EMITTANCE COMPENSATION FOR MAGNETIZED BEAMS

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Abstract

Emittance compensation is a well established technique [1] for minimizing the emittance of an electron beam from a RF photo-cathode gun. Longitudinal slices of a bunch have a small emittance, but due to the longitudinal charge distribution of the bunch and time dependent RF fields they are not focused in the same way, so that the direction of their phase ellipses diverges in phase space and the projected emittance is much larger. Emittance compensation reverses the divergence. At the location where the slopes of the phase ellipses coincide the beam is accelerated, so that the space charge forces are reduced. A recipe for emittance compensation is given in [2].

For magnetized beams (where the angular momentum is non-zero) such emittance compensation is not sufficient because variations in the slice radius lead to variations in the angular speed and therefore to an increase of emittance in the rotating frame. We describe a method and tools for a compensation that includes the beam magnetization.

INTRODUCTION

A magnetized beam is a beam with an angular momentum is non-zero. Such beams are useful for the generation of flat electron beam, as are needed for the International Linear Collider, and in an electron cooler, where the electron temperature inside a solenoid must be minimized. According to Busch's theorem the magnetization is (outside solenoid fields) a constant of motion, i.e. an existing unmagnetized bunch cannot be magnetized. It must be born that way inside a longitudinal magnetic field.

A RF Photo-cathode electron gun is an efficient device to produce low emittance bunched electron beams. The electrons are produced by a pulsed laser beam, which eliminates the beam choppers used for bunched beams from DC electron guns. The disadvantage of the RF gun is that particles in different longitudinal positions of the bunch do not see the same external and internal fields as they would in a DC gun. This leads to an apparent emittance blow up. However, the information of how the bunch was deformed is still available and the process of emittance compensation, devised by Serafini and Rosenzweig [1], makes it possible to reverse the emittance blow-up to a large degree.

For this purpose it is useful to cut the bunch into longitudinal slices and consider their motion. When a slice leaves the cathode it occupies a horizontal line in the x-x' phase space. The length of this line is given by the spot size of the laser and the thickness comes from the transverse temperature of the electrons leaving the cathode.

As the bunch progresses through the electron gun the lines rotate in phase space with different speed, depending on the local current density (space charge) and the time dependence of the RF fields. The projection of all lines will then produce a butterfly shape.

The beam is then focused by a solenoid. Because of the space charge defocusing the position of envelope waist of each slice depends quadratically on the convergence and it is possible to minimize the spread of the waist positions by adjusting the solenoid strength. At the waist the beam is accelerated to higher energies where the space charge forces become less important. The phase space lines may have changed their length, but they are all approximately horizontal which minimizes the projected emittance.

For a magnetized beam the emittance compensation is not that simple. The phase space of a slice leaving the cathode is also a line. This line turns into an ellipse when the slice leaves the gun solenoid and experiences the solenoid fringe field. Of course the emittance of the slice does not change, since the application of another solenoid field with

$$B_s = \frac{r_0^2}{r^2} B_{s0} \tag{1}$$

will reproduce the initial area in phase space. Here r is the radius of the slice and the index 0 is used for the field and radius on the cathode.

Equation 1 shows that the appropriate solenoid field depends on how much the slice changed the radius. If the slices of the bunch expand at different rate then a single field strength can not recover the emittances of all slices.

The magnetization is creating a different kind of emittance blow-up of the projected emittance. Magnetized emittance compensation must therefore try to line up the radius expansion as well as the slopes of the envelopes.

The well known [3] differential equation for the slice envelope is:

$$\begin{aligned} & r'' + \frac{\gamma'}{2\beta^2\gamma} r' + \frac{\gamma''}{2\beta^2\gamma} r + \left(\frac{eB}{2m_e c\beta\gamma}\right)^2 r \\ & - \left(\left(\frac{M}{\beta\gamma}\right)^2 + \left(\frac{\varepsilon_n}{\beta\gamma}\right)^2\right)\frac{1}{r^3} - \frac{I}{I_0(\beta\gamma)^3\sigma_r^2} r = 0 \end{aligned} \tag{2}$$

Here r is the radius of the beam, $\gamma = E/m_e + 1$, $\beta = v/c$, ε_n is the normalized thermal emittance and $M = \langle \beta\gamma(x\cdot y' - y\cdot x')\rangle$. B is the

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solenoid field and m_e is the electron mass. I is the local beam current of a slice and I_0 is the Alven current

In [2] we have given a recipe for designing non-magnetized emittance compensation. It assumes that the magnetization and the thermal emittance is negligible. Then it is possible to solve the differential equation and find an approximate position of the beam waist. This does not work for magnetized beams.

