BEAM-BASED NON-LINEAR OPTICS CORRECTIONS IN COLLIDERS*

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Abstract

A method has been developed to measure and correct operationally the non-linear effects of the final focusing magnets in colliders, that gives access to the effects of multi-pole errors by applying closed orbit bumps, and analyzing the resulting tune and orbit shifts. This technique has been tested and used during 4 years of RHIC (the Relativistic Heavy Ion Collider at BNL) operations. I will discuss here the theoretical basis of the method, the experimental set-up, the correction results, the present understanding of the machine model, the potential and limitations of the method itself as compared with other non-linear correction techniques.

INTRODUCTION

The IR bump technique has been developed to correct operationally local errors in the interaction regions (IRs). The dynamic aperture of a hadron collider is typically determined by beam-beam effects and by the errors from the magnets in the low-β* sections. Non-linear effects become significant at RHIC only for β*< 2m. Local non-linear corrections can improve dynamic aperture, beam lifetime and operability. Non-linear corrections, together with emittance reduction by electron cooling, a planned RHIC upgrade, can allow decreasing β* from the present 1m to 0.5m, a potential 50% increase in luminosity. We describe here a general formulation of the IR bump valid for all orders, followed by examples of application to the normal and skew sextupole and octupole effects. Then we will outline the experimental set-up, procedures and results of the correction achieved so far in RHIC, mainly sextupole and octupole, followed by a discussion on the feasibility of higher orders measurements. Finally, we will discuss the correlation of observed non-linear optics distortions with field measurements of the IR magnets

IR BUMP METHOD: THEORY

The magnetic errors in an accelerator magnet can be described in terms of the multipole errors a_n and b_n defined as:

$$B_y + iB_x = B_N \sum_{n=0}^{\infty} (b_n + ia_n) \left(\frac{x+iy}{R_r} \right)^n \quad (1)$$

B_N represents the main field of a magnet measured at a reference radius R_r.

A local orbit excursion through a region with nonlinear fields generates feed-down effects to lower order harmonics. The most useful observables come from the feed-down to the zero and first order harmonics, which affect respectively the beam closed orbit and betatron tunes. The magnitude of the orbit and tune changes depends on the size of the orbit excursion. For an orbit bump built with 3 dipole correctors (3-bump), the bump amplitude A_{bump} is the measure of the orbit excursion and is defined as the orbit excursion at the location of the dipole corrector in the middle of the bump. The tune shift arise either from the feed-down to the normal gradient or from the effect of linear coupling. The tune shift ΔQ and the linear coupling term Δc for different bump planes (H and V) and for different multipole errors can be expressed as follows:

$$\begin{aligned} &\Delta Q(H, norm) = g(b_n, x_{co}) \\ &\Delta Q(V, skew, even) = -1^{n/2} g(a_n, y_{co}) \\ &\Delta c(V, norm, even) = -1^{(n-1)/2} h(b_n, y_{co}) \\ &\Delta Q(V, norm, odd) = -1^{(n-1)/2} g(b_n, y_{co}) \\ &\Delta c(H, skew) = h(a_n, x_{co}) \\ &\Delta c(V, skew, odd) = -1^{(n+1)/2} h(a_n, y_{co}) \end{aligned} \quad (2)$$

where the functions g and h are:

$$\begin{aligned} g(c_n, z_{co}) &= \frac{n}{4\pi} \frac{1}{B\rho} \int \beta_z B_N c_n \frac{z_{co}^{n-1}}{R^n} ds \\ h(c_n, z_{co}) &= \frac{n}{2\pi} \frac{1}{B\rho} \int \sqrt{\beta_x \beta_y} B_N c_n \frac{z_{co}^{n-1}}{R^n} e^{i(\mu_x - \mu_y)} ds \end{aligned} \quad (3)$$

c_n stands for either the a_n or b_n and z stands for x or y. The tune shifts for horizontal and vertical tunes are then as: $\Delta Q_x \, \alpha \, \Delta Q$, $\Delta Q_y \, \alpha \, -\Delta Q$.

