# NOVEL METHOD OF EMITTANCE PRESERVATION IN ERL MERGING SYSTEM IN PRESENCE OF STRONG SPACE CHARGE FORCES*

D. Kayran, V.N. Litvinenko, BNL, Upton, NY 11973, U.S.A..

*Abstract*

Energy recovery linacs (ERLs) are potential candidates for the high power and high brightness electron beams sources. The main advantages of ERL are that electron beam is generated at relatively low energy, injected and accelerated to the operational energy in a linac, and after the use is decelerated in the same linac down to injection energy, and, finally, dumped. A merging system, i.e. a system merging together high energy and low energy beams, is an intrinsic part of any ERL loop. One of the challenges for generating high charge, high brightness electron beams in an ERL is development of a merging system, which provides achromatic condition for space charge dominated beam and which is compatible with the emittance compensation scheme. In this paper we present principles of operation of such merging systems. We also describe an example of such system, which we call a Zigzag or a Z-system. We use a specific implementation of the Z-system for R&D ERL at Brookhaven as the illustration.

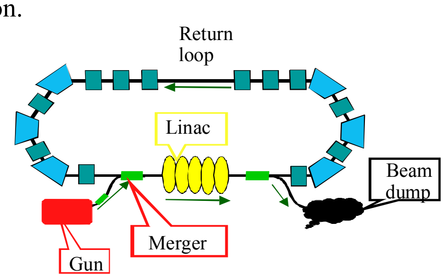


Figure 1.Schematics view of ERL.

## MOTIVATION

Each ERL loop (Fig. 1) has at least one merging system, which includes dipoles. Any dipole is a source of coupling between $(\delta = (E - E_o)/E_o, \xi = s - vt)$ longitudinal and (x, $p_x$) transverse phase space planes.

The space charge forces repel particles from each other. Hence, the particles in the head of the bunch gain energy, while the particles at the tail of the bunch lose energy (see Fig. 2). This effect is very significant for low energy beams with high charge per bunch. Low injection energy (well below 10 MeV) is strongly desirable in high current ERLs to lessen the radiation hazards and to reduce requirements for RF power.

The use of achromatic system for a merger will decouple the motion, but only in the absence of the longitudinal space charge forces. In the presence of the space charge forces, the coupling resulting from the variation of the particle's energy can cause significant growth of the transverse emittance in a traditional achromatic system.

In addition, the emittance compensation schemes [1] do not allow using a strong focusing in a merger. This requirement limits even further the number of available merger schemes used for high charge, high brightness electron beams.

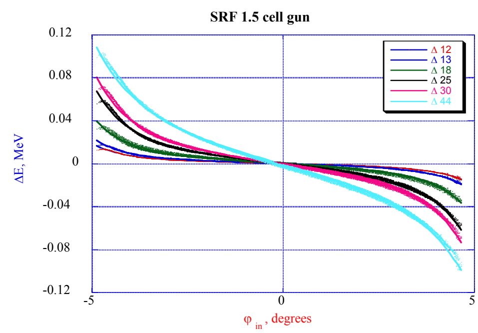


Figure 2. Variation of electron energy, ΔE= E(s) - E(o), in a merger (caused by the space charge forces of 1 nC bunch) is a function of its position in the bunch (shown in the units of a relative 703.75 MHz RF phase). Different colours indicate different locations along the beam line, which indices (12,13,18,25,30,44) showing the element numbers in lattice file for PARMELA [2]. Fitting is defined by eq. (1).

One of the keys to a successful solution of the above problem is finding a correlation between electron's energy change and its location in the bunch. For example, Fig. 2 shows results of PARMELA [2] simulation of a 1 nC bunch with initial "beer-can" distribution [3]. The other beam parameters at the cathode were: duration 10º (39.5 psec), radius 4 mm, 1.5 cell RF gun with energy gain of 3.7 MeV. The energy change of the particle is defined by longitudinal electric field **E**:

$$\frac{dE}{ds} \cong e\,\mathrm{E}(\zeta),$$

where $\zeta$- is longitudinal coordinate of the particle relative of the bunch center. The resulting dependencies (see Fig. 2) fit very well with analytical formula for the field of the homogeneously charged cylinder [3]:

$$\mathrm{E}(\zeta_o) = \frac{2Q}{r^2 \cdot 2l}\left(2\zeta_o - \sqrt{r^2 + (\zeta_o + l)^2} + \sqrt{r^2 + (\zeta_o - l)^2}\right) \quad (1)$$

where $Q$ is the charge, $r$ is the radius and $l$ is the length of the beam.

