HIGH PRECISION MEASUREMENTS OF LINAC COUPLED CELLS*

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Abstract

For an assembled structure (module, tank) of a Linac, the single cells, when coupled, loose their individuality and in cooperation contribute to the generation of the structure modes (resonant frequencies) F_m. On the other end these modes are the only measurable quantities. The system of the coupled cells can be modelled, in a narrow frequency band, as a lumped constant circuit. The modes are solution of an equation obtained equating to zero the determinant relevant to the lumped circuit representation. This is an algebraic equation of the same order as the number N of cells. A plausible question can be posed: is it possible from a manipulation of the measurable quantities (F_m) to draw the lumped circuit parameters, namely coupling constants and single cell resonant frequencies? The answer is positive if a certain degree of symmetry is satisfied. The coefficients of above mentioned equation can be easily related to the measured modes F_m. By varying, by means of tuners, the tune of a single cell of a small unknown amount, any couple of equation coefficient moves on a straight line. Therefore, we have *N(N-1)/2* known straight line coefficients which may give the unknowns with extremely high accuracy.

INTRODUCTION

It has been already demonstrated that a coupled cavity system is well represented by a lumped constant circuit [1-5]. This representation was extremely fruitful to describe the behaviour of these cavities in a bandwidth sufficiently small with respect to the central frequency. It is worthwhile to remind that, once assembled the system, the cavities loose their individuality and that the system frequency modes are generated by a rather complex cooperation of the cell frequencies. The problem that generally arises is to extract from the frequency modes all the parameters characterizing the structure, in order to predict the system behaviour in a variety of cases. This tool would allow to easily optimize the structure, once some quality index are fixed.

THE THEORY

Even if a linac module is formed by a large number of cells, we will refer to a subset having the minimum of cells sufficient for the linac characterization. Let us allow for a SCL full cell symmetric subset, which is formed by two accelerating end cells, two coupling cells and a central accelerating one; we foresee that there is a coupling of the first order (k_1) and of the second order (k_a,

k_c).The equivalent circuit is reported in Fig 1. The resonant frequencies F_m of the circuit can be found equating to zero the relevant determinants, in the assumption of vanishing losses. The frequencies F_m are supposed to be measurable.By equating to zero the circuit
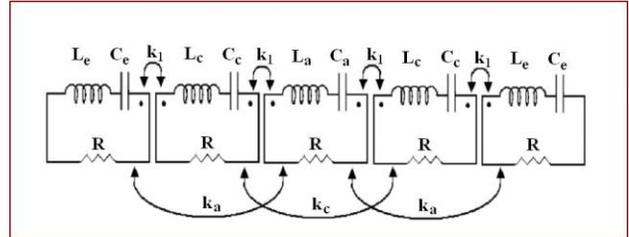

Figure 1: The equivalent circuit

determinant, we get the dispersion relation which is a polynomial of the 5th order,

$$D_5(f_e, f_a, f_c, k_1, k_a, k_c, F_m) = 0 \; ; \quad 1 \le m \le 5 \qquad (1)$$

whose solutions are functions of six parameters:

$$\boldsymbol{F_m = F_m(f_e, f_a, f_c, k_1, k_a, k_c) \quad 1 \le m \le 5}$$

Because of the symmetry one can demonstrate that, for any order of the system, Eq. (1) may be factorized as the product of two equations of lower order. In our case:

$$D_5(.....) = G_2(.....) \times G_3(.....) \qquad (2)$$

Therefore, the resonant frequencies can be derived by a separate solution of the two equations:

$$G_2(f_e, f_c, k_1, k_c, F_m) = 0 \; ; \; m = 2,4 \qquad (3)$$

$$G_3(f_e, f_a, f_c, k_1, k_a, k_c, F_m) = 0 \; ; \; m = 1,3,5 \qquad (4)$$

First of all we underline that in Eq. (3), the resonant frequency of the central cell f_a and the coupling constant k_a do not appear. This aspect makes simpler the calculation of the remaining unknowns.

In algebraic equations the coefficients of any power can be represented as a combination of the roots. This representation depends on the order of the equation and the position of the coefficients. Allowing for equation $G_2(…)=0$, this representation gives:

$$G_2(…) = (F^2)^2 - (F_2^2 + F_4^2)F^2 + F_2^2 F_4^2 = 0 \qquad (5)$$

On the other side we have from the factorization of Eq. 3:

$$G_2(…) = (F^2)^2 - \frac{4f_c^2 + (4-2k_c)f_e^2}{4 - 2k_c - k_1^2}F^2 + \frac{4f_c^2 f_e^2}{4 - 2k_c - k_1^2} = 0 \qquad (6)$$

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A similar procedure can be adopted for the factorized equation $G_3(\ldots)=0$.