Whether the feed-down from a given multi-pole affects the normal gradient or the linear coupling depends on the plane of the bump, on whether the multi-pole is normal or skew and on the parity of the multi-pole order n as summarized in Table 1.

Table 1: Measurable quantities for multipoles

Bump	b_2	a_2	b_3	a_3	b_4	a_4	b_5
H	ΔQ	Δc	ΔQ	Δc	ΔQ	Δc	ΔQ
V	Δc	ΔQ	ΔQ	Δc	Δc	ΔQ	ΔQ

In order to simplify the identification of individual multipoles from the observed tune shifts, the measurements conditions should be such that the tune shifts produced by coupling are negligible compared with the tune shifts from the normal gradient change. For this condition to be valid, the residual coupling in the machine has to be well corrected. The coupling compensation and the tune separation must satisfy the relation:

$$\left| C_0 + \Delta c \right|^2 / 4\Delta Q_{xy} << \Delta Q \quad (4)$$

Where C_0 is the residual component of the coupling and Q_{xy} is the betatron tune separation.

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As seen from Table 1 some of the nonlinear errors, like the a_3 error, do not produce tune shifts due to the normal gradient feed-down when either horizontal or vertical orbit bump is used. In these cases a diagonal bump can be used.

IR bump: multipole by multipole correction

Here we apply the IR bump relations, (2) and (3), explicitly to low-order non-linear errors to allow easier discussion later of the experimental results for sextupole and octupole. The essence of these corrections is the minimization, respectively; of the linear and quadratic tune shifts from an orbit bump.. Here we discuss explicitly the sextupole, skew sextupole, octupole, and skew octupole nonlinear effects and will show that the correction of the tune shift versus bump amplitude is equivalent to the compensation of resonance strength driven by the given nonlinearities. Two assumptions are crucial for this analysis. First, that the betatron phase advances across the triplets on each side of the IR are negligible, which is equivalent that sources of nonlinear errors and correctors are at the same betatron phase. Second, that the betatron phase advance over the IR between the triplets is very close to π. Both assumptions are typical for low-β^* IRs and certainly are satisfied at RHIC.

The perturbation Hamiltonian for a sextupole error is given by

$$\Delta H = \frac{k_2}{6}\cdot(x^3 - 3xy^2) \tag{5}$$

The sextupole strength k_2 is proportional to the b_2 magnet error and also describes the sextupole corrector strengths,

$$k_2 = \frac{1}{(B\rho)}\frac{\partial^2 B_y}{\partial x^2} = \frac{2B_N b_2}{(B\rho)R_r^2} \tag{6}$$

In order to evaluate the resonance strengths, we use action-angle coordinates,

$$\begin{cases} x = \sqrt{2J_x\beta_x}\cos\psi_x \\ y = \sqrt{2J_y\beta_y}\cos\psi_y \end{cases} \tag{7}$$

From substituting (7) into (5) and expanding, it follows that normal sextupoles can drive the $Q_x=p$, $3Q_x=p$, and the $Q_x+2Q_y=p$ resonances. For full correction, the following resonance strengths need minimization:

$$\begin{aligned} &\sum_j\left[k_2\beta_x^{1/2}\beta_y ds\exp(i\psi_x)\right]_j \to 0;\; \sum_j\left[k_2\beta_x^{3/2}ds\exp(i\psi_x)\right]_j \to 0 \\ &\sum_j\left[k_2\beta_x^{3/2}ds\exp(i3\psi_x)\right]_j \to 0;\; \sum_j\left[k_2\beta_x^{1/2}\beta_y ds\exp(i(\psi_x \pm 2\psi_y))\right]_j \to 0 \end{aligned} \tag{8}$$

The sums are taken over all quadrupoles and sextupole correctors in the IR. The correction weight factors for the resonances are $\beta_x^{3/2}$and $\beta_y\beta_x^{1/2}$. In the approximation of negligible phase advance in the triplets and π phase advance between them, from (8) one obtains:

$$\begin{cases} \sum_L k_2\beta_x^{3/2}ds - \sum_R k_2\beta_x^{3/2}ds = 0 \\ \sum_L k_2\beta_x^{1/2}\beta_y ds - \sum_R k_2\beta_x^{1/2}\beta_y ds = 0 \end{cases} \tag{9}$$

where L and R denote the left and the right side triplet.

