CALCULATING OF COUPLING FACTOR OF MICROWAVE ELECTRON GUN

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Abstract

To design the coupler of a designing microwave electron gun, we use "energy method" proposed by Derun Li, et al. The intrinsic Q of the electron gun cavity is very high: about 20000. The method is to calculate the intrinsic and external Q values of a cavity coupled to a waveguide, using MAFIA code in time domain. The comparisons between simulation and experimental results are given for a set of different coupling lengths. The result shows that "energy method" works efficiently for high Q cavities.

INTRODUCTION

Microwave electron gun has been widely used nowadays because it is a kind of good injector. Hot cathode microwave electronic gun has the advantages of simple structure, low cost, high luminosity and steady work state. Accelerator Lab of Tsinghua University is investigating a new type of on axis coupling hot cathode microwave electronic gun, which can reduce the backward bombing [1]. The gun is formed by 3 cavities, which work at standing-wave mode. The power is introduced by the waveguide connected with the third cavity, shown in Figure 1. Only when the frequency of the cavity chain is equal to that of the power, and the coupling between the cavity chain and the waveguide is proper, the power can be efficiently sent to the cavities. So the design of the coupling iris between the cavity and the waveguide is very crucial.

Figure 1: A 3D model of the third cavity coupled to a waveguide through an open iris.

Numerical simulation is mainly used to calculate the coupling factor, either in frequency domain or in time domain. In frequency domain, KROLL and YU have offered a method that can be used on computer according to Slater's theory [2]. The method is effective to low Q, high β cavity. In time domain, either steady state method or transient state method can be used. Steady state method is to record the system's whole attenuation course to get β. This method is suitable for travelling wave in which field building time is short or for standing wave structure in which β is very high. Transient state method is to record the field distribution of several times in the attenuation course to get β. This method can be used to calculate β under every condition.

According to transient state method, Derun Li et al. offered a method---Energy Method [3]. The method is to simulate the time domain field signal in the cavity and waveguide, to fully use the functions inside MAFIA code, so it can calculate coupling factor of high Q cavity. Since Q of the cavity used in our microwave gun is very high: about 2×10^4, and the Energy Method is simpler than KROLL-YU method, we use the former to design the coupling iris of our microwave electron gun.

THE METHOD

We assume that the following equations are true at any moment when the field is built up in the coupler cavity:

$$Q_0 = \omega_0 \frac{U(t)}{P_\omega(t)}, Q_{ext} = \omega_0 \frac{U(t)}{P_{out}(t)} \tag{1}$$

ω_0 is intrinsic angular frequency of the system, $U(t)$ and $P_\omega(t)$ are respectively the stored energy and power loss on the cavity averaged over one period at time t, $P_{out}(t)$ is the microwave power flowing out through the waveguide over one period at time t. It is also the power flowing out through the coupling iris. First we calculate the intrinsic frequency of the system in time domain, then we use the frequency to get β in the twice simulation in time domain.

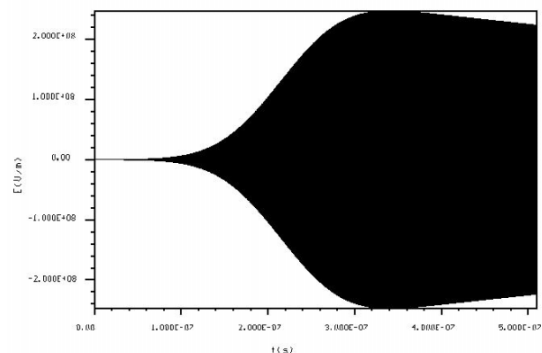*Calculating of the system frequency*

Figure 2: An electric field component is built up by the driving source in the cavity.

The resonant cavity to be calculated is a cell of the disk loaded waveguide cavity chain. It works at TM010 mode. So we can know the approximate frequency of the system. We set a pulse signal which has a proper

bandwidth in the T module of MAFIA. The central frequency of the signal is the approximate frequency. The polarization direction is along the axis of the cavity. The signal will build a electric field in the cavity which has many modes. TM010 mode will be left after other modes have attenuated. We use the monitor function in T module to record the electric field nearby the driving signal. The field recorded is shown in Figure 2.

