

# MUON ACCELERATION WITH THE RACETRACK FFAG\*

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## Abstract

Muon acceleration for muon collider or neutrino factory is still in a stage where further improvements are likely as a result of further study. This report presents a design of the racetrack non-scaling Fixed Field Alternating Gradient (NS-FFAG) accelerator to allow fast muon acceleration in small number of turns. The racetrack design is made of four arcs: two arcs at opposite sides have a smaller radius and are made of closely packed combined function magnets, while two additional arcs, with a very large radii, are used for muon extraction, injection, and RF accelerating cavities. The ends of the large radii arcs are geometrically matched at the connections to the arcs with smaller radii. The dispersion and both horizontal and vertical amplitude functions are matched at the central energy.

The recent optimization results [3] set three NS-FFAG accelerators: the first accelerator energy range is from 2.5-5 GeV, the second from 5-10 GeV, and the third from 10-20 GeV. This report is trying to reduce the number of the NS-FFAG to two: first NS-FFAG from 3.67 – 8.57 GeV and the second NS-FFAG from 8.57-20 GeV with momentum range of  $\delta p/p = \pm 40\%$ .

The NS-FFAG major advantage with respect to any other fixed field accelerator is the small aperture size. The scaling FFAG would require almost two orders of magnitude larger aperture. The aperture size at low muon energies is unfortunately dominated by the muon emittance. This all depends on the results of three dimensional muon cooling. The NS-FFAG provide extreme focusing resulting in a very small beam amplitude functions and very small dispersion.

## INTRODUCTION

The NS-FFAG lattice in this example is made of repetitive triplet cells with two linear gradient combined function magnets. Two focusing gradient magnets surround a larger defocusing magnet in the center of the triplet cell. They are smaller size with an opposite bending magnetic field while the larger defocusing combined function magnet in the center does most of the bending. Details of the NS-FFAG design principles and examples made for muon acceleration have been presented earlier [1]. The future “neutrino factory” or the “muon collider” [2] required a large number of muons in a very short time due to the muons fast decay. The superconducting linac is the best solution but the most expensive one. Numerous cost optimization studies have been performed with a selection of the NS-FFAG as the optimum solution [3]. Muon acceleration has additional constraints due to the fixed frequency of the superconducting cavities. This requires “gutter” acceleration at crest of the sine RF wave. The muon time of flight has to be as short as possible of the order of 5-7 ps per cell. The NS-FFAG are not asynchronous and their time of flight changes with energy following a function close to parabola with the minimum close to the center of the energy range. This required additional optimization in the design. More details on the gutter acceleration have been presented elsewhere [4]. This report is a continuation of an effort to make the racetrack structure with the NS-FFAG. This is another attempt improving on the energy range for muon acceleration with possible reduction in cost.

## ACCELERATION

The recent optimization study assumed accelerating gradient of 10 MeV/m, already achieved with the 201.25 MHz cavities at operating temperatures of 4.5 K. This gradient required a ring with 91 cells, with 83 cells reserved for the cavities. The total energy gain is 10 GeV during the 17 turns [ $10 \text{ GeV}/17 = 588 \text{ MeV per turn}$ ]. The total RF voltage of 621 MeV with 83 cavities makes 7.48 MeV per cavity. This report assumes lower operating temperature with accelerating gradient of 17 MeV/m. The racetrack has a total of  $N=56$  cells with a drift size of 2 m. Fifty cells are assumed for the cavity placement. A parameter  $a = V/(\omega\Delta T\Delta E) \sim 1/12$ , where  $V$  is the total RF voltage muon particle would gain passing through all the cavities ( $V \sim 621 \text{ MV}$ ),  $\omega = 2\pi f_{\text{rf}} = 2\pi * 201.25 * 10^6 = 1.264 * 10^9 \text{ 1/s}$  (with  $f_{\text{rf}}$  as the RF frequency),  $\Delta T \sim 91 \text{ cells} * 6.6 \text{ ps} = 6 * 10^{-10} \text{ s}$  is the difference in the parabola between the value at the highest and the lowest energy to the minimum for the total ring, and  $\Delta E = E_{\text{max}} - E_{\text{min}} = 10 \text{ GeV}$ , previously described [4]. The required total voltage and the number of turns for the racetrack ring could be estimated from the parameter  $a \sim 1/12$ . Importance of the optimization for the time-of-flight deviation could be easily observed by comparison of the two lattice designs: in the first example the time of flight per cell is 6.6 ps [4] while in another [1] is 5.8 ps. This difference in the time-of-flight reduces the required voltage (or the number of turns) from 621 MV to 545 MV. The present racetrack design total time of flight is  $\Delta T \sim 7.0 \text{ ps}$  due to larger number of cells  $N=112$ . The energy gain required is  $\Delta E = 11.43 \text{ GeV}$ . The larger energy gain and the larger total number of cells should be compensated by the larger cavity gradients 17 MV/m.