The most importantly, the fit for the energy changes with the formula allows to separate variables and to express it as a function of initial longitudinal coordinates $(\delta_o, \zeta_o)$ and the azimuth $s$ along the orbit (Fig. 3): $\delta(s) \cong \delta_o + f(\zeta_o) \cdot (s + \alpha \cdot s^2)$.

*Work performed under the auspices of the U.S. Department of Energy and partially funded by the US Department of Defence
#dkayran@bnl.gov

Thus, energy dependence vs $s$ for any electron is well described by its dependence on two parameters – the initial energy and the initial phase. Hence, in general case, we are seeking a 2-parameter dependence:

$$\delta_i(s) = a_i \cdot g_1(s) + b_i \cdot g_2(s) \qquad (2)$$

where: $i$ is the index of the particle, $E_i$ is the energy of i[th] particle, $a_i$ and $b_i$ are individual parameters for i[th] particle (presumably some functions of the initial $(\delta_{io},\zeta_{io})$), $g_1(s)$ and $g_2(s)$ are the function of azimuth $s$, which are the same for all particles in the beam. It is important that results of following method are completely independent from the specific dependence of parameters $a_i$ and $b_i$ on $(\delta_{io},\zeta_{io})$/

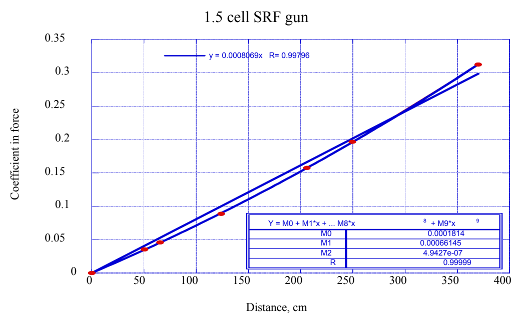


Figure 3. Dependence of the energy gain on the azimuth $s$. Red dots are the results of simulations; the blue lines are linear and second order polynomial fits.

## CONCEPT

Horizontal betatron oscillations around the ideal trajectory are described by well-known homogeneous linear equation:

$$\frac{dX}{ds} \equiv X' = D(s)\cdot X(s); \quad D = \begin{bmatrix} 0 & 1 \\ -K_1(s) & 0 \end{bmatrix}; \qquad (3)$$

where $X^T = [x, p_x]$ and $K_1(s)$ is defined by focusing strength of magnets and the space charge of the beam. The solution of equation (3) can be expressed in matrix form:

$$X(s) = M(s)\cdot X(0)$$

where $X(0)$ is initial transverse phase space coordinates. The $M(s)$ is the 2x2 transport matrix from $0$ to $s$ which is satisfied conditions:

$$M' = D(s)\cdot M \ ; \ \det M = 1 .$$

For a particle with energy deviation $\delta(s)$ the equation of motion becomes inhomogeneous:

$$\Psi'(s) = D(s)\cdot \Psi(s) + K_o(s)\cdot \delta(s)\cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad (4)$$

where $K_o(s) = eB_y$. Using $\Psi(s) = M(s)\cdot A(s)$

$$A' = K_o \cdot \delta \cdot M^{-1}\cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow A' = K_o \cdot \delta \cdot \begin{bmatrix} -m_{12} \\ m_{11} \end{bmatrix}$$

and zero initial conditions $\Psi(o) = 0$ one gets:

$$A^T(s) = \left[ -\int_0^s K_o(s')\cdot \delta(s') m_{12}(s') ds', \int_0^s K_o(s')\cdot \delta(s') m_{11}(s') ds' \right] .$$

Hence, for the decoupling of transverse and longitudinal motions (it is actually the condition on 4x4 symplectic matrix [5]) at the end of the system, $s_f$, one should request that for each electron two conditions are satisfied:

$$A(s_f) = 0. \qquad (5)$$

Using parameterization (2), one can rewrite (5) in the form of four conditions:

$$\int_0^S K_o(s')\cdot g_1(s)\cdot m_{11}(s')ds' = 0; \int_0^S K_o(s')\cdot g_2(s)\cdot m_{11}(s')ds' = 0; \qquad (6)$$
$$\int_0^S K_o(s')\cdot g_1(s)\cdot m_{12}(s')ds' = 0; \int_0^S K_o(s')\cdot g_2(s)\cdot m_{12}(s')ds' = 0;$$

which provide sufficient for the decoupling of transverse and longitudinal motion for all electrons within the bunch.

