# NONLINEAR PERMITTIVITY EFFECTS IN DIELECTRIC ACCELERATING STRUCTURES\*

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#### Abstract

New low loss ferroelectric ceramic materials possessing large variations in the permittivity as a function of the electric field present interesting and potentially useful applications for dielectric loaded accelerating structures, both wakefield-based and driven by an external rf source. We will consider X-band cylindrical dielectric structures and report numerical results on frequency multiplication, wave steepening and shock formation, and the effect of nonlinearities on the mode structure of these devices. We will examine applications of nonlinear dielectric devices to high gradient acceleration, rf sources, and beam diagnostics.

#### **INTRODUCTION**

Technology based on nonlinear optical phenomena has had a significant impact on the laser field, where harmonic generation and other effects are routinely and productively used. Similar effects have been employed at rf frequencies where the nonlinear properties of ferrite loaded transmission lines have been used to produce short rf pulses [1]. Given the serious interest in dielectric loaded accelerating structures (DLAs) [2] operating at Xband and above it seems worthwhile to examine potential applications of nonlinear dielectrics in these devices.

The properties of wakefields in a nonlinear dielectric waveguide were initially studied a number of years ago [3]. Numerical results showed that some wave steepening did occur and could act to enhance the wakefield acceleration gradient. Further development of these results into a working technology was hampered by the unavailability of suitable low loss, low permittivity dielectrics with fast response times and suitably high beam currents for wakefield experiments.

A reexamination of the potential applications of nonlinear dielectric waveguides has been prompted by substantial progress in the area of microwave dielectrics, particularly ferroelectric-based ceramic materials [6,7]. A ferroelectric ceramic possesses an electric-field-dependent dielectric permittivity that can be rapidly varied by an applied bias voltage pulse. Ferroelectrics have unique intrinsic properties that make them extremely interesting for a number of high-energy accelerator and microwave applications. Response times of  $\sim 10^{-11}$  sec for the crystalline form and  $\sim 10^{-10}$  sec for ceramic compounds have been measured. Ferroelectrics allow control of their dielectric properties in two directions using a single

external control pulse, offering unique capabilities for high-power switching and tuning devices intended for accelerator applications.

When ferroelectrics are used for tuning accelerating structures [6], the permittivity of a slab or cylindrical shell of the material is adjusted with an applied DC bias voltage. In this case the electric field of the rf signal is much smaller than the bias field strength and has a negligible effect on the permittivity. Typical values of the tunability (change in relative permittivity with a change in the electric field) are roughly 30% and can be up to 80% at 4-5 MV/m [8] with a reasonable loss factor of  $5 \times 10^{-3}$  at X-band. We will consider in this note the case where the nonlinearity in the ferroelectric loading of a DLA is induced by a wakefield or external rf pulse rather than by a DC bias field.

The high dielectric constant of ferroelectrics (~500) is not desirable for many applications. In particular the use of high permittivity materials leads to enhanced wall losses in DLAs. Lowering the permittivity (and the loss tangent) through the use of ferroelectric-low loss tangent dielectric composites is one approach. Recent theoretical work [4] has shown that ferroelectric composites can be designed that also preserve or even enhance the tunability of the material, and DC permittivities ~100 are feasible.

We have however assumed for computational efficiency that the DLA loading materials have zero field permittivities ~10. We expect that the tunability of the material is the more important effect and that appropriate scaling of the dielectric and structure geometry will result in equivalent nonlinear effects being observed in laboratory. An alternative approach is the use of a double layer structure consisting of an inner linear dielectric with  $\varepsilon$ ~4-6 and an outer nonlinear shell, giving an average permittivity ~10. We plan to explore these options in our future research.

We will consider DLAs with weak and strong nonlinearities and examine the phenomena of wakefields in these structures. We suggest a possible experimental test of a nonlinear device at the Argonne AWA facility.

## NUMERICAL EXPERIMENTS

We used the *Arrakis* code [5] as our principal tool to investigate electromagnetic fields in nonlinear dielectric structures. This is a FDTD code based on a Lax-Wendroff algorithm and is specially suited for handling nonlinear problems. For the first set of calculations we assumed a

weak quadratic nonlinearity  $D(E) = \varepsilon_1 (1 - \frac{tE^2}{2E_{max}^2})E$  with

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 $E_{\rm max} = 100$  statvolts/cm = 3 MV/m, t = 0.1 is the fractional change in the permittivity at  $E = E_{\rm max}$ , and  $\varepsilon_1 = 10$ . The absolute tunability is defined as the ratio  $n(E) = \varepsilon(0)/\varepsilon(E)$  and is plotted as a function of the electric field strength is shown in Figure 1. This is similar to the experimental results we have obtained on the tunability of ferroelectric ceramics.

Consider a beam-excited cylindrical resonator loaded with this material, with an outer radius of 1 cm, beam channel radius 0.4 cm, and length 4 cm. The beam parameters correspond to those available at the Argonne Wakefield Accelerator high current linac:  $\sigma_z = 1.5$  mm, and maximum beam intensity ~ 40 nC.



Figure 1: Absolute tunability as a function of the electric field strength used in the calculation of the nonlinear resonator.



Figure 2: Segment of the axial electric field waveform (r=0, z=2 cm) induced in the nonlinear resonator by a 1.6 nC bunch (solid, multiplied by a factor of 10 for clarity) and a 16 nC bunch (dotted). A phase shift and extra harmonic content are clearly visible in the high current result.

Figure 2 shows a comparison of the longitudinal electric field induced in the structure by the high and low current beams. The effect of the nonlinearity is to create higher frequency Fourier components (Fig. 3) and other waveform changes. (Note that the low current waveform

is multiplied by the ratio of the bunch intensities to make the effects of the nonlinearity more apparent.)

A straightforward experiment at the AWA to study dynamic nonlinearities of ferroelectrics at high frequencies would consist of instrumenting this structure with field probes and measuring the waveforms as a function of beam intensity and bunch length. The heterodyne wakefield rf measurement system at the AWA [9] is capable of diagnosing very high frequency waveforms like those present in this experiment.

We also studied a case of a strong nonlinearity in a wakefield device. Here we modeled a 32 cm structure, outer radius 1 cm, and beam channel radius 6 mm. The electric field dependence of the dielectric displacement was taken as an arctangent function with a small field permittivity of 10, rolling off to 2 at high fields. Figure 4 shows a comparison of low current (negligible nonlinear effects) and high current beam wakefields. A clear enhancement of the accelerating gradient with respect to the linear case caused by bunch steepening is observed.



Figure 3: Fourier spectrum of the wakefield signal in the nonlinear resonator. Solid: 1.6 nC bunch, multiplied by 10. Dotted: 16 nC bunch. Additional high frequency components are clearly visible in the high current bunch spectrum.



Figure 4: Snapshot of the axial electric field on axis for the strong nonlinearity wakefield simulation. Solid: 1.6 nC bunch, scaled by the charge ratio of 40. Dotted: 64 nC bunch.

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## **POTENTIAL APPLICATIONS**

The recent development of nonlinear ferroelectric ceramics has opened up a new field of advanced nonlinear devices for accelerator and rf applications. Beam diagnostics based on nonlinear waveguides are one possibility, since the frequency spectrum is a function of both beam intensity and pulse shape. Nonlinear structures may also find applications in rf sources for frequencies (such as sub-mm waves) not accessible by conventional technologies. Electromagnetic shock formation [1] can be used to produce intense short broadband rf bursts. Finally, application of wave steepening/pulse compression effects in nonlinear waveguides to enhance the performance of wakefield accelerators is an exciting possibility.

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