According to Table 1, a horizontal orbit bump should be used to produce the tune shift due to normal gradient feed-down from sextupoles. The tune shifts from a sextupole is proportional to the horizontal beam orbit offsets,

$$\begin{cases} \Delta Q_x = \frac{1}{4\pi}\sum k_2\beta_x x_{co}ds \\ \Delta Q_y = -\frac{1}{4\pi}\sum k_2\beta_y x_{co}ds \end{cases} \tag{10}$$

Again in the stated approximation, the horizontal orbit in the triplet from an IR bump is approximately proportional to $\beta_x^{1/2}$:

$$x_{co} \approx A_{bump}\frac{\beta_x^{1/2}}{\beta_{xc}^{1/2}} \qquad x_{co} \approx -A_{bump}\frac{\beta_x^{1/2}}{\beta_{xc}^{1/2}}$$

respectively on the left and right triplets in the IR, A_{bump} is the orbit bump amplitude and β_{xc} is the beta-function at location of the central corrector of the bump. The total tune shift from the sextupoles in the triplets region from one IR, due to the horizontal IR bump are then given by:

$$\begin{cases} \Delta Q_x \propto \left(\sum_L k_2\beta_x^{3/2}ds - \sum_R k_2\beta_x^{3/2}ds\right)\cdot\frac{A_{bump}}{4\pi\beta_{xc}^{1/2}} \\ \Delta Q_y \propto -\left(\sum_L k_2\beta_x^{1/2}\beta_y ds - \sum_R k_2\beta_x^{1/2}\beta_y ds\right)\cdot\frac{A_{bump}}{4\pi\beta_{xc}^{1/2}} \end{cases} \tag{11}$$

The sextupole field errors in the triplets can be corrected by minimizing the linear coefficients of the tune shifts with the horizontal bump amplitude with two local sextupole correctors. Comparing (11) and (9), one concludes that for a normal sextupole the tune shift correction is equivalent to the resonance compensation.

A similar demonstration of the equivalence of tune shift from bump amplitude and resonance compensation has been done for skew sextupole, normal and skew octupole and can be found in [1]. We will report here only the final results.

The tune shifts from skew sextupoles are produced by a vertical bump and are given by:

$$\begin{cases} \Delta Q_x = \frac{1}{4\pi}\sum k_{2s}\beta_x y_{co}ds \propto \left(\sum_L k_{2s}\beta_x\beta_y^{1/2}ds - \sum_R k_{2s}\beta_x\beta_y^{1/2}ds\right)\frac{A_{bump}}{4\pi\beta_{yc}^{1/2}} \\ \Delta Q_y = -\frac{1}{4\pi}\sum k_{2s}\beta_y y_{co}ds \propto -\left(\sum_L k_{2s}\beta_y^{3/2}ds - \sum_R k_{2s}\beta_y^{3/2}ds\right)\frac{A_{bump}}{4\pi\beta_{yc}^{1/2}} \end{cases} \tag{12}$$

Two IR skew sextupoles can eliminate the linear dependence of the tune shifts from the vertical bump amplitude. For octupoles, if only a horizontal IR bump is used, the tune shifts from the IR octupoles are proportional to the square of the bump amplitude:

$$\begin{cases} \Delta Q_x = \frac{1}{8\pi}\sum k_3\beta_x x_{co}^2 ds \propto \left(\sum_L k_3\beta_x^2 ds + \sum_R k_3\beta_x^2 ds\right)\frac{A_{xbump}^2}{8\pi\beta_{xc}} \\ \Delta Q_y = -\frac{1}{8\pi}\sum k_3\beta_y x_{co}^2 ds \propto -\left(\sum_L k_3\beta_x\beta_y ds + \sum_R k_3\beta_x\beta_y ds\right)\frac{A_{xbump}^2}{8\pi\beta_{xc}} \end{cases} \tag{13}$$

