Self-Consistent Computation of Electromagnetic Fields and Phase Space Densities for Particles on Curved Orbits [1]

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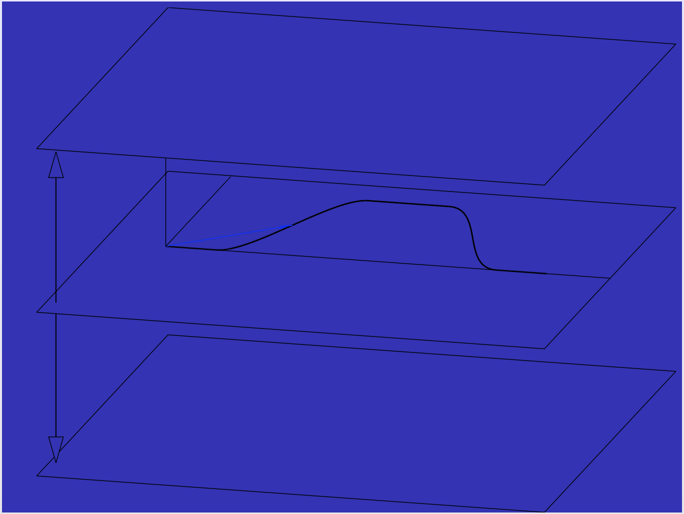
Outline

1. Vlasov-Maxwell with Shielding: Planar Case
2. Maxwell in Lab Frame with fixed vertical density
   Field Formula and its computation
3. Vlasov in Beam Frame
   Beam Frame Phase Space Density and Relation to Lab Charge/Current Densities
4. Two Numerical Approaches:
   Method of Local Characteristics, in progress
   Self-Consistent Monte Carlo, code developed
5. Self-Consistent Monte Carlo Results

Statement of Problem - I



Planar Vlasov-Maxwell System in Lab Frame

$$(\triangle - \partial_u^2)\mathcal{E} = H(Y)S(\mathbf{R}, u), \qquad \mathcal{E} = (E_Z, E_X, B_Y)$$
$$\partial_u \Psi + \dot{\mathbf{R}} \cdot \nabla_{\mathbf{R}} \Psi + \dot{\mathbf{P}} \cdot \nabla_{\mathbf{P}} \Psi = 0, \qquad u = ct,\ \mathbf{R} = (Z, X)$$
$$\mathcal{E}(Y = \pm h/2) = 0, \qquad \mathbf{P} = m\gamma\mathbf{V}$$

- Vertical Charge Density $H(Y)$ is fixed
- $\dot{\mathbf{R}}$ and $\dot{\mathbf{P}}$ are given by the Lorentz Equation

Planar Vlasov-Maxwell Continued

- Continuity Equation $\Leftarrow$ Vlasov Equation
- Poynting Theorem and Energy Conservation, and CSR vs. ISR
  Heinemann, et. al. FRPMN101 Friday
- 1D charge/current model
  Warnock FRPMS083 Friday
- Self Consistent Monte Carlo Results
  Bassi, et. al. THPAN084 Thursday

Solution of 3D wave equation with Shielding BC

Solve: $(\triangle - \partial_u^2)\mathcal{E} = H(Y)S(\mathbf{R}, u), \qquad \mathcal{E}(Y = \pm h/2) = 0$

1. Eigen Expansion in $Y \longrightarrow$ 2D Klein-Gordon with no BC
   Trick $\longrightarrow$ 3D Wave Equation with no BC
2. 3D wave equation solved with retarded Green's function
3. Make temporal argument of source a variable of integration

Solution:

$$\mathcal{E}(\mathbf{R}, Y, u) =$$

$$-\frac{1}{4\pi}\int_{-u}^{u} d\eta\, G(\eta, Y)\int_0^{u-|\eta|} dv \int_0^{2\pi} d\theta\, S(\mathbf{R} + \sqrt{(u-v)^2 - \eta^2}\,\mathbf{e}_\theta, v)$$

where $\mathbf{e}_\theta = (\cos\theta, \sin\theta)$ and no singularity.