### *System with bilateral symmetry (Zigzag)*

The simplest case of parameterization in eq. (2) is that of a "frozen" longitudinal motion:

$$\delta' = g(\zeta_o) \Rightarrow \delta_i(s) = \delta_{io} + s\cdot g(\zeta_{io}),$$

which is a good approximation for variety of the processes relevant to the space charge effects (see Fig. 3, where linear approximation fits $g_2(s)$ rather well, or see Refs. [6,7] where similar considerations were applied for coherent synchrotron radiation effects);

The decoupling conditions in this case are:

$$\int_0^S K_o(s')\cdot m_{11}(s')ds' = 0; \qquad \int_0^S K_o(s')\cdot s\cdot m_{11}(s')ds' = 0; \qquad (7)$$
$$\int_0^S K_o(s')\cdot m_{12}(s')ds' = 0; \qquad \int_0^S K_o(s')\cdot s\cdot m_{12}(s')ds' = 0;$$

Let's consider a system, which we call Zigzag, with symmetrical focusing $K_1(s) = +K_1(s)$ and asymmetrical curvature $K_o(s) = -K_o(s)$ (see Fig. 4).

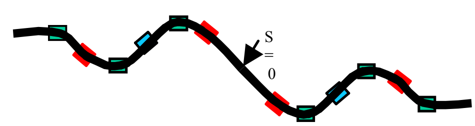


Figure 4. Schematic of a Zigzag: green boxes are the dipoles, red and blue boxes are focusing and defocusing lenses.

The elements of the transport matrix for such systems are coupled by the conditions for the bilateral symmetry:

$$M(-s) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} M(s) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} .$$

$$\Rightarrow m_{11}(-s) = m_{11}(s); m_{12}(-s) = m_{12}(s)$$

Hence for Zigzag system two achromatic conditions from (7) are automatically satisfied:

$$K_o(-s)\cdot m_{11}(-s) = -K_o(s)\cdot m_{11}(s) \Rightarrow \int_{-L}^{L} K_o(s')\cdot m_{11}(s')ds' \equiv 0$$

$$K_o(-s)\cdot(-s)\cdot m_{12}(-s) = -K_o(s)\cdot(s)\cdot m_{12}(s) \Rightarrow \int_{-L}^{L} K_o(s')\cdot m_{12}(s')s'\cdot ds' \equiv 0$$

The rest of achromatic conditions (7) can be rewritten as:

$$\int_0^L K_o(s')\cdot m_{12}(s')ds' = 0; \qquad \int_0^L K_o(s')\cdot s\cdot m_{11}(s')ds' = 0. \qquad (8)$$

Example: A simplest system consists of 2K short dipoles (with bending angle $\theta_k$ and position $s_k$ each) without focusing in horizontal direction. In this case the elements

 0-7803-8859-3/05/$20.00 

of transport matrix are: $m_{11} = 1$, $m_{12} = s$ and only one condition remains:

$$\sum_{k=1}^{K} s_k \cdot \theta_k = 0 \qquad (9).$$

For K=2 the condition (9) gives a simplest Zigzag with $s_2 = 2s_1$ , $\theta_1 = -2\theta_2$ [8].

