GEANT SIMULATION OF SIX-DIMENSIONAL COOLING OF MUON BEAMS IN RING COOLERS

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Abstract

The reduction of the phase-space volume of the beam (cooling) is essential for both muon colliders and neutrino factories. In a muon collider in particular, the six-dimensional (6D) emittance must be reduced by six orders of magnitude. Cooling the beam in all phase space dimensions can be done through emittance exchange, in which the beam loses energy by traversing wedge-shaped absorbers in a dispersive magnetic field, designed in a way such that more energetic muons go through more absorber material than less energetic ones, thus losing more energy. The longitudinal momentum is then regained using RF cavities. We simulate ring coolers, in which the beam undergoes 6-dimensional cooling through emittance exchange while rotating several times in the ring. The simulation software is a Geant3-based package, specially designed for this purpose, with changing electric fields in RF cavities treated correctly. Magnetic fields are read from external maps. Two ring cooler designs and cooling simulation results are presented here.

INTRODUCTION

The idea of ring coolers was first proposed by V. Balbekov [1] with the Tetra Ring, made of four sectors, each consisting of a long focusing solenoid with a hydrogen absorber and rf cavities for transverse cooling, and bending dipoles and short solenoids for the dispersive field, with LiH wedges for emittance exchange. This first attempt was successful in cooling simulations, but only with hard-edge magnetic fields. Other ideas that came later were the quadrupole-focusing rings [2], that did not perform as well, and the RFOFO Ring [3], in which the focusing, bending and dispersion are achieved using tilted solenoids, and a single wedge absorber, in addition to 6 rf cavities in each of the 12 cells which are responsible for cooling in all six phase-space dimensions. The RFOFO Ring was also the first cooling ring that has realistic magnetic fields. We present here a new Geant simulation of the RFOFO Ring and compare its results with those of a previous simulation [4], done with ICOOL [5].

A relatively new idea is to use pressurized hydrogen gas instead of wedge absorbers for emittance exchange. This idea is used in the Small Dipole Ring, where dipoles are used for bending and edge focusing. There is an intention to use such a small ring to experimentally demonstrate six-dimensional cooling of a muon beam. It is important to balance between cost and performance in this demonstration ring, hence relatively modest, but measurable, cooling is all that is needed here. The Small Dipole Ring for cooling demonstration is also simulated with the same Geant package.

THE SIMULATION

Simulation Software

The simulation software, MUC_GEANT [6], is an application of Geant-3.21 [7] specially designed for muon cooling simulation. The Runge-Kutta routine was changed to include electric fields, so that rf acceleration would be simulated properly. The software is also data-driven, i.e. it allows the user to change the cooling channel parameters without changing the code. The magnetic fields are not generated by the simulation software and are read from external field maps.

The input data ("beam") and output (used for emittance calculation) are both in the same format as in ICOOL, for an easy and reliable comparison of the two codes.

Simulation Procedure

Closed orbits are obtained by simulating single muons in the ring with only the magnetic field on. The "reference particle" is a muon that has a period of rotation corresponding to an integer harmonic of the rf frequency and with a momentum around which the lattice is designed to have good acceptance. The reference particle defines the beam center and the entry time for each of the rf cavities in the ring.

Other parameters such as rf phases and gradient for best acceptance can be defined by turning on the rf fields and the absorbers, leaving out random processes (scattering and straggling). The following step is a full simulation of the ring with all the processes active.

THE RFOFO RING

Geometry

Figure 1 shows a Geant drawing of the ring.

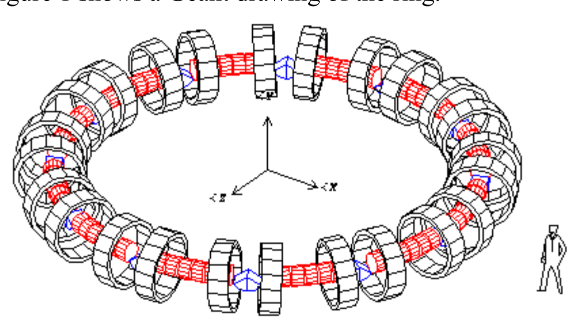

Figure 1: The RFOFO Ring. The large rings are the tilted solenoids, the blue wedges are the hydrogen absorbers, and the red cylinders represent the rf cavities (active volume only).

The ring is 33 m in circumference (ring central line) and consists of 12 identical cells. Each cell has two solenoids with axial fields opposite to each other, to focus the beam. Tilting the solenoids by 3º with respect to the vertical axis generates the vertical component for bending the beam. The solenoids are centered 10 cm away from the central line in a way that the beam follows the field lines. The magnetic fields are read from an external map and the solenoids shown in the drawing are for demonstration only.