Instead we have created a computer code named SLENV (Slice ENVelope) that integrates equation (2) for the slices of the bunch using the Runge-Kutta method. It calculates the emittance from the envelopes. A built-in optimizer minimizes the projected emittance.

SLENV is similar to the computer code HOMDYN [4]. It differs however in the following:

- SLENV does not try to simulate the motion close to the cathode, where space charge forces are strongly non-linear and the bunch length is changing rapidly. Instead, SLENV has an interface to read PARMELA [5] results for that region.
- SLENV allows calculating a subset of the slices, which degrades the results only slightly, but increases the computing speed. Nine equally spaced slices are used.
- SLENV allows ignoring the longitudinal space charge forces and the change of the bunch length, also to increase the computing speed.
- A built-in optimizer can be used to find the best emittance compensation. The package CONDOR [6] is used. A genetic algorithm will be added.

SLENV is not designed to be accurate, but to find an initial solution for emittance compensation, which can be used as a start point for further optimization with exact codes like PARMELA, ImpactT and ASTRA.

THE INNER WORKINGS OF SLENV

A measure for the beam quality is the 4-D emittance which is defined as the determinant of the 4-dimensional sigma matrix:

$$\varepsilon_{4D}^4 = \begin{vmatrix} <x^2> & <xx'> & <xy> & <xy'> \\ <xx'> & <x'^2> & <x'y> & <x'y'> \\ <xy> & <x'y> & <y^2> & <yy'> \\ <xy'> & <x'y'> & <yy'> & <y'^2> \end{vmatrix}$$

SLENV averages the sigma matrices for all slices to calculate the projected emittance. It is assumed that the beam is round. Also assumed is that the beam has a uniform density inside the bunch. Besides the total current and bunch lengths the input contains for each slice $\sigma_r, \sigma_r', \gamma, I, \varepsilon_n$ and M . r is replaced in Eq. (2) with $\sigma_r = r/2$ and Eq. (2) is converted into two equations of first order.

The sigma matrix for a single slice then has the elements:

$$<x^2>=<y^2>=\sigma_r^2/2 \qquad <xx'>=<yy'>=\sigma_r\sigma_r'$$
$$<xy>=<x'y'>=0 \qquad <xy'>=-<yx'>=M/2$$

$<x'^2>$ and $<y'^2>$ are calculated from $\varepsilon_{4D}^4 = |\sigma_{4D}|$ and it is assumed that ε_{4D} and M are constants of motion, i.e. the non-linearity of internal and external fields is ignored.

A switch allows calculating the path length and the energy of each slice. Divergence of the path length changes the current distribution of the bunch. For the change of the slice energy a simple longitudinal space charge model is used assuming a cylindrical charge distribution of the bunch. However, this has no visible effect on the envelopes and is barely visible in the emittance calculation.

RESULTS

Figure 1 shows the RMS beam envelope of a magnetized electron beam. The radius on the cathode is 4.7 mm, the magnetic field is 40 Gauss and the bunch charge is 3.2 nC. The PARMELA calculation uses 250K particles; the results at the exit of the gun are used as a start point for SLENV which uses 500 slices.

Figure 2 shows the corresponding projected emittances. While there is a good agreement in the envelopes, the SLENV emittances diverge from those of PARMELA. However, the change of the emittance as a function of the beam line parameters depends on the envelopes and the systematic error in the emittance calculation does not impede the usefulness of the program.

The emittance compensation in this example was optimized with SLENV. One can see from Figure 1 that the best emittance compensation for a magnetized beam is not obtained when the accelerating cavity is placed at the waist of the beam but at a maximum of the envelope. The beam is focused by the transverse electric field at the cavity entrance.

