# EFFECTS OF FRINGE FIELDS AND INSERTION DEVICES REVEALED THROUGH EXPERIMENTAL FREQUENCY MAP ANALYSIS\*

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## Abstract

Following the pioneering work at the ALS [1] frequency map analysis (FMA) was performed at BESSY. With 7 families of sextupole magnets available in the storage ring, amplitude dependent tune shifts can be made rather small. Therefore, the impact of fringe fields of dipoles and quadrupoles as well as systematic octupole and decapole field components of these magnets are visible in the maps. At BESSY, the FMA is used in preparation of the storage ring for topping-up operation. Insertion devices (IDs) with their dynamic and static field integrals distort the frequency maps (FM) and reduce the horizontal dynamic aperture significantly. Shimming can recover the aperture which is important for efficient injection. The current status of the experiments as well as the results of the theoretical modeling will be presented.

#### INTRODUCTION

The frequency map analysis of Hamiltonian systems was introduced to accelerators as a theoretical tool in 1993 [2] and shortly later the first experimental FMA of a storage ring was performed at the ALS [1]. Since than, FMA has turned into a well accepted method to analyse the non-linear single particle dynamics [3].

In brief, a frequency map is a collection of dots in the tune space representing the fundamental frequencies of the beam motion for different horizontal and vertical oscillation amplitudes. For well behaving systems, one expects a smooth and regular relation between the amplitude- and tune-space. Therefore, FMA requires systematically chosen transverse initial conditions and an algorithm - like the Fourier transformation or the NAFF algorithm [3] - for extracting the fundamental frequencies from the recorded beam motion.

In the following two sections the setups for experimental and theoretical FMA will be described and their results are compared. Section 4 presents the impact of IDs on the FMA.

# **EXPERIMENTAL SETUP**

At BESSY, two fast half-wave kicker magnets are available to give the beam the desired horizontal and vertical initial conditions. Single kicks are synchronised to



Figure 1: Spectra of the vertical and horizontal beam motion (top and bottom) with the fundamental tunes:  $dQ_x$  (yellow),  $dQ_y$  (green) and combinations of them.

the 50 Hz of the mains. Each kicker can drive the beam to the aperture limits. Even though we want to study single particle dynamics, in reality only an ensemble of particles can create detectable signals. Therefore, a train of 50 bunches with initially  $5 \cdot 10^8$  electrons per bunch is used. This keeps collective effects at a tolerable level. The motion of the beam is detected and recorded with a bunch-by-bunch and turn-by-turn beam position monitor (BPM) as described in [4]. The key component of this BPM is a digital oscilloscope which samples the stripline signals of each bunch for up to 500 turns. Usually, the decay of the coherent motion is much faster due to collective effects. Averaged over the 50 bunches the resolution is 6 µm per turn and is only limited by the 8bit-ADC of the scope. For further analysis the pin-cushion distortion due to the stripline geometry is corrected and the horizontal and vertical beam motions are Fouriertransformed. The fundamental frequencies in the two planes are found with a refined peak finding algorithm.

With this setup, high quality beam spectra can be obtained. Fig. 1 shows results for an increasing vertical and a small horizontal kick. Each spectrum is shifted proportional to the square of the vertical kick, which should lead to linear shifts of the plotted spectral lines. Due to the chosen operating conditions, the tune shifts are quite large. The fundamental frequencies  $dQ_x$  and  $dQ_x$  and many combinations of them are clearly visible. These lines are created by higher order resonances and their appearance already at small kick amplitudes is an early indication of their impact in the "FM" at larger kicks, see Fig. 2. For example, the distortion of the regular map

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Figure 2: Tune shift for increasing vertical kick amplitudes. The initial tune is in the upper right corner. Dots in red represent individual measurements with more than 2% losses of particles.

close to the  $3Q_x-2Q_y$ -resonance is a consequence of the  $2dQ_x-2dQ_y$ -component in the spectrum [5].

The spectra presented in Fig. 1 were obtained by filtering the data with a Hanning-window. This improves the visibility of additional smaller spectral components. With experimental data, windowing does not necessarily improve the accuracy of the extracted fundamental frequencies. In Fig. 3 a complete FM is shown, which was taken under normal operating conditions with all 4 superconducting wavelength shifters (sc WLSs) and at positive chromaticities. Without Hanning-window the synchrotron sidebands are more clearly visible. This sideband structure has been observed for the first time and proves again the high quality of the experiments.