Each cell has one liquid hydrogen wedge absorber that points up with an opening angle of 110º. The tip of the wedge is 9.5 cm above the ring central line. The energy lost in the absorber is restored by six 201.25 MHz rf cavities that are 28.5 cm long with iris radii of 25 cm. The simulated rf field is flat, axial and sinusoidal in time. The axis of each cavity is tangential to the ring central line.

The Reference Particle

The closed orbits for various muon momenta are shown in Figure 2. The reference particle is a 201 MeV/c muon with a period that is the 25th harmonic of the rf frequency.

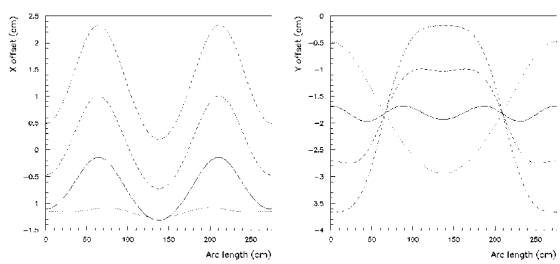

Figure 2: Closed orbits in a single RFOFO cell, deviation from the ring's central line, in radial direction (left) and vertical direction (right). The different line types represent different energies: solid – 227 MeV (reference particle with p = 201 MeV/c), dotted, dashed and dot-dashed show 200, 250 and 270 MeV muons, respectively.

Cooling a Muon Beam in the Ring

After defining the ring parameters, we turned on the cavities with a gradient of 12.3 MV/m (which gave the best acceptance) and the absorbers, including random processes, for the actual cooling simulation.

The beam that was used here is identical to the one used by ICOOL [4] and consists of 1,000 initial muons that are "injected" at the center of the absorber. Measuring the beam was done using virtual detector planes placed in the middle of each cell (between the 3rd and 4th rf cavities). Figure 3 shows the reduction of the beam volume in all dimensions as it rotates in the RFOFO Ring.

The merit factor, which measures the performance, is defined as

$$M = (\varepsilon_{6D}^{initial}/\varepsilon_{6D}^{final})\cdot T,$$

where $\varepsilon_{6D}^{\text{initial}}$ and $\varepsilon_{6D}^{\text{final}}$ are the initial and final 6-dimensional emittances, respectively , and T is the beam transmission, taking into account beam losses as well as muon decays.

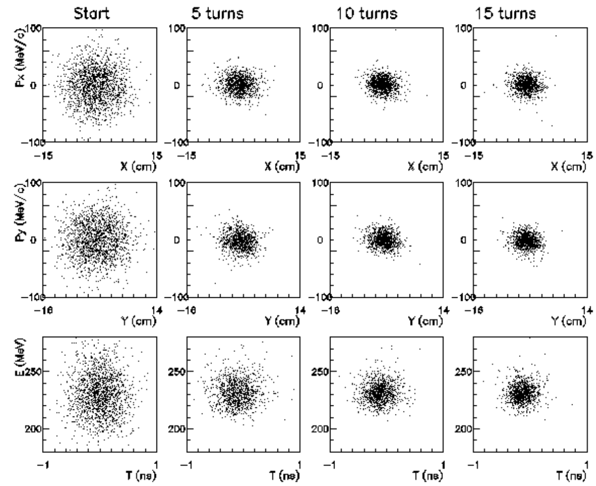

Figure 3: The distributions of p_x vs. x (upper row), p_y vs. y (middle row) and t vs. E (lower row) in four "stations" along the beam path: initial beam (left column), after 5 ring turns (second-to-the-left column), after 10 turns (second-right column) and 15 turns (right column).

Figure 4 shows a comparison of the Geant simulation and the ICOOL simulation of the RFOFO Ring. The merit factor in both simulations is around 100 after 15 turns, where the beam transmission is around 50%, and the overall agreement between the codes is very good.