Only assumption: Planar Motion

Approximate Field Formula

Approximations (to decrease computational time):

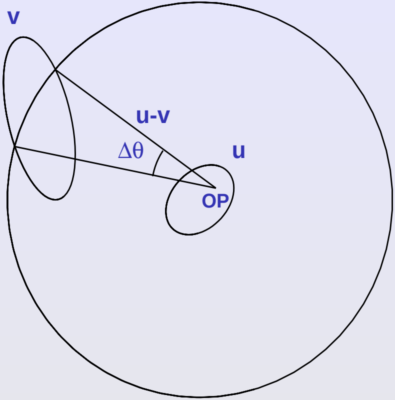
- Average fields over $H(Y)$ and assume Y extent of beam small compared to h

Without shielding the formula is:

$$\mathcal{E}(\mathbf{R}, u) \approx -\frac{1}{4\pi}\int_0^u dv \int_{\theta_{min}}^{\theta_{max}} d\theta S(\mathbf{R} + (u - v)\mathbf{e}_\theta, v)$$

1. Need to find reasonable θ limits
2. θ integration: superconvergent trapizoidal rule
3. v integration: adaptive Gauss-Kronrod

Evaluation of Integral

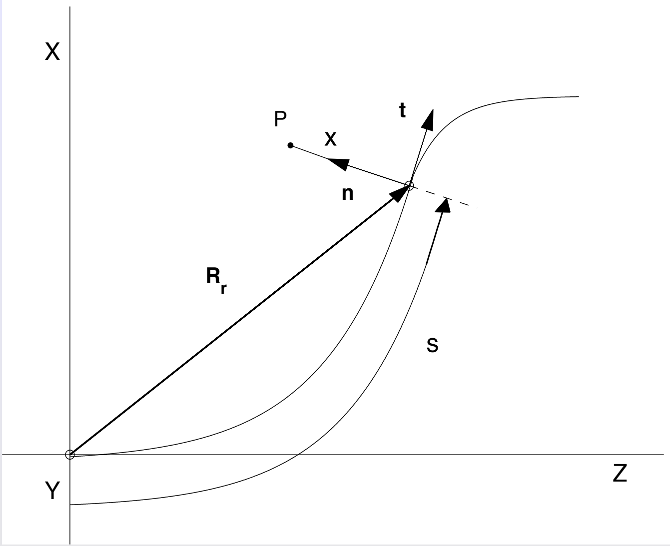


- $\mathcal{E}(\mathbf{R}, u) \approx -\frac{1}{4\pi}\int_0^u dv \int_0^{2\pi} d\theta S(\mathbf{R} + (u - v)\mathbf{e}_\theta, v)$
- circle is domain of dependence of OP at **R** due to source at time v

Beam Frame Coordinates

$\mathbf{R} =$

$\mathbf{R}_r(s) + x\mathbf{n}(s)$



Beam Frame EOM

- Lab to Beam $(Z, X, P_Z, P_X; u) \leftrightarrow (z, x, p_z, p_x, ; s)$

  $z = s - \beta_r u, \quad p_z = \frac{\gamma - \gamma_r}{\gamma_r}$
- EOM

$$\begin{aligned} z' &= -\kappa(s) x \qquad p_z' = F_z \\ x' &= p_x \qquad\qquad p_x' = \kappa(s) p_z + F_x \end{aligned}$$

where the self forces are

$$F_z \approx \frac{q}{P_r c}(\mathbf{E}_\parallel(z,x,s)\cdot\mathbf{t}(s) + p_x \mathbf{E}_\parallel(z,x,s)\cdot\mathbf{n}(s))$$
$$F_x \approx \frac{q}{P_r c}(\mathbf{E}_\parallel(z,x,s)\cdot\mathbf{n}(s) - cB_Y(z,x,s)),$$

EOM with $F = 0$ have been linearized

Relation Between Lab and Beam Frame Densities

- Subtle relation between phase space densities

  $(Z, X, P_Z, P_X; u) \leftrightarrow (z, x, p_z, p_x, ; s)$

  $F_{Lab}(Z, X, P_Z, P_X; u) = \frac{\beta_r^2}{P_r^2} f_{Beam}(z, p_z, x, p_x; s)$

