# SPIN TRANSPORT IN THE INTERNATIONAL LINEAR COLLIDER*

Jeffrey C. Smith †, CLASSE, Cornell University, Ithaca, NY, USA


### Abstract

Polarized positron and electron beams are ideal for searching for new physics at the International Linear Collider (ILC). In order to properly orient and preserve the polarization of both beams at the interaction point (IP) the beam polarization must be manipulated by a series of spin rotators along the beam line. Furthermore, the polarization for both beams should be known with a relative uncertainty of about 0.5% or better, therefore, all sources of depolarization along the ILC should be identified. We report on a spin rotator design for the ILC and polarization studies between damping ring extraction and the interaction point.


## SPIN ROTATOR

After exiting the damping ring with a vertical polarization, the polarization must be rotated to whatever orientation is required at the IP. There should be complete flexibility in the IP polarization and this is performed in the spin rotator. The ILC spin rotator design discussed below is an adaptation of the design for the Next Linear Collider [1].

### Design Options

The are several different options for a spin rotator. One option is to use a set of horizontal and vertical chicanes in a configuration referred to as a "half-serpent" [2]. The most significant issue with such a system is the vertical bending magnets which introduce synchrotron radiation. Requiring that the emittance doesn't degrade by more than 2%, the minimum length of each bend must be 1115 meters. Another problem is the momentum compaction, $R_{56}$, which will introduce a longitudinal energy dependence. Requiring the emittance dilution to be only 2% results in $R_{56} = 823$ meters. This is unacceptable compared to the 800 meters in the bunch compressor.

The main source of all the problems with the serpent scheme is the use of vertical bending magnets which result in large vertical dispersion. The serpent method rotates the spin about the x-axis using vertical bending magnets. The only way around the use of vertical magnets is to rotate the spin about the longitudinal axis. This requires a longitudinal field such as in a solenoid. However, solenoids not only rotate the spin about the longitudinal axis but also the orbit, causing x-y coupling. The beam roll through a solenoid is equal to one-half the spin precession, so if the spin must rotate by 90 degrees then the orbit will roll by 45 degrees. It has been found [4] that with an appropriate combination of solenoids, FODO cells and horizontal bends, the spin can be manipulated while preserving the other beam parameters.

**Emma Rotator** One method of removing the solenoid coupling is dividing the solenoid in half and introducing a canceling symmetry between the two halves. This cancellation takes place in a so-called "Emma Rotator". The first solenoid rotates the spin by half the desired total. It also rotates the beam by a quarter of the same amount. The center of the Emma Rotator is a FODO cell transfer line which is +1 in x and -1 in y thereby reflecting the beam about the y-axis. The second solenoid, of equal strength to the first, will rotate the spin the rest of the way as it rotates the beam back to the flat state. The transfer matrix for a solenoid that rotates the spin by $\frac{\theta_s}{2}$ is [3]

$$\mathbf{R}_s = \begin{pmatrix} C^2 & \frac{SC}{k} & SC & \frac{S^2}{k} \\ -kSC & C^2 & -kS^2 & SC \\ -kSC & -\frac{S^2}{k} & C^2 & \frac{SC}{k} \\ kS^2 & -SC & -kSC & C^2 \end{pmatrix} \tag{1}$$

where $C = \cos kL_s$, $S = \sin kL_s$, $k = \frac{B_s}{2B_0\rho} \approx \frac{\theta_s}{4L_s}$, $B_0$ is the field in the solenoid and $L_s$ is the length of the solenoid. Inserting the Emma Rotator between two such solenoids results in

$$\mathbf{R}_{emma} = \mathbf{R}_s \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \cdot \mathbf{R}_s \tag{2}$$

$$= \begin{pmatrix} \cos\frac{\theta_s}{2} & \frac{\sin\frac{\theta_s}{2}}{k} & 0 & 0 \\ -\sin\frac{\theta_s}{2} & \cos\frac{\theta_s}{2} & 0 & 0 \\ 0 & 0 & -\cos\frac{\theta_s}{2} & -\frac{\sin\frac{\theta_s}{2}}{k} \\ 0 & 0 & \sin\frac{\theta_s}{2} & -\cos\frac{\theta_s}{2} \end{pmatrix}. \tag{3}$$

The coupling is canceled independent of the solenoid strength or length provided the parameters are the same for both solenoids. There is a finite focusing that is inherent in a solenoid that still remains but this can be compensated with a linear optics matching section. Rotation of the spin about the longitudinal axis is then achieved simply by adjusting the strength of the two solenoids by an equal amount.

