THE EXTREME VALUE THEORY TO ESTIMATE BEAM LOSSES IN HIGH POWER LINACS

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Abstract

The influence of random perturbations of high intensity accelerator elements on the beam losses is considered. This influence is analyzed with the help of the Extreme Value Theory (EVT) to allow loss estimates for a very low fraction of the beam. Many fields of modern science and engineering have to deal with events which are rare but have significant consequences. EVT is considered to provide the basis for the statistical modeling of such extreme events (extreme variations of financial market for insurance companies or extreme wind speed for electric companies).
To illustrate the application of this theory to beam losses estimates, the SPIRAL2 driver is used. This 5 mA deuteron accelerator is simulated from the output of the source to the target with a high resolution PIC modelisations (up to 1.3 million macro-particles) by using realistic external fields.

INTRODUCTION

Once the reference design for the accelerator with perfect elements respects the requirements, it is necessary to evaluate the effects of imperfect elements. This evaluation permits to define tolerances for the construction of the linac and to test the robustness of the achieved architecture. To correct such errors, a system based on correctors and diagnostics has to be designed taking into account that the diagnostics are also imperfect (misalignments, measurement,...).
Several authors studied the effects of imperfect ion linacs on the beam [1, 2, 3, 4, 5, 6]. In the references [2, 3], it is shown how manufacturing errors modeled with multipoles components could induce an emittance growth but the effect of non linear space charge force is not treated. The halo induced by these effects is then underestimated and the loss prediction becomes distorted. The approach in [1] is helpful if the Coulomb force is negligible but is inaccurate for high power linac at low energy. To tend to "realistic" simulation of a high intensity linac, it is necessary to perfom start to end transport to be capable to estimate the impact of halo produced at low energy on the beam losses at the high energy part of the accelerator. The references [4, 5, 6] detail start to end simulations to take into account this point. In these references, the main mechanisms to produce the beam halo are the space charge and/or the non linear external fields. These studies used macroparticles to estimate the beam distribution and to record the losses at the beam pipe. The discrete recorded losses at different locations in the linac allow to build Cumulative Density Function (CDF) to provide a probability to deposite more than a certain fraction of the beam. But the discrete form of this CDF induces that the probability to loose more than the more extreme recorded loss becomes null. We are not capable then to predict very extreme events.
The Extreme Value Theory provides a firm theoretical foundation to perform such a goal (Fisher and Tipett (1928) and Gnedenko (1943)). Combining this theory with the bootstrap technique, we propose in this paper, after a short introduction to EVT, to detail a procedure to compute average probability of occurence of extreme events such a very low beam loss (10^{-5}) including a confidence interval (error bar) associated to this evaluation. To illustrate the method, the SPIRAL2 linac is used.

EXTREME VALUE THEORY

Some of the most frequent questions concerning risk management in several fields involve extreme quantile estimation. This corresponds to the determination of the value a given variable exceeds with a given (low) probability. In many fields of modern science, engineering and insurance, extreme value theory is well established [7, 8, 9]. When modelling the maxima of a random variable, this theory plays the same fundamental role as the central limit theorem plays when modelling sums of random variables. In both cases, the theory tells us what the limiting distributions are.
Generally there are two related ways of identifying extremes in data. Let us consider a random variable which may represent daily losses. The first approach then considers the maximum the variable takes in successive sequences or periods, for instance months or years. These selected observations constitute the extreme events, also called block maxima. The second approach focuses on the realizations which exceed a given (high) threshold [9]. It is called Peaks Over Threshold (POT). This two ways to treate data are complementary and can be compared but we will only use the first one in this manuscript.

Distribution of Maxima (GEV)

The limit law of the block maxima, which we denote by M_n, with n the size of the subsample (block), is given by the following theorem:

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Theorem 1 (Fisher and Tipett (1928), Gnedenko (1943)) *Let (X_n) be a sequence of random variables. If there exists two series of real constants $C_n > 0 \, \forall \, n$ and d_n and some non-degenerate distribution function H such that*

$$\frac{M_n - d_n}{c_n} \longrightarrow H,$$

then H belongs to one of the three standart extreme value distributions: Fréchet, Weibull or Gumbel.