## RESULTS OF THE TESTS

Detailed results of the test of the concept can be found in [5,8]. Here we present only main results by comparing a traditional achromat (chicane) with a Z-system (see Fig. 4). To make a fair comparison, both systems have the same focusing strength and are made of chevron with 86 cm radii or curvature. Both configurations are achromatic for particle with constant energy. This resulted in following parameters:

ZigZag lattice: 10º bend, 40 cm drift, -20º bend, 81.6 cm, 20º bend, 40 cm drift, 10º bend

Chicane lattice: 12.4º bend, 47.5 cm drift, -11.36º bend, 96.6 cm, 11.36º bend, 47.5 cm drift, -12.4º bend

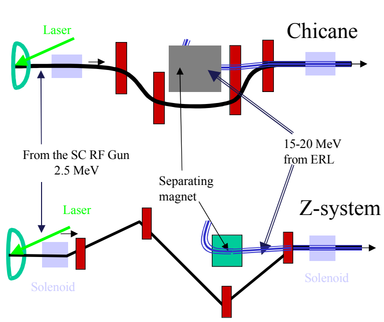


Figure 4. Schematics of traditional chicane and a Zigzag-system for an ERL.

In the numerical test performed with PARMELA, a 1 nC electron bunch from the 1.5-cell RF gun was propagated through the above merging systems followed by a 15 MeV 703.75 MHz linac. The electron beam energy at the gun exit was $\gamma mc^2$=4.2MeV. Initial beam has "beer-can distribution": duration of 12º (47 psec) and radius 4 mm.

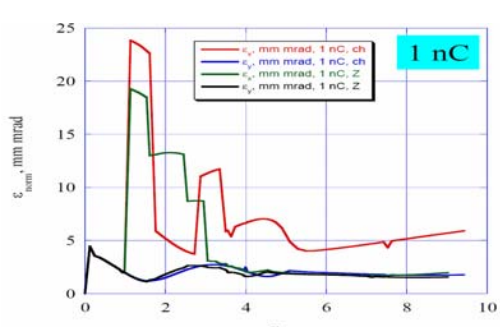


Figure 5: Results of PARMELA simulation for 1nC.

Results of the PARMELA simulation are shown in Fig. 5. In both cases, for the chicane and the Z-system, vertical normalized emittances are equal to about 1.8 mm*mrad at the linac exit, the indication of the equivalence of the systems for the process of emittance compensation. In contrast, the horizontal emittance behavior is very different for two systems. After passing the Z-system horizontal emittance and vertical emittance have practically the same value, while the chicane results in a doubling of the horizontal emittance.

## CONCLUSIONS

We developed the new concept to the ERL merging system compatible with the emittance compensation schemes for generating high brightness electron beam. To our surprise this simple concept, some version of which were intuitively used previously [6,7], works very well for many processes, including space charge dominated magnetized beams [10] and coherent synchrotron radiation [6].

## ACKNOWLEDGEMENTS

The authors are grateful to Ilan Ben-Zvi for fruitful discussions and for bring this problem to our attention, to Rama Calaga, Xiangyun Chang and Jörg Kewisch for their help with the simulation of the RF gun and with additional tools for PARMELA simulation, and for the test of the concept with magnetized beams.

## REFERENCES


[1] J. B. Rosenzweig and L. Serafini, Phys. Rev. E 49, 1599 (1994).

[2] L. M. Young, J. H. Billen, PARMELA documentation, LA-UR-96-1835.

[3] "Beer-can" distribution is an uniformly charged cylinder.

[4] V.V. Batygin, I.N. Taptygin, Collection of E&M problems, Second Edition, Nauka, Moscow, 1970

[5] D. Kayran, V. Litvinenko, Novel merging system preserving the emittance of high brightness high charge beams in ERLs, in preparation
D. Kayran, V.N. Litvinenko, Optimum merger for an ERL, ERL 2005, March 2005, Newport News, VA.

[6] David H. Dowell, Compensation of bend-plane emittance growth in a 180 Degree Bend, PAC97

[7] Ryoichi Hajima First-Order Matrix Approach to the Analysis of Electron Beam Emittance GrowthCaused by Coherent Synchrotron Radiation, Jpn. J. Appl. Phys. Vol. 42 (2003) pp. L 974–L 976

[8] V. N. Litvinenko et al., Proceedings of the 2004 FEL Conference, p. 570
http://accelconf.web.cern.ch/AccelConf/f04

[9] I. Ben-Zvi at al., Extremely High Current, High-Brightness Energy Recovery Linac, these proceedings.

[10] Electron Cooling of RHIC, I. Ben-Zvi et al., these proceedings
Electron Beam Generation and Transport for the RHIC Electron Cooler, J. Kewisch et al., these proceedings.