In the simulation we assume that the system is loss free. The cavity and the waveguide both are set ideal conductor. In the cavity is vacuum. When the time is so long that the driving signal affects little, the field in the cavity will attenuate with the time constant $\tau = \frac{2Q_L}{\omega_0}$. Q_L is the loaded quality factor. And

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} \tag{2}$$

The system frequency is near 2856MHz and $Q_L > 5000$, so the time constant $\tau > 0.25 \mu s$. It is so long that the CPU time will be very long. It is unpractical to find the time constant by the steady state method. So the transient state method is chosen.

We can use the FFT in the P module to get the system frequency from the signal recorded. But in our practice, to get the average frequency from the last 20 periods in time domain signal is more accurate.

Calculating of Coupling Factor β

We set f_0 for the frequency of driving signal, which is got from the former calculation. So we can get TM010 mode in the cavity. In this condition, the power flows to the waveguide through the coupling iris.

To calculate the average stored energy and the loss at time t_0, we set

$$\Delta t = \frac{1}{4 f_{drv}} \tag{3}$$

$f_{drv} = \frac{\omega_0}{2\pi}$, it is the driving frequency, ω_0 is the angular frequency. The monitor function in T module can record the field at any time in any space. The field distribution at t_0 and $t_0 \pm \Delta t$ should also be record besides the field nearby the signal. t_0 is the time when the driving signal's energy contributed can be omitted compare to the whole stored energy.

The electromagnetic field is built up by the driving signal. The following equation is true:

$$\Phi(t) = \Phi_\infty (1 - e^{-\frac{t}{\tau}}) \sin(\omega_0 t + \theta_0) \tag{4}$$

$\Phi(t)$ denotes $\vec{E}(\vec{r}, t)$ or $\vec{B}(\vec{r}, t)$, Φ_∞ is the field amplitude at steady state. θ_0 is the initial phase. For a high Q_L system, it is impractical to calculate β until the system reaches steady state. Derun Li found that we can calculate Q_0 and Q_{ext} easily by the field recorded at time t_0 and $t_0 \pm \Delta t$.

When $\frac{t}{\tau} << 1$, from equation (4) we can get:

$\Phi(t) \approx \frac{t}{\tau} \Phi_\infty \sin(\omega_0 t + \theta_0)$. From this equation we can know that the field is linearly built up by the time. The electrical field signal is shown in Figure 3.
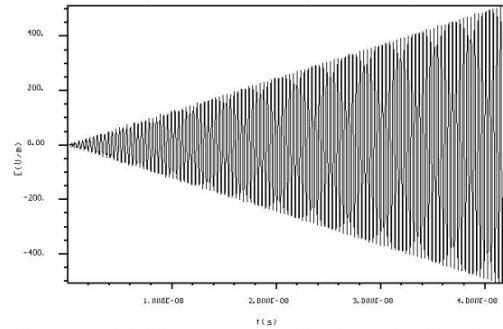

Figure 3: An electric field component is built up linearly as a function of time.

As pointed out before, t_0 must be the time when the driving source could be omitted. Generally speaking, t_0 should be chosen after 30 periods. To calculate $U(t)$, $P_\omega(t)$ and $P_{out}(t)$, P module in MAFIA must be used. The energy calculating in P requires the phase information of the field, but this will disappear after we set $\Delta t = \frac{1}{4 f_{drv}}$. That's because:

$$\Phi^2(t_0) + \Phi^2(t_0 \pm \Delta t)$$

$$= (\frac{t_0}{\tau})^2 \Phi_\infty^2 \sin^2(\omega_0 t_0 + \theta_0) + (\frac{t_0 \pm \Delta t}{\tau})^2 \Phi_\infty^2 \sin^2[\omega_0(t_0 \pm \Delta t) + \theta_0)]$$