### Spin Rotator Design

The solenoid solution will only rotate a beam about the longitudinal axis so a horizontal dipole magnet must still be used for rotation about a second axis in order to orient

* Work supported by the US National Science Foundation under contract PHY-0202078 and the US Department of Energy under contract DE-FG02-04ER41352.
† js344@slac.stanford.edu

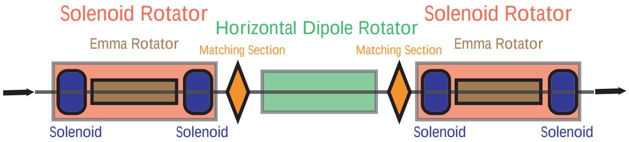


Figure 1: The ILC Spin Rotator schematic.

the beam along the longitudinal plane. The outgoing spin orientation must also be fully flexible and such a system can be designed using the configuration given in figure 1. If each pair of solenoids are capable of rotating the spin by ±90 degrees and the bend section can rotate the spin by a fixed amount of 90 degrees then this system will provide arbitrary control of the spin orientation at the interaction point if it is assumed that the incoming spin orientation is either up or down ($\pm y$). The resultant spin orientation will be

$$\bar{S} = R_{spin\_rotator} \cdot \begin{pmatrix} 0 \\ \pm 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \mp \sin\theta_{em2} \cos\theta_{em1} \\ \pm \cos\theta_{em2} \cos\theta_{em1} \\ \pm \sin\theta_1 \end{pmatrix} \tag{4}$$

where $\theta_{em1(2)}$ is the spin rotation angle through the first (second) solenoid pair. If the solenoid strengths are reversible and each solenoid can rotate the spin by 45 degrees then any arbitrary outgoing spin orientation is attainable simply by adjusting the strength of the solenoids in pairs. Any incoming longitudinal component of the polarization limits the range of outgoing polarization.

An alternate spin rotator design has been proposed by Nick Walker and Peter Schmid [5]. This spin rotator design results in much smaller emittance growth in the Emma Rotators compared to the NLC Zeroth Order design. The Walker/Schmid Emma Rotator was therefore inserted into the NLC spin rotator to get the final design and best performance, which performs better in emittance preservation than the two designs on their own.

The Emma Rotators will cancel the coupling only for on-energy particles. Off-energy particles, by virtue of having a different phase advance through the reflection cell, will not be transfered through a unit transfer matrix. Increasing the number of FODO cells will decrease the overall chromaticity, but a point of diminishing returns is reached and the number of FODO cells per Emma Rotator was set to six. For a nominal energy spread of 0.107%, the total normalized vertical emittance growth is negligible at less than 0.01 nm.

The final design is an Emma/Walker/Schmid/Smith hybrid as illustrated in figure 2. This new hybrid design utilizes the best elements of the various configurations. The $R_{56}$ for this design is -6 mm. This is small compared to the -800 mm in the bunch compressor and will not effect the longitudinal profile of the beam. The spin rotator is located at the end of the turnaround so that the spin rotator arc is the final bend in the turnaround arc, limiting the overall length of the RTML. If it is decided that there will be two interaction regions or fast helicity reversal for

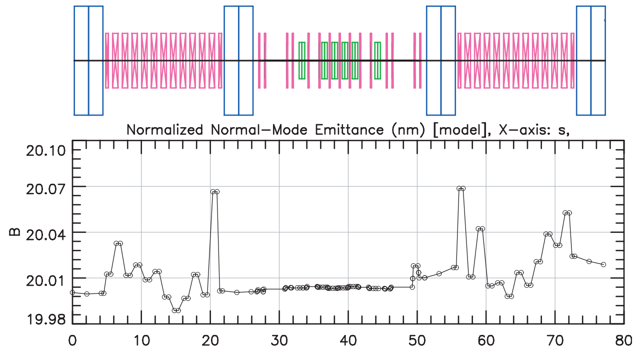


Figure 2: Vertical normalized emittance growth in the Spin Rotator (in nm).

the positron beam, then a configuration similar to Schmid's may be more desirable to allow for two spin rotators to be stacked side-by-side. One such configuration is discussed by Moffeit [6].

### *Spin Rotator Emittance Tuning*

The spin rotator design discussed above preserves emittance very well for a perfectly aligned lattice. The effects of misalignments were also studied. The full results are found in [7]. In general, emittance dilution in the spin rotator is not a concern, however, misalignments and strength errors in the quadrupoles can result in large amounts of coupling and a coupling correction section is essential downstream of the spin rotator. Skew quadrupoles placed within the Emma Rotators could also be used to mitigate the coupling effects but are not necessary provided the downstream coupling correction can remove the coupling.

## DEPOLARIZATION STUDIES IN THE ILC

### *Spin Transport in BMAD*

Spin Tracking has been implemented in BMAD [8] using a spinor-quaternion transfer map method. It is more efficient to use a SU(2) representation rather than SO(3) when describing rotations of spin. In spinor notation, the T-BMT equation describing spin motion can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\Psi = -\frac{i}{2}\left(\sigma \cdot \mathbf{\Omega}\right)\Psi. \tag{5}$$

and the solution can be written as

$$\Psi = \left(a_0 \mathbf{1_2} - \mathbf{i}\boldsymbol{a} \cdot \sigma\right)\Psi_i \tag{6}$$

where $\Psi$ = Spinor = $(\psi_1, \psi_2)^T$ and $\psi_{1,2}$ are complex numbers. The Four-Vector, $\mathbf{A} = (a_0, \boldsymbol{a})$, describes the transfer map for each element. Tracking through any element is simply achieved via the application of these quaternions in sequence. This results in very fast tracking times. A numerical spin integrator has also been implemented. It is an extension of the Boris-like numerical integrator developed by Stoltz *et al.* [9]. The Boris method is second order accurate, requires only one force calculation per particle per step and preserves conserved quantities more accurately over long distances than a Runge Kutta integration scheme.