- Lab frame charge and current densities from f_{Beam}

  $\rho_{Lab}(\mathbf{R}; u) \approx \int dp_z dp_x f_{Beam} = \rho_{Beam}(z, x; s),$

  $\mathbf{J}_{Lab}(\mathbf{R}; u) \approx Q\beta_r c[\rho_{Beam}(z, x; s)\mathbf{t}(s) + \tau(z, x; s)\mathbf{n}(s)]$

  where $\tau = \int dp_z dp_x p_x f_{Beam}$ and $s = z + \beta_r u \rightarrow \beta_r u$.

Method of Local Characteristics-I

2D Lab Frame Fields and 4D Beam Frame Vlasov are

$$\mathcal{E}(\mathbf{R},u) \approx -\frac{1}{4\pi}\int_0^u dv \int_0^{2\pi} d\theta S(\mathbf{R}+(u-v)\mathbf{e}_\theta, v)$$
$$\partial_s f + z'\partial_z f + x'\partial_x f + p_z'\partial_{p_z} f + p_x'\partial_{p_x} f = 0,$$

Basic Idea

1. f, $\mathcal{E}$ and history of charge/current density are known at s
2. Freeze fields at s, then Vlasov equation becomes a Liouville equation on $[s, s+\Delta]$
3. Find f at $s+\Delta$ by integrating backward along characteristics
4. Calculate the fields at $s+\Delta$ from f at $s+\Delta$ and history
5. iterate

Remarks

- Interpolation needed at step 3

Method of Local Characteristics-II

Remarks

- It's hard to imagine a better approach, since this approach preserves the geometry of the solutions.
- Method developed by Warnock and extended by Venturini in 2D
- Extended by Sobol to 4D in collective beam-beam interaction
- Less "noise" than Monte Carlo
- Our 4D work here is in progress

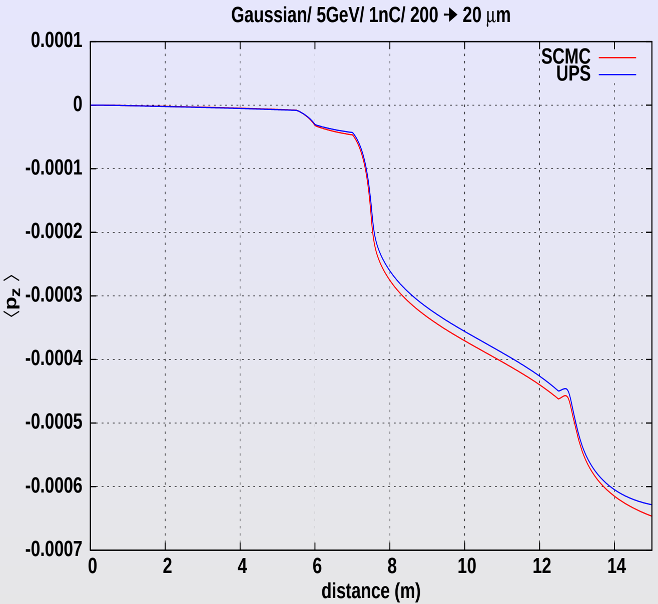
Self Consistent Monte Carlo Method-I

- Outline and comparison with PIC for Vlasov-Poisson
  1. Monte Carlo generation of initial conditions from $f_{Beam}(z, x, p_z, p_x; 0)$ (Similar in VP PIC)
  2. Create a smooth Lab Frame charge density from scattered beam frame phase space points. We use a Fourier method used in statistical estimation. (Charge deposition in VP PIC)
  3. Calculate fields from history of Fourier coefficients using our field formula (Solve Poisson Equation in VP PIC)
  4. Use 3) to move the phase space points (Same in VP PIC)
  5. Go to 2) (Same in VP PIC)