A tune shift correction using only a single plane bump does not compensate all octupole resonance. One has to also correct tune shifts caused by a vertical orbit bump:

$$\begin{cases} \Delta Q_x = \dfrac{1}{8\pi}\sum k_3\beta_x y_{co}^2 ds \propto \left(\sum_L k_3\beta_x\beta_y ds + \sum_R k_3\beta_x\beta_y ds\right)\dfrac{A_{ybump}^2}{8\pi\beta_{yc}} \\ \Delta Q_y = -\dfrac{1}{8\pi}\sum k_3\beta_y y_{co}^2 ds \propto -\left(\sum_L k_3\beta_y^2 ds + \sum_R k_3\beta_y^2 ds\right)\dfrac{A_{ybump}^2}{8\pi\beta_{yc}} \end{cases} \quad (14)$$

A minimum of three IR octupole correctors is needed.
For the skew octupole, according to Table 1 separate horizontal or vertical orbit bumps generate to linear coupling *Δc,* not *ΔQ*. Simultaneous horizontal and vertical orbit bumps can however create the needed *ΔQ* due to gradient feed-down. Assuming equal bump amplitudes, $A_{xbump} = A_{ybump} = A_{bump}$, the tune shifts produced by this diagonal bump are:

$$\begin{cases} \Delta Q_x = \dfrac{1}{4\pi}\sum k_{3s}\beta_x x_{co} y_{co} ds \propto \left(\sum_L k_{3s}\beta_x^{3/2}\beta_y^{1/2} ds + \sum_R k_{3s}\beta_x^{3/2}\beta_y^{1/2} ds\right)\dfrac{A_{bump}^2}{4\pi\beta_{xc}^{1/2}\beta_{yc}^{1/2}} \\ \Delta Q_y = -\dfrac{1}{4\pi}\sum k_{3s}\beta_y x_{co} y_{co} ds \propto -\left(\sum_L k_{3s}\beta_x^{1/2}\beta_y^{3/2} ds + \sum_R k_{3s}\beta_x^{1/2}\beta_y^{3/2} ds\right)\dfrac{A_{bump}^2}{4\pi\beta_{xc}^{1/2}\beta_{yc}^{1/2}} \end{cases} \quad (15)$$

A diagonal IR bump and two skew octupole correctors are therefore necessary to cancel skew octupole effects, and that is equivalent the skew octupole resonance strengths compensation.

IR BUMP METHOD: OPERATIONS

An operational environment has been developed at RHIC for local IR corrections. We will describe the IR layout, the correction tools, procedure and results. Sextupole IR correction is now operational and part of the machine set-up for the RHIC run. IR sextupole corrections have been validated by lifetime observations and dynamic aperture measurements. Higher order correction is subject of beam experiments.

The RHIC IR layout and correctors

The 6 IRs β* are tuneable from 10m to 0.9m. The typical operation configuration has the 6 IR 's tuned to β*=10m at injection. β* is squeezed on the to 1m (or 0.9m) in IR6 and IR8 (high-luminosity IRs) and to 3m in IR10 and IR2. The beams are horizontally separated after the interaction point (IP) by the DX and D0 magnets and focused by the triplets (Q1 Q2 and Q3). Every triplet has three 4-layers corrector packages, as schematically described in Figure 1.