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### RACETRACK DESIGN

We follow previous work where the principle of the NS-FFAG racetrack lattice design [5]. The racetrack is made of four arcs: two equal arcs with smaller radii at the opposite sides and two equal arcs with larger radii connected between as shown in Figure 1.

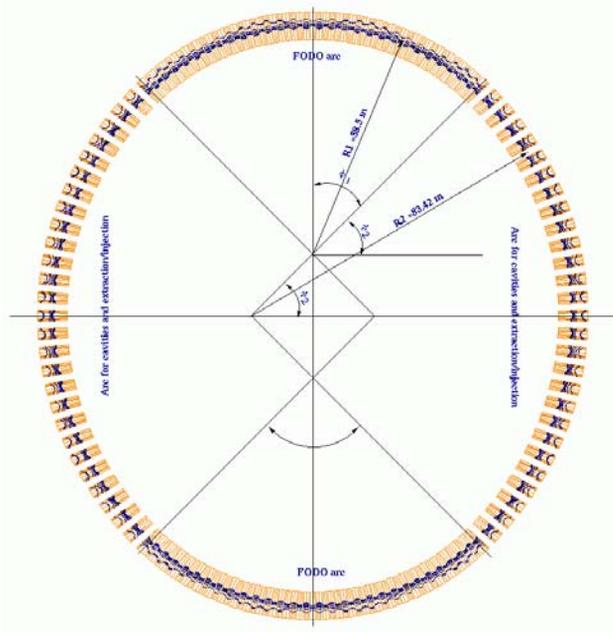


Figure 1: Muon acceleration 6.7-20 GeV with the “racetrack”.

The larger arcs are a part of the N=112 cells NS-FFAG ring. Each cell includes large 1.98 m long drifts for the placement of the RF cavities and extraction and injection kickers. Details of this ring are presented in Figure 2. The upper and lower arcs, as presented in Figure 1, are densely packed FODO cells but with small magnets and short drifts between. The connection between two arcs is established at the center of the major bending element where the dispersion and  $\beta_x$  have minimum values.

There are geometrical constraints for the two rings to be able to make a “matched” racetrack: a tangent at the point of connection of both arc are perpendicular to the radii of the arcs. This makes a connection between the two angles as:  $\theta_1 + \theta_2 = \pi/2$ . Two rings with quite different radii are designed separately. The radii of the two arcs are  $r_1 = 58.507$  m and  $r_2 = 83.423$  m. The length of the arcs, defined by the  $\theta_1$  and  $\theta_2$  angles, determines a number of cells participating from the ring with a larger radius or a number of cells missing from the arc of the ring with a smaller radius. There are 112 cells in each ring. If the two angles are equal  $\theta_1 = \theta_2 = \pi/4$  the number of the cells is the arcs is equal  $N_1 = N_2 = 28$ , and the racetrack is defined.