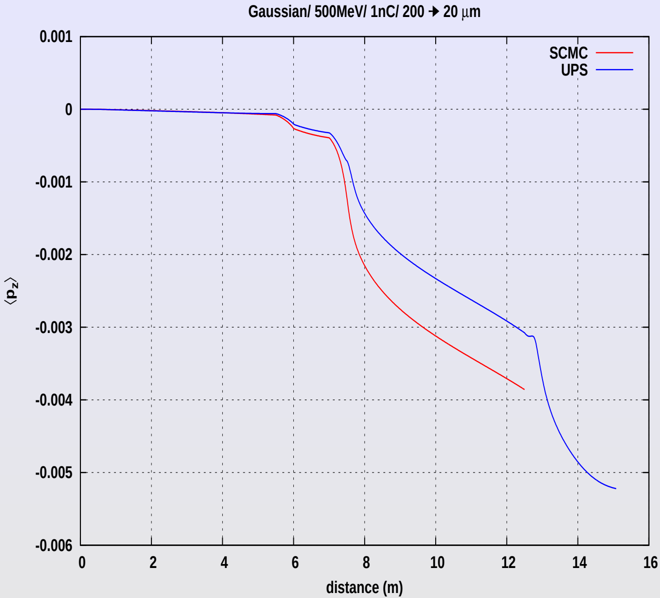
- Unperturbed Source Model
  Fields calculated up front from source evolved with no self forces, F.
  Very fast in Gaussian case, but not self-consistent.

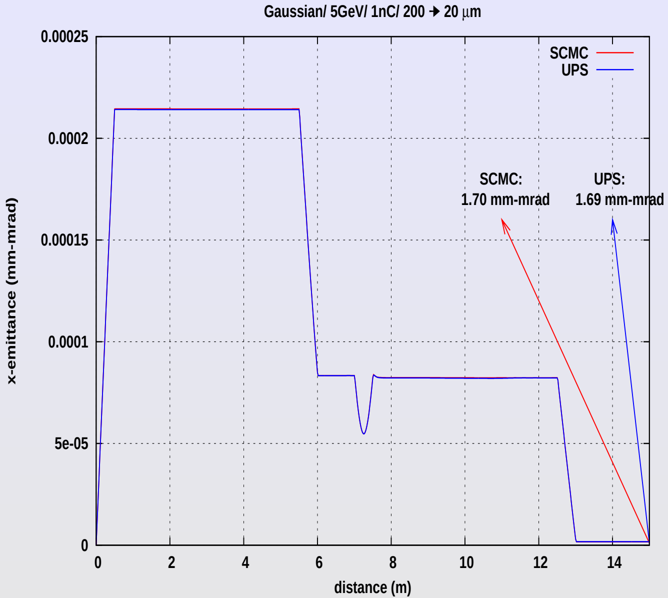
Self Consistent Monte Carlo Method-II

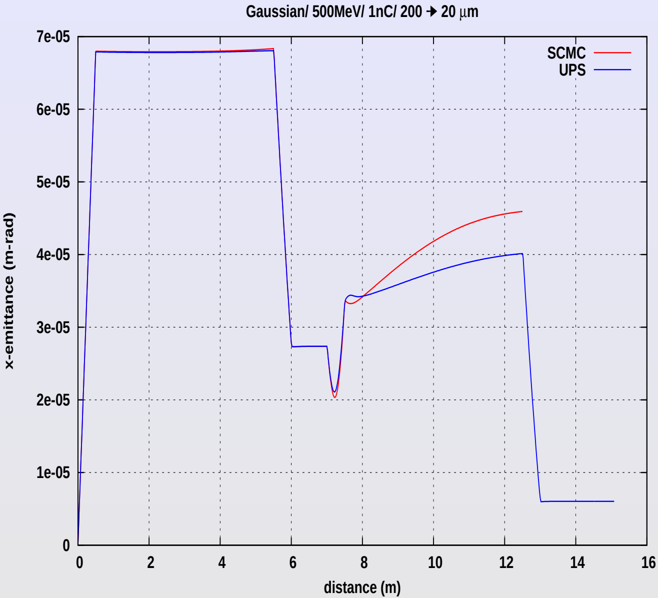
- A parallel code has been developed and results for the Zeuthen benchmark bunch compressor will be presented in Bassi, et. al. THPAN084 Thursday
- Can follow 2D densities; 4D probably beyond current computing capability
- A few results follow (Thanks to the UNM High Performance Computing Center and NERSC)