Jenkison and von Mises suggested to represent these three distributions with the following representation:

$$H_{\xi\sigma\mu}\,(p) = exp\left(-\left(1+\xi\frac{p-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right) \qquad (1)$$

with μ, the location parameter, σ, the scale parameter and ξ a form parameter. When $\xi \rightarrow 0$, the GEV tends to the Gumbel distribution.This general representation is very useful as at the beginning of the treatement, we don't know in advance the limiting distribution type of the sample.

Bootstrap technique

If we admit that large-sample theory holds for our estimates, we can construct confidence intervals for each fitted parameters of the GEV or a particular return level.The bootstrap technique is a very simple and efficient technique in order to build such confidence intervals. It is a type of Monte Carlo method applied based on observed data [10]. The fundamental idea of this model-based sampling theory approach to statistical inference is that the data arise as a sample from some conceptual probability function, f. Uncertainties of our inferences can be measured if we can estimate f. The most fundamental idea of the bootstrap method is that we compute measures of our inference uncertainty from that estimated sampling distribution of f.

In practical application, the bootstrap means using a *resampling* with *replacement* from the actual data X to generate B bootstrap samples X^*. Often, the data or sample consist of n independent units and it then suffices to take a simple random sample of size n; with *replacement*, from the n units of data, to get one bootstrap sample. The set of B bootstrap samples is a proxy for a set of B independent real samples from f (in reality we have only one actual sample of data). Properties expected from the replicate real sample are inferred from the bootstrap samples by analysing each bootstrap sample exactly as we first analyzed the real data sample. From the set of results of sample size B, we measure our inference uncertainties (variance). The bootstrap can work well for a very large sample sizes (n), but may not be reliable for small n (let us say 5, 10 or 20), regardless of how many bootstrap samples, B, are used.

BEAM LOSSES DATA BASED ON LARGE SCALED COMPUTATIONS

Usually, EVT is used to analyse real events (measurements) but, to study the construction tolerances of a high power linac, it would correspond to the analysis of a huge number of similar built linacs. For obvious reasons, it is necessary to produce the data set with virtual accelerators. One limit of this strategy is the resolution which can be achieved with the simulation tools. Differently, it has to be shown that the relevant physics is present in the codes. We proposed here to use the present state of art in our laboratory to simulate this set of virtual linacs with imperfections. To illustrate the application of EVT to beam losses estimates, the SPIRAL2 driver is used. This design has been presented at the EPAC 2004 conference [11]. The transverse rms normalized emittance at the input is 0.2 π.mm.mrad. The beam current is 5 mA. A deuteron beam is considered to estimate the most critical beam losses. Multiparticle simulations are performed from the Low Energy Beam Transport (LEBT) line to the target through the radio frequency quadrupole (RFQ), the Medium Energy Beam Transport line (MEBT), the superconducting linac (SCL) and the High Energy Beam Transport (HEBT) line. The transport of the beam through the RFQ is computed with the code TOUTATIS [12]. The rest of the linac is simulated with the TraceWin/PARTRAN package[13]. To manage the necessary huge number of runs for the Monte Carlo study, we implemented in Tracewin a software package that permits to pilot a heterogenous collection of PCs [13]. The package is based on a client/server architecture to distribute the different independent runs. This is a multiparameter scheme and not a paralell scheme which is less optimal as each run can be performed by a single PC (less communication between each node). The figure 1 shows the beam density projection per plane in the linac for the perfect structure.

Depending on the linac section, errors with different amplitudes have been used. For an error of amplitude A, the value has a uniform probability to be between $-A$ and $+A$. Two types of error have to be coped for: static and dynamic. For each, it bas been considered, for instance, cavity or quadrupole misalignments or field and phase errors. We implemented a correction scheme to minimize the effects of the static errors. For example, steerers and beam position monitors to correct the misalignement of cavities and quadrupoles have been used. Usually, each defect is first studied separately and is amplified until an unacceptable threshold is reached. Second, the defaults are combined and amplified until the threshold is reached again. The weighting for the combination has to take into account the relative sensitivity and the capacity to respect the induced tolerances. The main threshold for the SPIRAL2 project is to avoid losses in the superconducting section above 1 W per cavity. As this threshold is exceeded without error, collimators in the MEBT are used to control the beam losses. The amplitudes of errors have been chosen after iterations