THE STRAIGHT LINE METHOD

By comparing Eq.s (5) and (6) one may write the following two equalities:

$$F_2^2 + F_4^2 = \frac{4 f_c^2 + (4 - 2 k_c) f_e^2}{4 - 2 k_c - k_1^2}$$

$$F_2^2 \cdot F_4^2 = \frac{4 f_c^2 f_e^2}{4 - 2 k_c - k_1^2} \tag{7}$$

Our method consists in isolating in one of the two equations an unknown, e.g. f_c, and inserting it in the other one. We obtain a linear equation of following kind:

$$F_2^2 \cdot F_4^2 = f_e^2 \left(F_2^2 + F_4^2\right) - \frac{f_e^4 \left(4 - 2 k_c\right)}{4 - 2 k_c - k_1^2} \tag{8}$$

which may be shortened as:

$$y = A \cdot x + B \tag{9}$$

where the variables x and y are defined as:

$$\begin{cases} x = F_2^2 + F_4^2 \\ y = F_2^2 \cdot F_4^2 \end{cases} \tag{10}$$

In this way, we got rid of the frequency f_c which does not appear in Eq. (8). As a consequence, by varying the resonant frequency of the coupling cells, the two variables must move on a straight line, since the known terms are not sensitive to the variation of f_c. It is worthy of note that with this method we need not to know the amount of variation of f_c. Likewise we may proceed by coupling the coefficients of equation $G_3(\ldots)=0$ between them and with the ones of equation $G_2(\ldots)=0$. As a whole we will obtain ten equations similar to Eq. (8). The method hereby proposed is a generalization of the one described in Ref. [6] where it was used to measure the first order coupling constant.

THE MEASUREMENTS

The above results are valid under the assumption of symmetrical system. However, in the case of asymmetry (symmetric cells not having the same frequency), it can be shown that the behaviour described by Eq. (2) is stationary at first order of this asymmetry. If the asymmetry is kept below 10^{-4} (which may be attainable) the perturbation induced is largely negligible. The first module of PALME has been conceived in such a way that each cavity has two threaded frequency tuners, with 0.8mm pitch. The maximum excursion in frequency for each tuner is about 6 MHz. The frequencies of the cells were measured to within the same systematic error (if any); therefore it has been possible to equalize the symmetric cells. Afterwards, the five resonant mode frequencies F_m were measured for different values of the frequency f_c changed by means of the coupling cavity tuners.
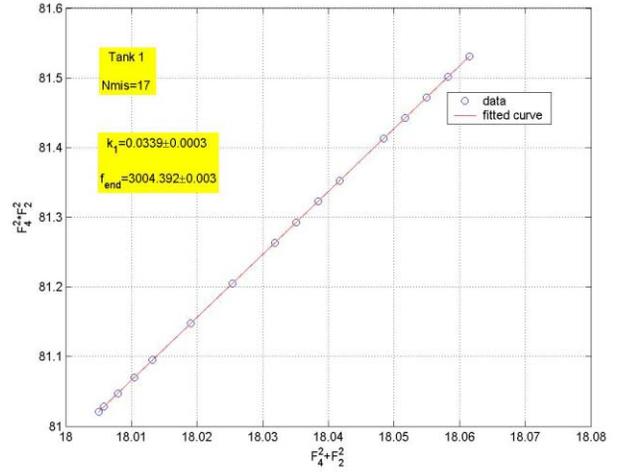

Figure 2: The straight line method according to Eq. (10)

In Fig. 2 the pattern obtained with the measured values of the frequencies F_2 and F_4, according to Eq. (10), is reported. It is apparent that the points match very well with a straight line. The angular coefficient A and the known B term are calculated by means of the least square method which delivers, as well as, the measurement errors. According to Eq. (9), the angular coefficient gives, without intermediate steps, the value of f_e. Conversely the value of k_1 is given to within the factor $(1-k_c/2)$. According to the design calculations, the constant $k_c/2$ is barely larger than 10^{-4}, therefore as one may see from Table 1, the correction would be immaterial.