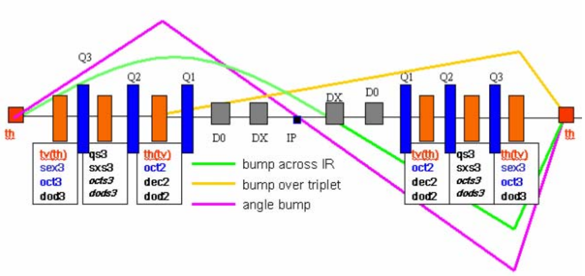

Figure 1. Schematic view of the IRs and bumps

The multi-layered packages include 2 octupole, 2 dodecapole and 1 sextupole per triplet. Correction packages are installed in every IR, presently only the correctors in IR6 and IR8 are powered by bipolar 50 A corrector supplies. The skew quadrupole correctors in package C2 are set to compensate the coupling effect of the IR triplets arising from residual roll errors. [2]

IR bump application

A control room application has been developed to set-up IR bumps and to analyse data for IR corrections.
For high-resolution and continuous tune measurement we use the phase lock look (PLL), a 245 MHz system, that outputs tune data at 100 Hz and with a resolution can be better than $5x10^{-5}$ [3].

The application provides communication to power supply and ramp managing software for IR bump set-up and activation, reads in and plots PLL and orbit data in real time and allows online data analysis to support operational corrections..

We can set-up several types of IR bumps for different purposes, as seen in Figure 1. Triplet bumps, 3-bump centred at one triplet, used for local correction at the individual triplet. The bumps across the IR span 2 triplets: in this case we measure the integrated effect of the IR quadrupoles. It is worth noticing that in order to correctly measure errors from the IR magnet only, the dipole correctors must not be interleaved with the IR magnets, so for the bump across the IR we still use only 3 dipole correctors, using the natural property of the IR optics to produce a bump anti-symmetric with respect to the IP.

The application allows to set up bumps at every IR, to define the maximum bump amplitude and the duration. Tune (100Hz) and RMS orbit data from the arcs (0.25-1 Hz) are collected and displayed as a function of time and as a function of bump amplitude. The application allows polynomial fitting of the tune shift and orbit data as a function of bump amplitude.

Experimental set-up for IR correction

Good operational preparation is needed for successful IR measurements and correction, in particular:

1. The 2 rings must be longitudinally separated, by at least 3 RF buckets to avoid beam-beam effects.
2. Measurements are performed at the beginning of a ramp, when the transverse emittance is smaller and with 6 bunches, to avoid risking magnet quenches in case of accidental beam loss.
3. The machine must be well decoupled, with coupling corrected to a minimum tune separation $\Delta Q_{min} < 0.002$.
The tunes are separated before the measurements by 0.01-0.012 to further minimize coupling effects.
4. Good overall orbit correction with horizontal and vertical orbit rms < 1mm, and good (<2mm) centring of the orbit in the IR triplets, to insure symmetry during the measurement.
5. The choice of bump amplitude is a critical one. Large bump amplitude is desirable to enhance the measured effect but this must be weighted with practical aperture considerations. Maximum bump amplitudes of 5mm proved enough to resolve sextupole effects, 10mm for octupole effects, and 15mm for higher orders.

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IR sextupole effects and correction

The sextupole correction proved to work for all machine optics we have run so far at RHIC [4]. Table 3 summarizes the IR sextupole correctors strengths necessary to operationally cancel the linear dependence of tune shift versus bump amplitude, that typically is in the range of 0.00003-0.0003/mm for different IRs and different machine configurations.

Table 2. Sextupole corrector strengths

IR sextupole corrector	Run-3 d-Au β* 2m 4500A	Run-3 p-p β* 1m 2000A	Run-4 Au β* 1m 4500A	Run-4 p-p β* 1m 2000A	Run-5 Cu β* 0.9m 4500A
Yo5-sx3	-0.0014	-0.003	-0.006	-0.001	-0.007
Yi6-sx3	+0.004	0.0	+0.003	+0.001	+0.0035
Yi7-sx3	+0.003	+0.007	+0.0005	+0.001	+0.003
Yo8-sx3	-0.01	-0.038	0.0	-0.0012	-0.003
Bi5-sx3	+0.0012	+0.001	+0.0011	-0.0022	+0.0025
Bo6-sx3	-0.004	-0.003	-0.001	+0.001	-0.005
Bo7-sx3	0.0	-0.003	0.0	-0.007	-0.005
Bi8-sx3	0.0	-0.0005	0.0	+0.002	+0.0025