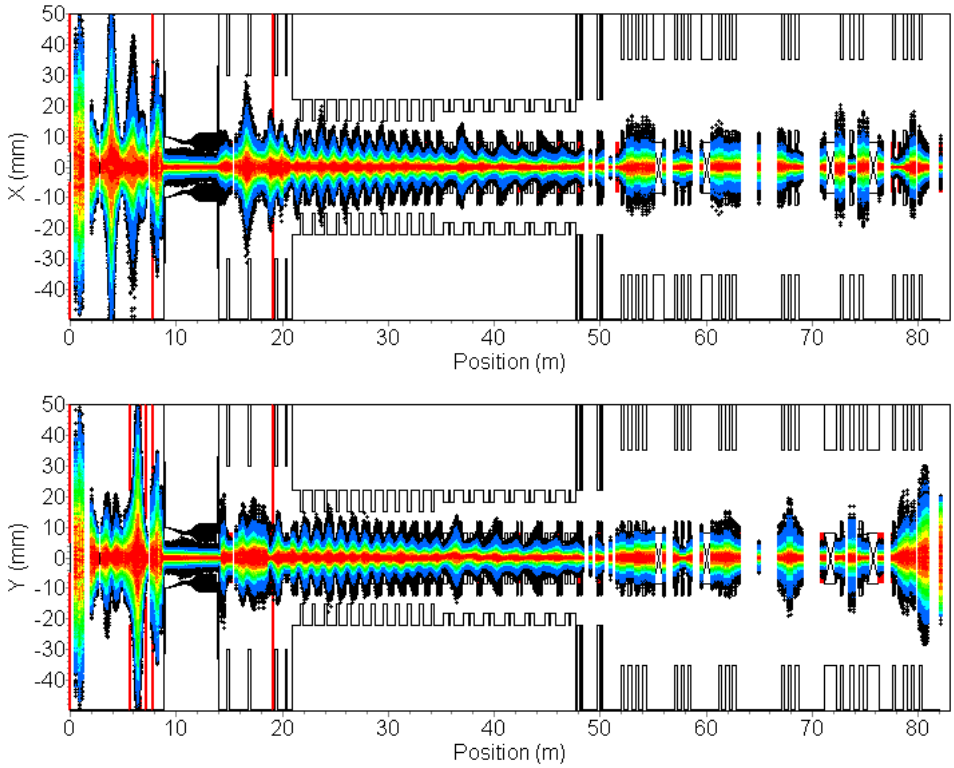

Figure 1: The deuteron density projection in the transverse plane in the SPIRAL2 linac (the black line is the aperture).

with the engineering teams and the background from previous studies on high intensity linacs [15]. For each run, a 1,300,000 macroparticle $4 \times \sigma$ gaussian distribution is used at the input of the LEBT line to provide the required resolution to detect beam losses lower than 1 Watt in the whole linac. We randomly generated up to 341 runs to obtain a sufficient size of events per element or block.

APPLICATION OF THE EXTREME THEORY FOR THE LOSS ESTIMATE

Introduction

To model the tails of our deposited beam power in the SPIRAL2 linac, we will apply the following method:

- first, scan the mean deposited power for each element of the accelerator to detect the most critical components (our block maxima).
- second, fit the data with the Generalized Extreme Value (GEV) distribution.
- third, estimate confidence intervals for value of interest with the bootstrap method.

Figure 2 shows the average losses repartition along the structure for the 341 linacs and the corresponding dissipated power. These last data allow us to select the most critical component in a particular section. It is assumed that elements with a high standard deviation have also a high mean value. If we focus on the results for the SCL, we can observe two critical elements. The first one is the first quadrupole of the first super-conducting section and the second one is the first cavity of the $\beta = 0.12$ section.