Table 1- Results according to Eq. 9

f_e (MHz)	3004.392 ± 0.003
k_1	0.0339 ± 0.0003

The same procedure has been adopted allowing for the resonant frequencies satisfying the equation $G_3(\ldots)=0$. In a similar way we resort to an equation similar to Eq. (9):

$$y = A \cdot x + B \tag{11}$$

Since our aim was to find the values of f_a and k_a with the best approximation, we made an investigation among the ten pairs and the optimum choice seems to be the following:

$$\begin{cases} x = F_1^2 F_3^2 + F_3^2 F_5^2 + F_5^2 F_1^2 \\ y = F_1^2 \cdot F_3^2 \cdot F_5^2 \end{cases} \tag{12}$$

The plot, according to Eq. (12), is shown in fig. 3

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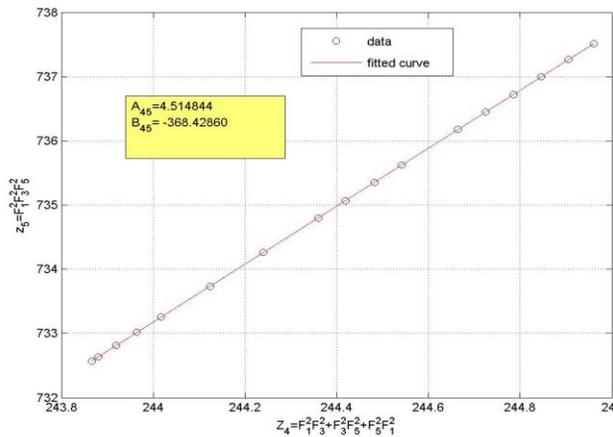
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Figure 3: The straight line method according to Eq. (12)

By adopting again the least square method we obtain the values of f_a, f_c and k_a with the relevant errors.

Table 2 Results according to Eq. 11

f_a (MHz)	3005.44 ± 0.02
f_c (MHz)	2996.46 ± 0.03
k_a	-0.0067 ± 0.0015
k_c	$-0.0004 \pm$ N.V.

Because of the propagation of the errors we have an increase of the relative approximation of about one order of magnitude. The frequency of the coupling cells is calculated in the case of removed tuners.

By means of the parameter values already calculated it has been possible to characterize the coupling cell tuners as a function of the progressive number (or fraction) of turns. The plot is reported in Fig. 4. The agreement with numerical results is excellent: the asymptotic gradient is 1.110MHz/turn while numerical calculations gives 1.140MHz/turn.
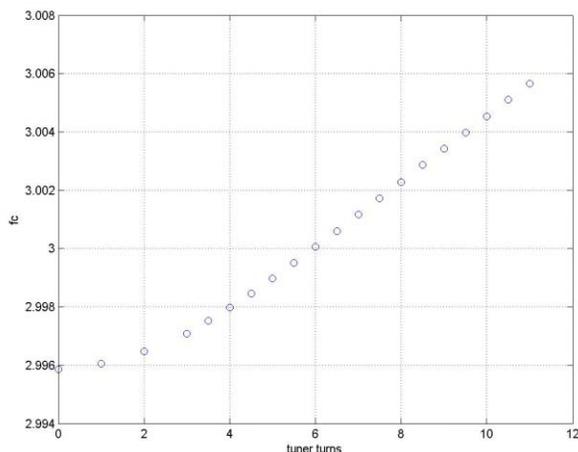

Figure 4: Tuner calibration plot .

CONCLUSIONS

Even if this method has been applied to a five cell system, where the unknown are only six, ($f_e, f_c, f_a, k_1,$ k_a, k_c), it is most general.

This method allowed us to derive five of the six unknowns. The sixth unknown (k_c) is so small that the error is of the same order of magnitude as the variable. However, just because the variable is so small, its role in the physics of the problem is negligible.

In principle the subset of the five cells analyzed will be formed by six tiles, on which two half cells are bored on the opposite faces; on the two end cells only one half accelerating cell is bored. The central pair of cells (# 3 and # 4) may be replaced by another pair so that we measure all the accelerating cells. Not only, but we may also operate a permutation of the tiles in the pairs in order to minimize the deviation of the frequency f_a from the nominal one.

An alternative and faster procedure could be to assemble a subset of a larger number of cells (e.g. 9) and to measure the modal frequency (F_m). From these measurements and forcing some results obtained by the method described (k_1, k_a), the remaining unknown can be derived by variational techniques. This procedure is viable just because the error propagation has, as a starting point, precise data (those measured and those forced).

Finally the output of this method is a useful guide to tune the cells in order to facilitate the field equalization when the bead pull method is adopted.

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