Table 3 compares the strengths during for different runs and IR optics configurations. There are 2 sextupole correctors per IR (Figure 1), one at a location of higher β_x that is more effective in the horizontal plane, the other at a location of higher β_y, more effective in the vertical plane. The sextupole correction is done with the bump spanning the entire IR, and that includes effects from all IR triplets separator dipoles. Although a bump of 5mm is enough to measure the linear effect, we typically use 10mm bump amplitude to check the residual octupole and higher order effects.

The Au-Au d-Au data are taken at the collision energy of 100GeV/u – with the main quadrupole bus powered at ~4500 A – while the polarized proton (pp) data are at 100 GeV, a lower energy, with the main bus at ~2000 A. The magnetic errors in the IR dipole and quadrupoles depend on current, so the errors configurations are different. The quality of experimental set-up and correction improved at every run.

Figure 2 is an example of measurement and correction of an almost purely sextupole effect in both planes.

IR skew sextupole effects and correction

As discussed, a vertical bump provides all necessary information for the correction of skew sextupole effects. That was tested in IR8 in the yellow ring where we measured a substantial tune shift vs. vertical bump amplitude. Tune shifts before and after corrections are compared in Figure 3:

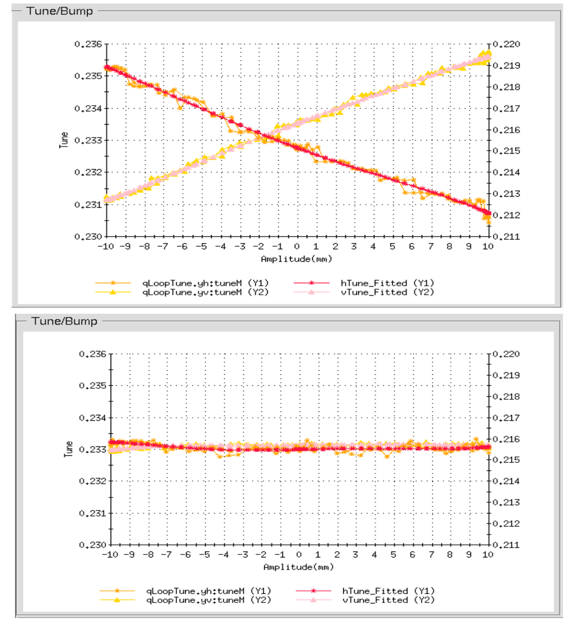

Figure 2. Tune shift versus bump amplitude before and after IR sextupole correction

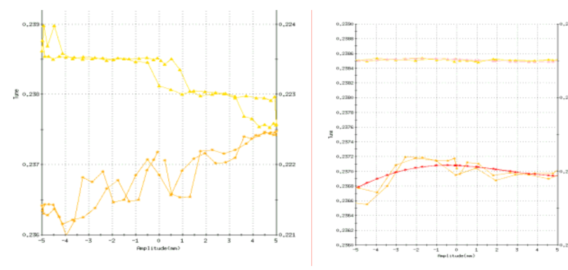

Figure 3. Skew sextupole correction

IR octupole effects and correction test

The application supports online fitting and plotting of the polynomial coefficients. We have 2 octupoles corrector layers per triplet, so it is possible to correct for the octupole effect – i.e. the quadratic term, at every individual triplet.