#### The larger radius ring for cavities

The basic cell of the NS-FFAG followed the previous design [1]. The 201 MHz superconducting cavities, been

developed at the Cornell University require ~2 m along the beam axis.

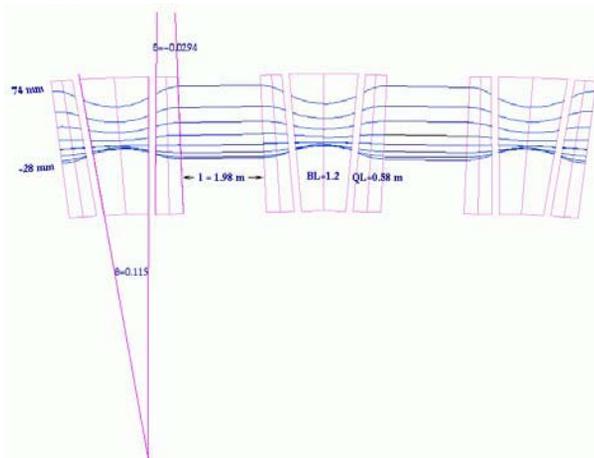


Figure 2: Three basic cells with orbit offsets for  $\delta p/p = +/- 40\%$  magnified 25 times.

The orbit offsets in the lattice vary between  $x_{off} = -13.7$  mm at the lowest energy  $\delta p/p = -40\%$  and of  $x_{off} = 48.7$  mm at the largest momentum  $\delta p/p = +40\%$ .

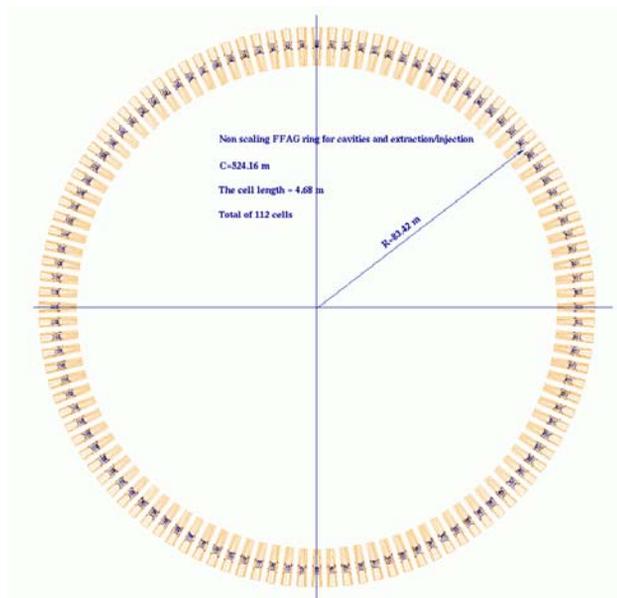


Figure 3: The larger radius ring – basic cell has 1.98 m long drift for cavity and injection extraction purposes.

The small orbit offsets reduce the size-price of the magnets. This is a consequence of the larger number of cells. There are  $2 * 28 = 56$  cells included in the racetrack design. The betatron functions and magnet blocks are presented in Figure 4.

#### Arcs made of packed FODO cells

The ring design where the request for the large drift was removed is presented in Figure 5. The orbit offsets and the magnet bending angles are presented in Figure 6. The

magnet sizes and magnetic fields are presented in Table 1. The circumference of this packed ring is 367.61 meters although there are 112 cells, 3.283 m long.

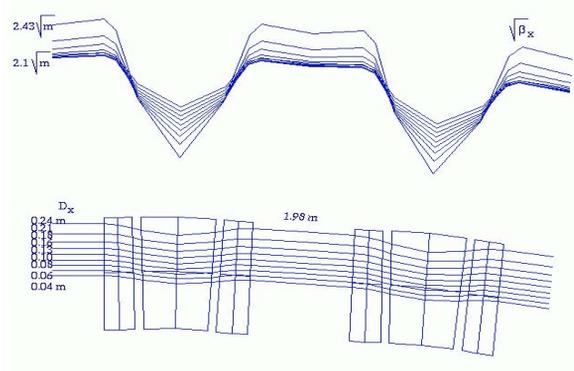


Figure 4: Horizontal betatron function and dispersion in two cells of the RF ring.