The collection as well as the data analysis are highly automated at BESSY. A complete experimental FMA requires 1 to 2 hours of beam time and less than 1 hour for the data analysis.



Figure 3: Appearance of synchrotron sidebands of the 4  $Q_y$ - and the  $3Q_x+2Q_y$ -resonance. The tune for small oscillation amplitudes is in upper right corner. With horizontal kicks the vertical tune shifts more strongly.

## THEORETICAL SETUP

BESSY is a 3<sup>rd</sup> generation light source with a doublebend achromat lattice with alternating high and low horizontal β-functions in the straight sections. There are 7 families of sextupole magnets and 4 of them, called harmonic sextupoles, are located in dispersion free regions and enable the control of the amplitude dependent tune shifts. In order to perform good non-linear calculations, a perfect knowledge of the coupled linear lattice, as obtained by the analysis of the measured orbit respone matrix [6], is required. The tracking code uses the small angle approximation, a 4<sup>th</sup> order symplectic integrator [7], and the known physical aperture limitations defined by the small vertical gaps of the ID vacuum



Figure 4: Comparison of experimental (top) and theoretical (bottom) FMA. The important resonance-structures are reproduced.



Figure 5: Comparison of experimental (left) and theoretical (middle: 2D- and right: 3D- calculations) FMA for normal operating conditions of the storage ring.

chambers and the septum magnet. As seen in Fig. 4, this approach leads to satisfactory agreement between the experimental and theoretical maps if the amplitude dependent tune shifts are large. Many resonances are visible and show up in the calculation due to the existence of linear focusing and coupling errors.

This coarse model disagrees with the observations as soon as the tune shifts are small. Improvements are possible if an octupole component of the quadrupole magnets [8] and fringe field effects of the dipole magnets - both based on the analysis of bench measurements - and fringe fields of the quadrupole magnets [9] are included. Sextupole magnets are modelled with 7 kicks of different strength in order to take the s-dependence of these short and large diameter magnets into account. The sextupoles are combined function magnets and also used as horizontal, vertical, and coupling correctors. This introduces normal and skew decapole and skew octupole components. Due to saturation, these correctors have an impact on the sextupole component, too [10]. All these effects were included in the calculation based on the actual settings of the correctors. Only with these refinements the agreement between experimental and theoretical FMA, as shown in Fig. 5, is very good.

3D-tracking was performed in order to model the synchrotron sideband structures visible in the experiment. If the chromaticity is non-zero, there is a second order path lengthening associated with the transverse oscillations of particles which leads to energy oscillations around a shifted mean energy [11]. This indeed produces synchrotron sidebands of the resonances covered by the map, however, due to the shifted mean energy the orientation of the map is slightly different (Fig. 4, right). Nevertheless, we have a good theoretical model for the 2D non-linear behaviour of the BESSY storage ring and

for speed reasons this will be used for the theoretical investigation of the impact of IDs.

# **IMPACT OF INSERTION DEVICES**

The impact of IDs on the single particle beam dynamics can be studied easily with the experimental FMA, because their influence can be turned on and off, opposite to the effect of the storage ring magnets and their intrinsic fringe fields. IDs can influence the dynamics in two ways. First, the wiggling motion leads to so called dynamic effects like the vertical focusing for an oscillation in the horizontal plane as described by L. Smith [12] or the impact of the finite pole width (field roll off) first seen and explained by the SPEAR group [13]. Second, the single particle dynamics can suffer from static field imperfections which may remain even after careful shimming of the ID-fields. Opposite to the first type, these field errors can be measured directly as straight line integrals through the ID. In the following subsections the



Figure 6: Impact of the linear dynamical focussing of 4 superconducting WLS on the experimental FMA.

impact of different IDs on the FMA is presented either dominated by the linear part of the dynamic effects, dominated by the static field imperfections, and influenced primarily by the non-linear dynamic effects. Finally, a FMA example is given for a successful cure.