Figure 4 shows the result of correction of the quadratic dependence of the tune shift at one individual triplet with

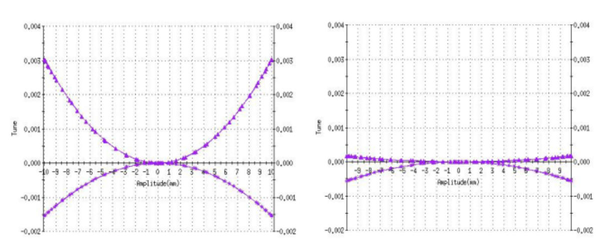

Figure 4. Octupole term before and after correction

the 2 local octupole correctors. The octupole correction at the individual triplets was not used so far in operations, as the relatively strong octupoles affected negatively beam lifetime and dynamic aperture. In operation we rather minimized in Yellow IR8 the octupole quadratic dependence across the entire IR. In this case the octupole strengths are small and we have measured no adverse effect on lifetime and dynamic aperture.

Higher order effects

On-line polynomial fitting allows us to evaluate the feasibility of measuring effects higher than octupole. For example, Figure 5 shows the polynomial fitting up to 4th order for the dependence of tune versus bump amplitude, for a 15mm diagonal bump centred in IR8, yellow ring, during Run-5 (Cu-Cu collisions).

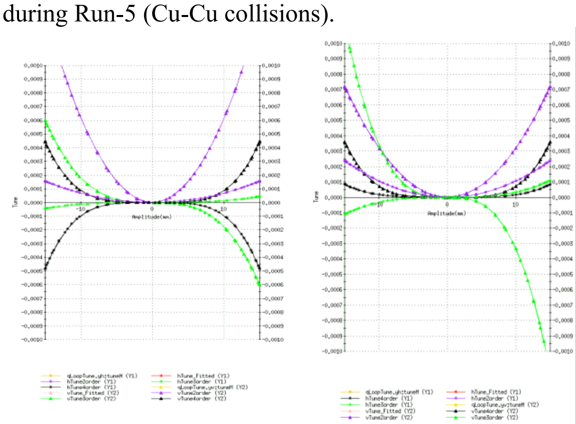

Figure 5. Measurement of higher orders

The left plot in Figure 8 shows the residual coefficients for the horizontal tune, (purple: octupole, green: decapole, black: dodecapole) after correction of the linear sextupole term. The plot on the right side of Figure 8 shows the change in the higher order harmonics after we powered up the yi7-dod3 dodecapole in IR8. A bump of 15mm was necessary to resolve the effect, and that requires careful preparation of the beam (6 bunches of low emittance, good initial orbit). This is a practical limit at top energy, to go beyond 15mm pencil beam could be used.

IR BUMP METHOD: MODEL

An effort is on going to compare the experimental non-linear effects measured in the IR with our present knowledge of the magnet data for the the IR triplets and separation dipole DX and D0.The off-line model has been used to analyse the dominant sextupole harmonics. The D0 separation dipoles have been identified as the major source of sextupole error in the IRs. The comparison between the operationally measured sextupole tune shift and that predicted by the magnet measurements is summarized in Table 4 for different IRs.

Table 3. Sextupole tune shift measured vs. predicted

IR section	Measurement by IR bump	Off-line Model
Blue IR6	$-1.1\ 10^{-3}$	$-4.2\ 10^{-3}$
Blue IR8	$-2.0\ 10^{-3}$	$-2.4\ 10^{-3}$
Yellow IR6	$-3.9\ 10^{-3}$	$-3.6\ 10^{-3}$
Yellow IR8	$-0.5\ 10^{-3}$	$-2.2\ 10^{-3}$

In 2 IRs the agreement is good. Analysis is in progress to understand the discrepancy in the two other IRs.

CONCLUSIONS

The IR bump method allows an order-by-order compensation of tune shifts. It is equivalent to the compensation of corresponding non-linear resonance strengths, and more operational than the action-kick minimization [5]. The sextupole, skew-sextupole and octupole correction have been experimentally demonstrated and are part of the RHIC machine operational set-up. Work on the measurement and correction of orders higher than octupole is in progress.

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