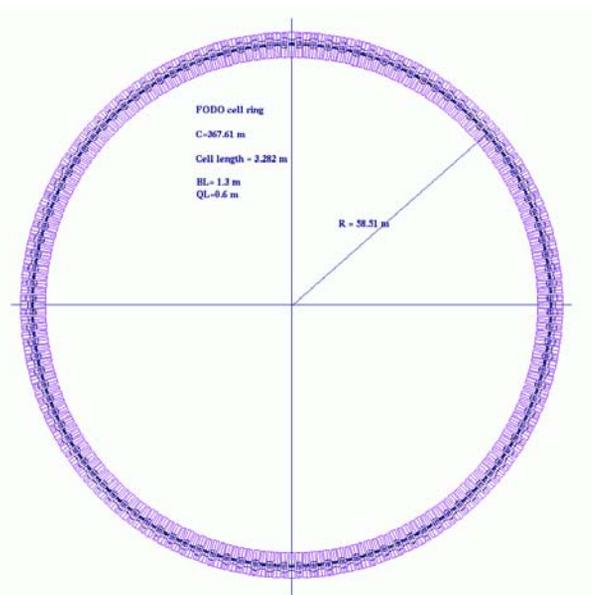


Figure 5: FODO like design of the ring made of packed combined function magnets. Beam orbits for momentum range of  $\delta p/p = \pm 40\%$  are presented in the middle of magnets with 25 magnifications.

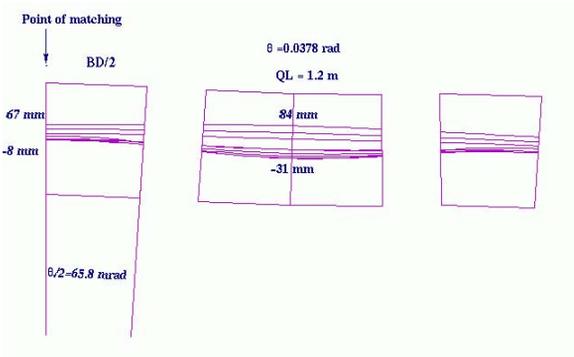


Figure 6: Maximum orbit offset within a single FODO cell.

“*Racetrack*” properties

The combined “*racetrack*” ring is presented in Figure 1. The geometrical conditions are also added to the tracked (Polymorphic Tracking Code-PTC [6] orbits of the muons for the momentum range of  $\delta p/p = \pm 40\%$ ).

Table 1. Magnet Specifications

Mag	L(m)	B <sub>0</sub> (T)	G (T/m)	A <sub>x</sub> (mm)	B <sub>max</sub> (T)
BD	1.2	4.56	-36.4	-7-+48	4.8-2.8
BF	0.58	-2.42	40.5	-28-74	-3.6-0.6
BD2	1.3	4.83	-22.3	-8-67	5.0-3.4
BF2	0.6	3.0	27.4	-31-84	2.5-5.3

SUMMARY

A continuation of the previous effort to make for the first times the NS-FFAG as a racetrack accelerator shows few improvements: all the amplitude functions and dispersion are matched at the central energy between two different NS FFAG arcs. The muon acceleration from an energy of 8.57 to 20 GeV is provided within a circumference of C~445.29 m. There are 56 cells available for cavities and extraction/injection kickers. The present solution clearly shows stable orbits from -40% to +40% in momentum and stable horizontal betatron function. It is important to note that magnets have significantly smaller gradients and magnetic fields. Values of the gradients and bending fields are reduced with respect to the previous designs.

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