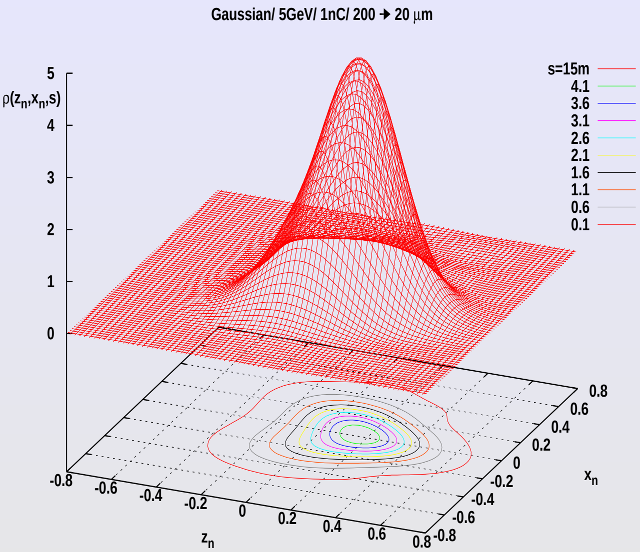


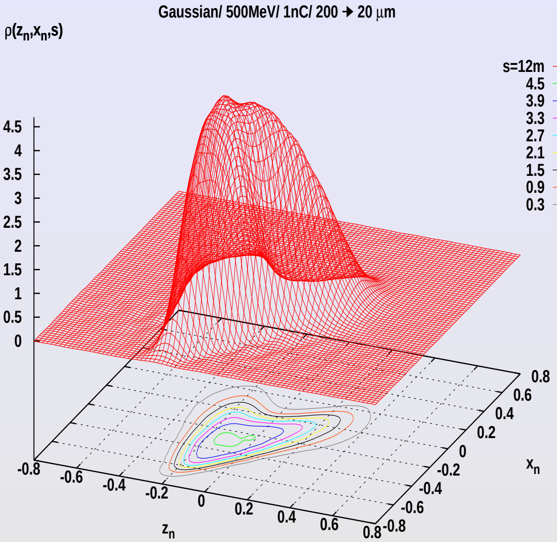
Gaussian/ 5GeV/ 1nC/ 200 → 20 μm
SCMC
UPS
0.0001
0
-0.0001
-0.0002
-0.0003
-0.0004
-0.0005
-0.0006
-0.0007
⟨pz⟩
0
2
4
6
8
10
12
14
distance (m)


Gaussian/ 500MeV/ 1nC/ 200 → 20 μm
SCMC
UPS
0.001
0
-0.001
-0.002
-0.003
-0.004
-0.005
-0.006
⟨p_z⟩
0
2
4
6
8
10
12
14
16
distance (m)


Gaussian/ 5GeV/ 1nC/ 200 → 20 μm
SCMC
UPS
SCMC:
1.70 mm-mrad
UPS:
1.69 mm-mrad
x-emittance (mm-mrad)
0.00025
0.0002
0.00015
0.0001
5e-05
0
0
2
4
6
8
10
12
14
distance (m)


Gaussian/ 500MeV/ 1nC/ 200 → 20 μm
SCMC
UPS
x-emittance (m-rad)
distance (m)
7e-05
6e-05
5e-05
4e-05
3e-05
2e-05
1e-05
0
0
2
4
6
8
10
12
14
16


Gaussian/ 5GeV/ 1nC/ 200 → 20 μm
ρ(z_n,x_n,s)
5
4
3
2
1
0
s=15m
4.1
3.6
3.1
2.6
2.1
1.6
1.1
0.6
0.1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
z_n
x_n


Gaussian/ 500MeV/ 1nC/ 200 → 20 μm
ρ(z_n,x_n,s)
s=12m
4.5
3.9
3.3
2.7
2.1
1.5
0.9
0.3
z_n
x_n