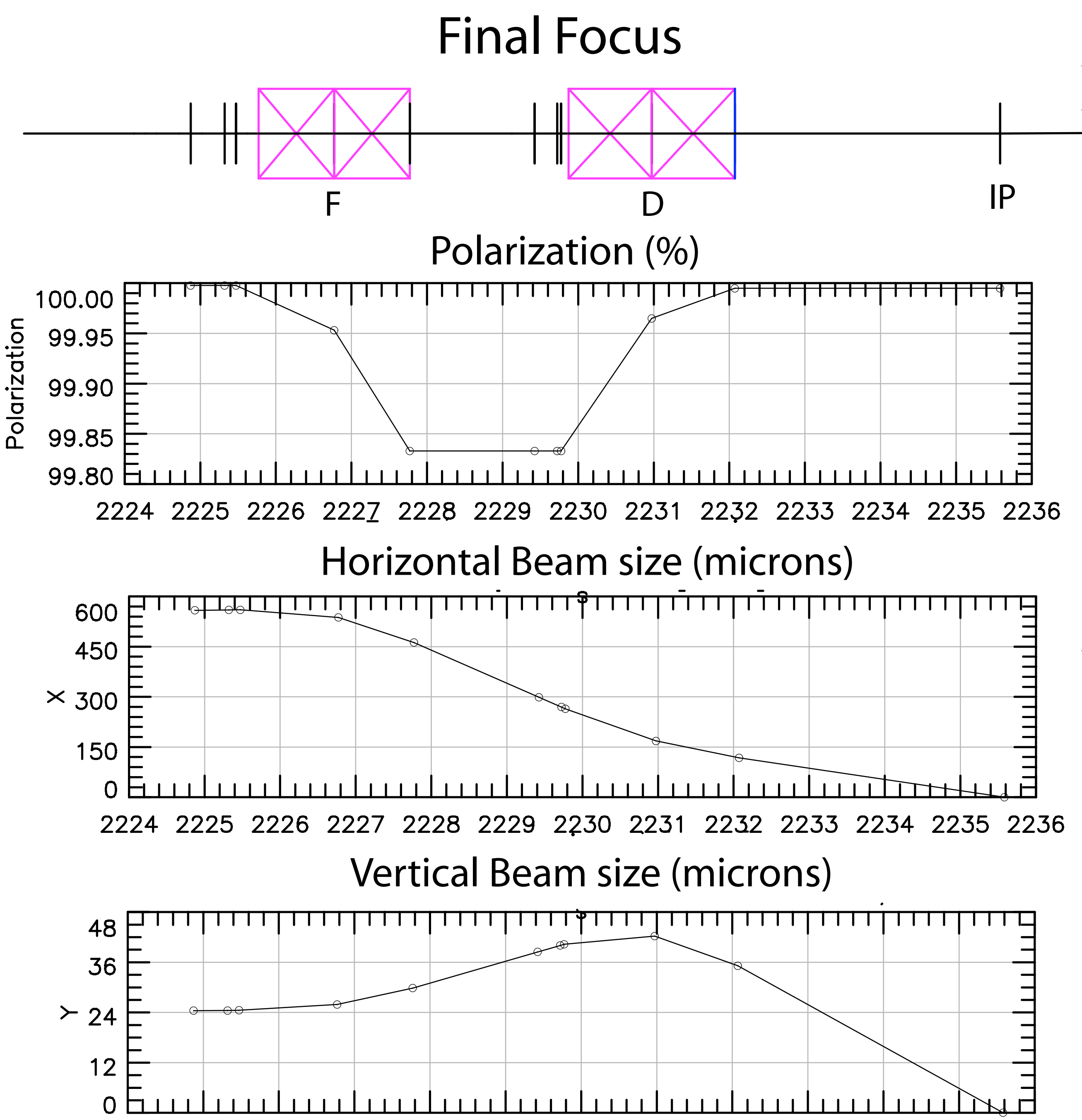


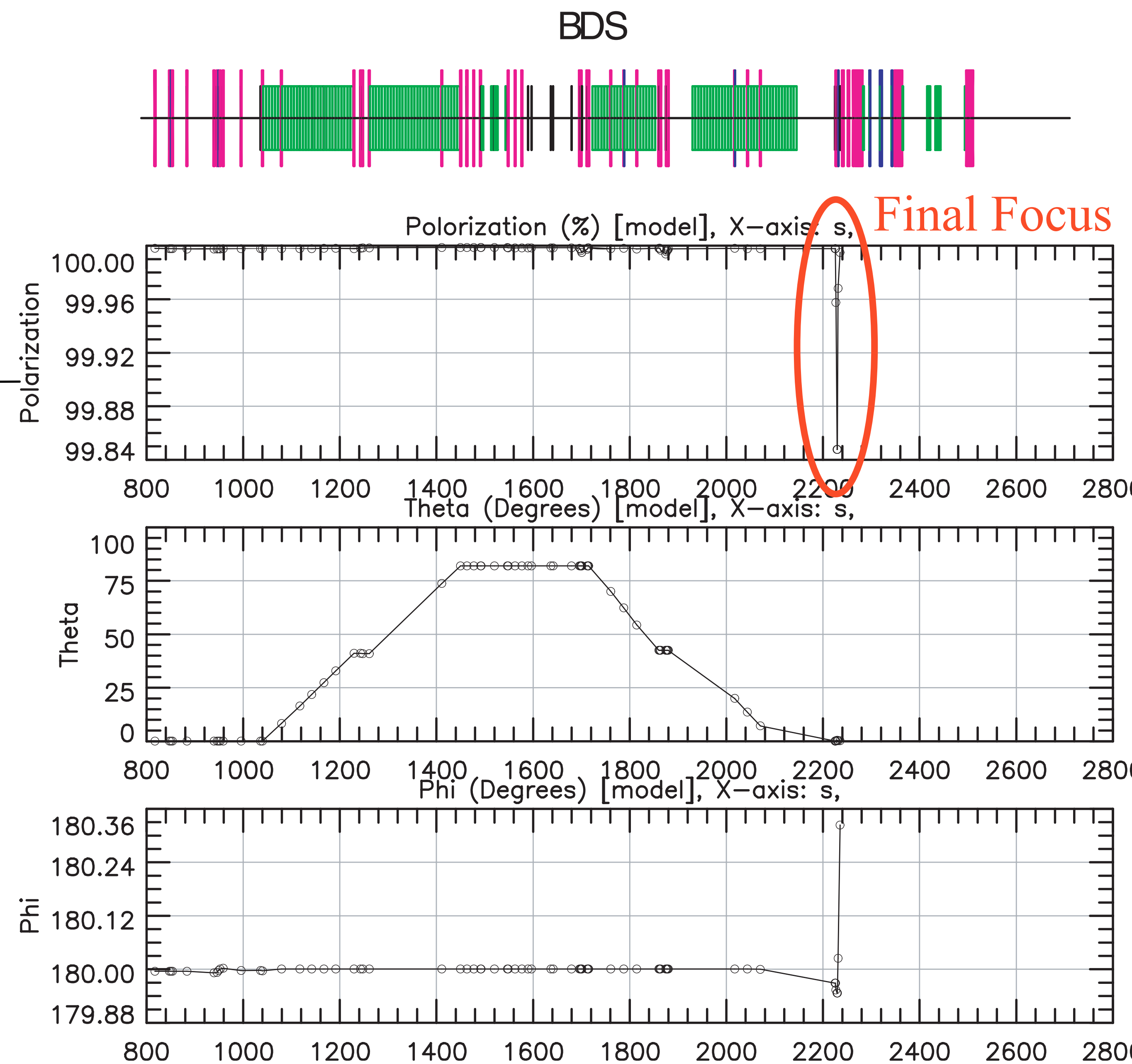
Helical Undulator

- What happens to the polarized electron beam as it passes through the helical undulator?
  - Previous analytical studies have shown there to be no problem.
  - Tracking simulations have yet to be performed.

Beam Delivery System

- Spin tune: G.gamma = 567 which is getting large
- Some effect seen in final focus





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IP Beam-Beam Interaction

- Studies have included both coherent and incoherent background processes.
- Current studies show depolarization at IP is less than 1% for all beam parameter sets
- Depolarization roughly equally split between T-BMT and Sokolov-Ternov (Spin Flip) effects at high energy.
- Details in poster THPMN083 presented by Duncan Scott.

Thank You

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