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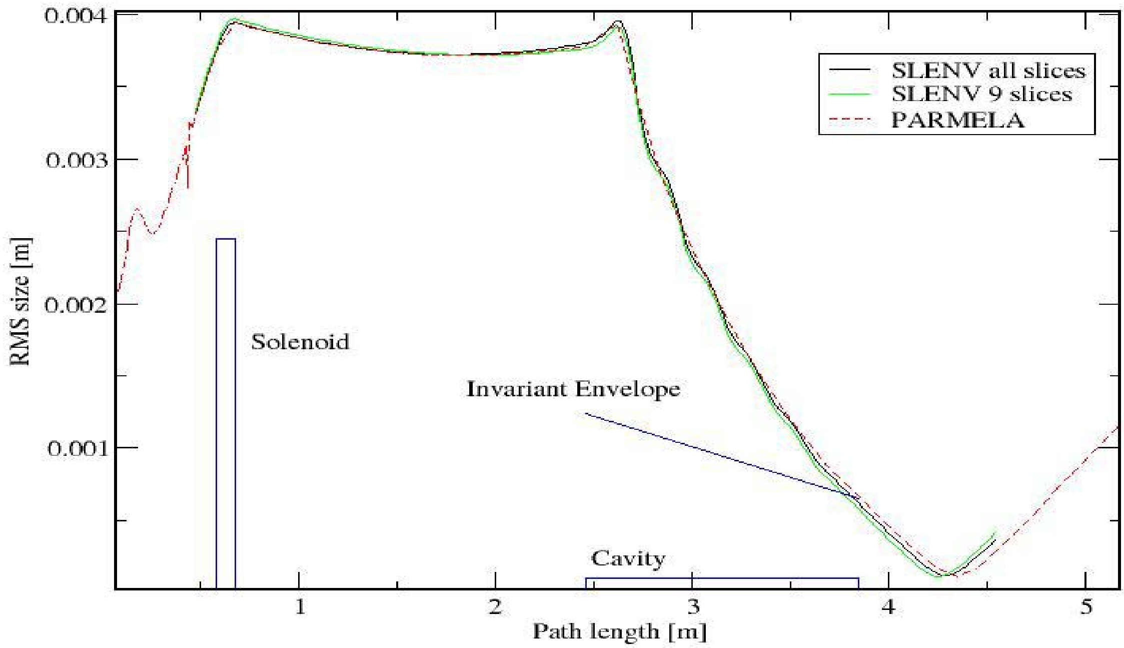

Figure 1: Beam Envelope calculated with PARMELA and SLENV

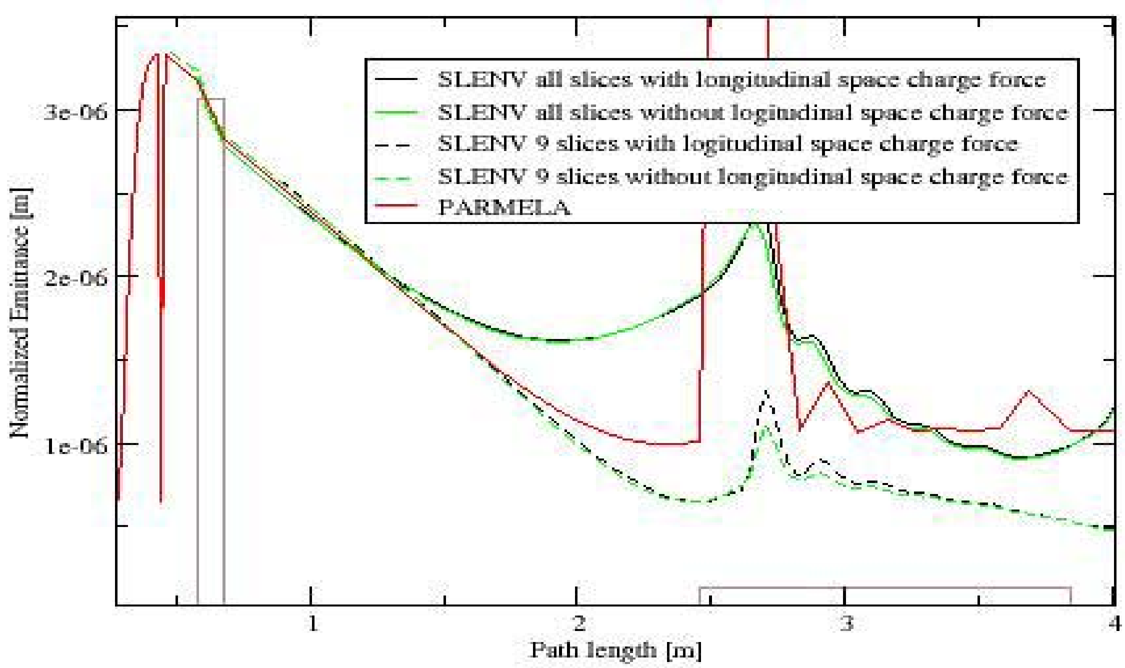

Figure 2: Beam Emittance calculated with PARMELA and SLENV in different modes.

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