### *Depolarization Studies*

For the ILC Low Emittance Transport section, the most significant factor in the depolarization of the beam is the large horizontal beam size. Spin-orbit coupling can be experienced in a linear collider as had occurred in the SLC [10]. The spin-orbit coupling was due to the strong bending magnets in the arcs but the ILC has no strong bending magnets downstream of the turnaround. There are some weak bends in the energy and polarimeter chicanes plus some geometry matching in the BDS but these are weak and pose no threat. If strong bends were added to the BDS that result in several revolutions of spin precession then further polarization studies would need to be performed.

The depolarization in the RTML is small. The one noticeable effect is from dispersion and the large beam size in the bunch compressor wigglers. However, the overall effect is less than 0.01%. The additional effect of misalignments on polarization is also very small. This emphasizes the relatively weak gamma factor enhancement to the anomalous gyromagnetic moment for the 5 GeV beam.

Given the small number of quadrupoles in the main linac, they pose little risk to the polarization. However, there are 7300 RF cavities per linac. Any individual cavity will have a small effect, but the combined effect of all cavities together may be problematic, so it was investigated using a simple model for RF cavities in the spin tracking as described in [11]. The effect due to the RF cavities was found to be small compared to the already small effect due to the quadrupoles. The precession in the cavities was found to be at least six orders of magnitude too small to effect the polarization. Earth's curvature and magnetic field result in spin precession but the effect on polarization is negligible.

Misalignments also have little effect on polarization in the main linac. The polarization precesses as the beam travels through the misaligned components and the polarization is not always longitudinal at the end, but this can be corrected with the RTML spin rotator, just as the effects of the Earth's curvature and field can be compensated using the same spin rotator. Further studies should be performed to analyze the variation of the polarization vector at the IP due to component and beam jitter. Jitter is expected to be small enough that the resulting polarization jitter will be well under 1 degree. Simulation studies have yet to be performed for the helical undulator.

The beam delivery system is at a rather large beam energy and the gamma-factor enhancement to the anomalous gyromagnetic moment is $G\gamma = 567.3$. For most of the BDS, depolarization was not found to be a concern. The only location where a slight depolarization occurs is in the final focus where a degradation to 99.84% appears between the two quadrupoles in the final doublet. However, the second quadrupole almost completely removes the depolarization caused by the first. The reason for the slight depolarization is due to the combination of large beam size and strong focusing magnets. Misalignments were found to have little effect on the depolarization, just as in the RTML and Main linac. This is because the dominant source of depolarization is the horizontal beam size which is relatively insensitive to misalignments.

## CONCLUDING REMARKS

A spin rotator has been designed for the ILC. It has the required versatility to allow for complete control over the IP spin orientation. Polarization and emittance are well preserved in the spin rotator. At 250 GeV, depolarization effects are beginning to occur where there are large beam sizes in strong magnets as exists in the Final Focus. Due to the configuration of the magnets, the net depolarization is virtually zero. The effects in the 1 TeV upgraded machine will be about a factor of 2 higher which is expected to still be insignificant. No depolarizing effects were observed in either the RTML or Main Linac. The helical undulator has yet to be studied.

## ACKNOWLEDGMENTS

The author is grateful to Ian Bailey, Des Barber, Georg Hoffstaetter, Larisa Malysheva, Peter Schmid and Peter Tenenbaum for providing support in implementing spin transport in BMAD and efforts in spin rotator design.

## REFERENCES

[1] *Zeroth-Order Design Report for the Next Linear Collider*, The NLC Design Group, SLAC-PUB-474 (1996)

[2] T. Fieguth, "Snakes, Rotators, Serpents and the Octahedral Group," SLAC-PUB-4195 (1987)

[3] A. W. Chao and M. Tigner, *Handbook of Accelerator Physics and Engineering, Second Printing*, World Scientific, Singapore (2002)

[4] P. Emma, "A Spin Rotator System for the NLC," NLC-NOTE-7 (1994)

[5] P. Schmid, "A Spin Rotator for the ILC," EUROTeV-Report-2005-024-1 (2005)

[6] K Moffeitt, *et al.*, "Spin Rotation Schemes at the ILC for Two Interaction Regions and Positron Polarization with both Helicities," LCC-0159, SLAC-TN-05-045 (2005)

[7] J. C. Smith, "The Preservation of Emittance and Polarization in the International Linear Collider," Ph. D. Thesis, Cornell University (2007)

[8] *see*: `www.lepp.cornell.edu/∼dcs/bmad`

[9] P. H. Stoltz, "Efficiency of a Boris-like integration scheme with spatial stepping," Phys. Rev. ST Accelerators and Beams, **5**, 094001 (2002)

[10] T. Limberg and P. Emma, "The North Arc of the SLC as a Spin Rotator," SLAC-PUB-6210 (1993)

[11] J. C. Smith, "Depolarization Studies in the International Linear Collider," ILC-NOTE-2007-012 (2007)