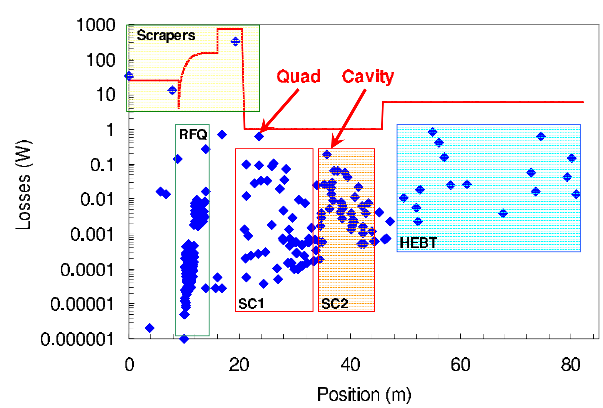

Figure 2: Average loss repartition along the structure. The most critical components are pointed with red arrows.

First quadrupole of the $\beta = 0.07$ section

Figure 3 shows the recorded losses distribution at the first quadrupole of the first super-conducting section. This

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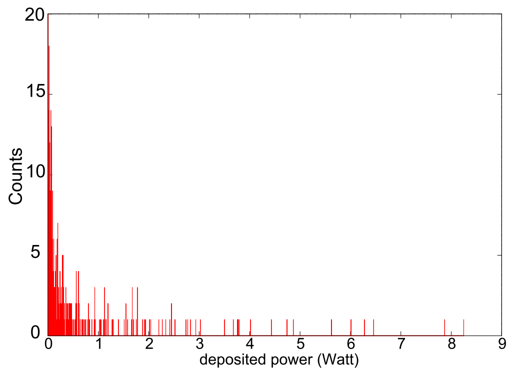

Figure 3: Unormalized probability density function for the losses at the first quadrupole of the first section. The deposited beam power (W) forms the abscissa and the number of counts the ordinate.

represents the unnormalized probability density function (PDF) computed with the results of all the different linacs with $1,300,000$ simulated macro-particles per linac. With this number of macro-particles, one particle represents ~ 8 mW at this location of the linac. Using this unormalized PDF, we can build a Cumulative Distribution Function (CDF) which will be our reference data to fit the parameters of the GEV function of the lost power p (see equation 1). To build the CDF, we used the following formula:

$$F_n\left(x_i^n\right) = \frac{i}{n} \quad \text{for} \quad i = 1, ..., n \tag{2}$$

which is the sample distribution function for a set of n observations, given in increasing order $x_1^n \leq ... \leq x_i^n$. For our case, n is equal to 341. The GEV fitted with these data is plotted in the figure 4. At this location of the linac, the requirements assume that less than 4 Watt should be deposited on the pipe. With the fitted GEV, we can estimate that the probability to loose less than 4 Watt is 0.97 which is very confortable. The fitted parameters are $\hat{\xi} = 0.223$, $\hat{\sigma} = 0.89$ and $\hat{\mu} = -0.86$. To see how sensible is this result in respect to the achieved statistics, we can calculate a confidence interval at 95% with the bootstrap method. We resampled 1000 times the recorded PDF and recomputes the expected return power level for a probability of 0.97. The figure 5 shows the empirical bootstrap distribution of the return level for this probability. The confidence interval at 95% is then $[2.3; 5.9]$ Watt. This indicates that the recorded losses are sufficiently numerous to estimate that, with a good accuracy, we kept the beam losses at an acceptable level. If we need to estimate probability for very high loss level, the same procedure has to be repeated. For instance, with the same set of events, we can estimate that for an average probability of occurence of 10^{-4}, the deposited power is 36 Watt with a confidence interval at 95% which is $[20; 52]$ Watt. It indicates that more recorded losses are required if we need to shrink the confidence interval around this value of 36 Watt.

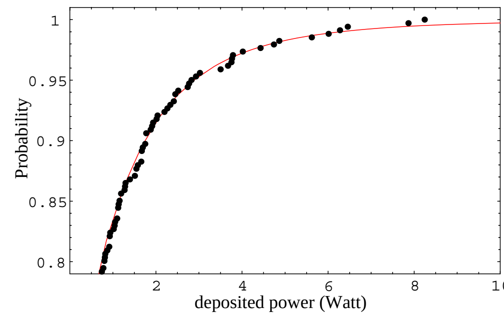

Figure 4: GEV fitted with the recorded losses for the quadrupole. The deposited beam power (W) forms the abscissa and the CDF the ordinate.