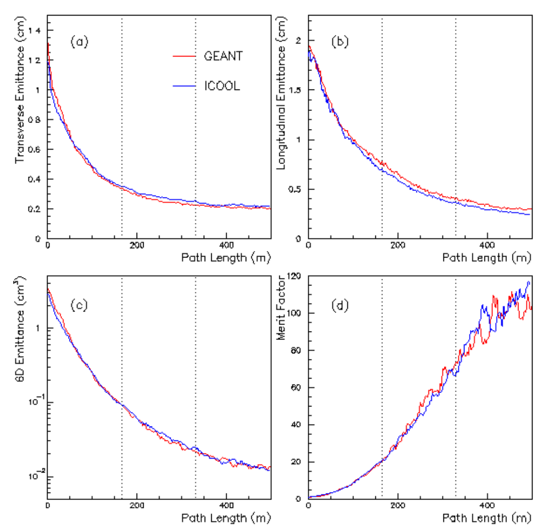

Figure 4: performance of the RFOFO Ring. The left plot shows beam transmission with and without decay (solid and dashed lines). The 4 plots on the right show a comparison between Geant (red) and ICOOL (blue) of (a) transverse, (b) longitudinal and (c) 6-dimensional emittance, and in (d) merit factor.

SMALL DIPOLE RING

This ring was proposed [8] as a way to demonstrate 6-D muon cooling experimentally, and was therefore designed to balance between cost and performance, but to produce measurable cooling.

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Geometry

Figure 5 shows a Geant drawing of the dipole magnets relative to human scale. The ring has 4 identical sectors, each containing a bending dipole with edge-focusing field. Four 201.25 MHz rf cavities are placed in the spaces between the magnets. The cavities were simulated as perfect pillboxes (Bessel function). The whole volume, including the rf cavities, is filled with hydrogen gas with a pressure of 40 atm at room temperature (10 atm at liquid nitrogen temperature).

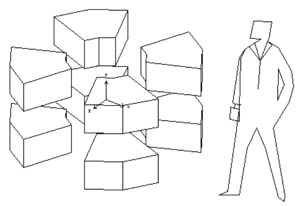

Figure 5: The Small Dipole Ring magnets relative to human scale.

Magnetic Field

The field map used for the simulation of the ring was produced with "shaped iron poles" to improve the horizontal acceptance of the ring. Figure 6 shows the vertical field at the center plane in one ring sector. The field itself is relatively weak (up to 1.8 T), allowing low-cost normal-conducting coils to be used.

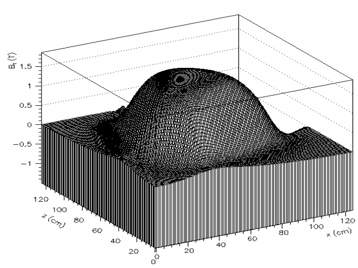

Figure 6: Vertical component of the magnetic field in a sector (1/4) of the Small Dipole Ring.

Reference Particle and Acceptance

Simulating only with the magnetic field on, the reference particle at the 3rd harmonic of the rf has a longitudinal (and total) momentum of 171 MeV/c and a distance of 56 cm at its nearest point to the center of the ring.

The transverse acceptance of the beam was defined by scanning the x-y and p_x-p_y planes. Figure 7 shows the acceptance of the ring in the transverse momentum plane.

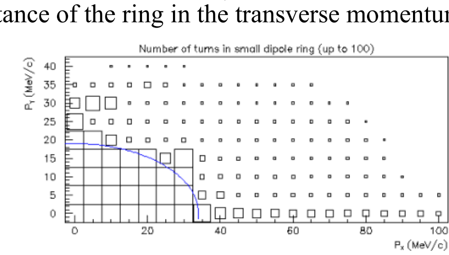

Figure 7: Transverse momentum acceptance of the Small Dipole Ring.

The longitudinal acceptance was measured for various rf gradients, with random processes in the absorber turned off. The best acceptance was achieved for an rf gradient of 16 MV/m.

Admittance and Equilibrium Emittance

A complete cooling simulation has not been performed because a well-matched beam has not yet been produced. However, the cooling potential can be estimated by comparing the admittance (emittance calculated from the ring acceptance) and the equilibrium emittance.

The transverse admittance was calculated to be about 8-10 π-mm, whereas the calculated longitudinal admittance is 15-20 π-mm. The equilibrium emittance is measured when a "point beam" is injected into the ring with all the random processes turned on in the hydrogen. The transverse equilibrium emittance is about 4 π-mm and the longitudinal is about 8 π-mm. This means that there is a potential of reducing the emittance by an order of 10, and measurement of 6-D cooling may be possible.

FUTURE PROSPECTS

More realistic features, such as cavity and absorber windows and dE/dx injection in both the RFOFO and the Small Dipole Ring can be studied relatively easily. In the case of the Small Dipole Ring, detectors should also be simulated, and their effect on the cooling channel studied.

Other cooling channels can also be simulated as the simulation software improves and becomes more flexible.

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