## Linear Dynamic Focusing of IDs

Fig. 6 shows the impact of all 4 sc WLSs on the experimental FMA under the same conditions as in Fig.4. Without harmonic sextupoles, there is a strong impact of the WLSs especially from the 7T 17 pole Wiggler [14] which introduces very strong vertical focusing. As expected, the dynamic aperture is dramatically reduced, however the shape of the frequency map remains unchanged. Under normal operating conditions, i.e. with the harmonic sextupole magnets set to their nominal settings, the impact of the sc WLSs is much smaller - see Fig. 7.

# Static Field Errors of IDs

Static field errors as a result of field imperfections are often a concern for the operation of IDs at BESSY [15, 16]. For example, the quasi-periodic, planar undulator U125ID2R reduces the lifetime of the beam [17] and the injection efficiency by 10-20%, if the gap is closed.

In Fig. 7, results of the experimental FMA are presented, comparing the situation with open and closed gap. All 4 sc WLSs as well as the harmonic sextupole magnets were turned on. The FMA shows the strong impact of non-linear fields, especially in the horizontal dynamics. The horizontal aperture is reduced by nearly 30%. A closer look at the horizontal beam dynamics reveals, that the pole width of the U125-undulator of 60 mm is sufficiently large. For small horizontal excursions of the beam, the effective dynamic field integrals due to the associated field roll off are less important than the integrals from the static field errors. Both contributions were included in the tracking code and it turns out, that



Figure 7: Impact of the U125ID2R on the experimental FMA under normal operating conditions. Left: gap open, right: gap closed to 15.7 mm

indeed with the U125ID2R-gap closed, the static field errors dominate the horizontal dynamics.

# Non-Linear Dynamic Field Effects of IDs

As an example for these kind of effects the UE52ID5R APPLE II-type undulator is chosen. In operation, this ID reduces the injection efficiency by  $\approx 20\%$ , if the rows of magnets are shifted for the emission of vertically polarised light. No lifetime shortening is observed under any operating condition of this ID. The results of the experimental FMA are shown in Fig. 8. The gap was closed to its smallest value and the shift parameters of 0, +13 and +26mm correspond to the emission of horizontally, circularly and vertically polarised light. The experimental results are independent of the sign of the shift parameter. As a function of this shift, the ID introduces strong non-linear fields and the frequency map appears very distorted. The horizontal dynamic aperture is reduced dramatically and the injection efficiency is reduced. The increased losses are a concern for the planned topping-up operation of the facility.

For this reason the problem was investigated in more detail. Fig. 9 shows the perfect agreement between the beam-based measurement and the calculation of the effective dynamic field integrals. In the measurement the tune shift was recorded as a function of the horizontal closed orbit offsets inside the ID. Since this tune shift is proportional to the derivative of the effective field gradient, the field itself can be obtained by integration. With these fields the tracking code yields tune variations identical to the measurement and the horizontal dynamic aperture is reduced.

Once the field errors are known, shimming can help to combat the detrimental horizontal effects [13, 18]. Eight shims with a total weight of a few grams change the beam



Figure 8: Impact of the shift parameter (0, +13 mm, +26 mm from left to right). Note the strong non-linear impact of the ID (bottom) and the 30% reduction of the horizontal aperture in the amplitude space of these maps (top).

dynamics completely. Their impact on the experimental FMA is presented in Fig.10. The horizontal dynamic aperture for a shift of 26mm (emission of vertically polarised light) is recovered at the cost of larger non-linear effects in the vertical plane. This does not matter with the vertical aperture determined physically by the small gap vacuum vessels for the IDs.



Figure 9: Result of the beam-based measurement of the effective dynamic field integrals of the UE52ID5R in comparison with calculations by J. Bahrdt (BESSY).

# CONCLUSION

At BESSY, excellent experimental and good theoretical setups for FMA are available. They were used for studies of the non-linear dynamics with subtle influences of fringe fields and higher order field components of the lattice magnets as well as the investigation of the stronger impact of IDs. Based on the gained understanding improvements are possible by (re-) shimming the IDs in order to reduce their static and dynamic effects.

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Figure 10: Impact of shimming on the shift dependent experimental FMA. UE52ID5R-gap=17mm and shift=0, +13mm and +26mm (from left to right).

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