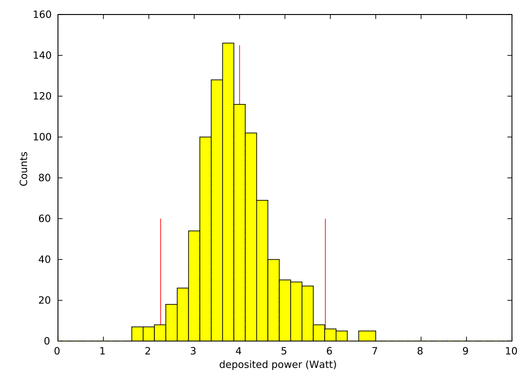

Figure 5: Empirical bootstrap distribution for the return level with a probability of 0.97. The two small red marks indicate the $\pm 2\sigma$ interval, the big red mark indicates the return level obtained with a direct estimate from the recorded losses.

First cavity of the $\beta = 0.12$ section

With the same procedure, we can construct a fitted GEV function with the recorded losses at the cavity location (figure 6). The fitted parameters are $\hat{\xi} = 0.465$, $\hat{\sigma} = 0.062$ and $\hat{\mu} = -0.061$. The probability to loose less than one watt is 0.99. With the bootstrap method, we can estimate a confidence interval for this probability. It is $[0.44; 1.33]$ Watt. The figure 7 illustrates the empirical bootstrap distribution of the return level for this probability. The table 1 gives a summary of the results for the most lossy quadrupole and cavity. To give an other example of the main interest to use EVT, we are capable to estimate that the average probability to loose more than 10 Watt in this cavity is 8.10^{-5}.

Table 1: Beam loss estimates (PE) and 95% bootstrap confidence intervals.

	CDF @ PE	Lower bound	Point estimate	Upper bound
Quad (W)	0.97	2.3	4	5.9
Cavity (W)	0.99	0.44	1	1.33

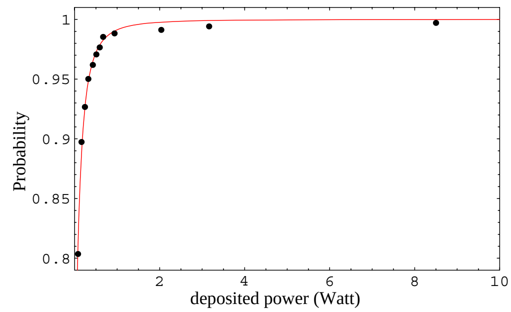

Figure 6: GEV fitted with the recorded losses for the most critical cavity. The deposited beam power (W) forms the abscissa and the CDF the ordinate.

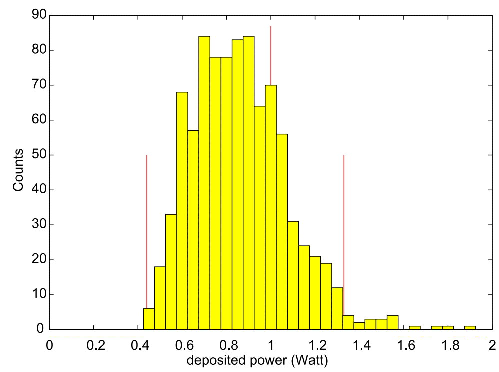

Figure 7: Empirical bootstrap distribution for the return level with a probability of 0.99. The two small red marks indicate the $\pm 2\sigma$ interval, the big red mark indicates the return level obtained with a direct estimate from the recorded losses.

CONCLUSIONS

This application of the Extreme Value Theory to beam losses estimates in the SPIRAL2 linac based on large scale Monte Carlo computations allowed us to provide low losses probability for this linac. The probability to loose more than one watt in a superconducting cavity predicted with the GEV is less than 10^{-2}. Differently, such an event will happen on average one linac over one hundred built linacs. The bootstrap technique has been used to estimate the precision of this prediction. A $\pm 2\sigma$ confidence interval equal to $[0.44; 1.33]$ Watt has been calculated for this probability To go further to "realistic" estimates of the beam losses, a more faithful modelisation of the linac is required. For instance, the output beam distribution of the ECR source is necessary to enhance the start to end modelisations and the beam interaction with the residual gas (neutralisation) has to be taken into account to simulate more accurately the space charge force especially at low energy.

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