$$= (\frac{t_0}{\tau})^2 \Phi_\infty^2 \sin^2(\omega_0 t_0 + \theta_0) + (\frac{t_0 \pm \Delta t}{\tau})^2 \Phi_\infty^2 \sin^2(\omega_0 t_0 \pm \frac{\pi}{2} + \theta_0)$$

$$= (\frac{t_0}{\tau})^2 \Phi_\infty^2 \sin^2(\omega_0 t_0 + \theta_0) + (\frac{t_0 \pm \Delta t}{\tau})^2 \Phi_\infty^2 \cos^2(\omega_0 t_0 + \theta_0)$$

$$\approx (\frac{t_0}{\tau})^2 \Phi_\infty^2$$

$$= |\Phi(t_0)|^2$$

So, the stored energy at t_0 is:

$$U_\Phi(t_0) = \frac{\alpha}{2} \oint_V \{|\Phi(t_0)|^2\} dv$$

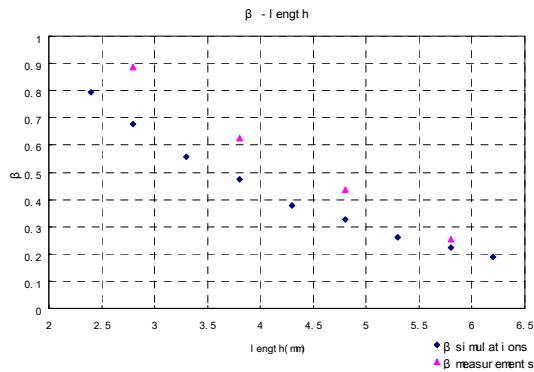$$\approx \frac{\alpha}{2} \oint_V \{\Phi^2(t_0) + \Phi^2(t_0 \pm \Delta t)\} dv \tag{5}$$

Figure 4: Coupling factor β is plotted as a function of iris length.

In this equation:
$$\alpha = \begin{cases} \varepsilon_0, when \quad \Phi = \vec{E} \\ \dfrac{1}{\mu_0}, when \quad \Phi = \vec{B} \end{cases}$$

Similarly the power loss is calculated by, (the material should be set as pure copper)

$$P_\omega(t_0) \approx \frac{R_s}{2} \oint_S \{H^2(t_0) + H^2(t_0 \pm \Delta t)\} ds \tag{6}$$

P module can calculate the electric energy $U_E(t_0)$ and the magnetic energy $U_B(t_0)$ in appointed volume at time t_0 and $t_0 \pm \Delta t$.

So we get,

$$Q_0 \approx \omega_0 \frac{U_E(t_0) + U_B(t_0)}{P_\omega(t_0)} \tag{7}$$

The group velocity $v_g = c\sqrt{1 - \left(\dfrac{f_c}{f_{drv}}\right)^2}$; f_c is the cut-off frequency determined by the geometry of the waveguide port; We set the waveguide length d in energy calculating. The electromagnetic field in d is recorded in T module and the stored energy is calculated in P module, then we get the energy density,

$$W_{wg}(t_0) = \frac{1}{d}[U_E(t_0) + U_B(t_0)]_d \tag{8}$$

And $$P_{out}(t_0) = v_g W_{wg}(t_0) \tag{9}$$

So we get β by equation (1):

$$\beta = \frac{Q_0}{Q_{ext}} = \frac{P_{out}(t_0)}{P_\omega(t_0)}$$

NUMERICAL RESULTS AND EXPERIMENT RESULTS

We short the two sides of the cavity in experiment. And we measure the $\rho - f$ curve from the waveguide port using network analyzer, then we get the coupling factor of the coupler.

Numerical simulations of the system have been performed for a set of different iris length, and compared with experimental measurements. The results are shown in Figure 4.

From Figure 4, we can know that β is exponential attenuated by the length. That means β is very sensitive to the length of the iris. The warp between simulation and measurement results is about 10%-30%. This is produced by both the error of the simulation and the measurement. The error of the simulation could be monished in the farther simulation by reducing the length of the mesh grid.

CONCLUSION

When the length of the iris is 2.1mm, the cross section is 9mm×20.1mm, the simulation result of β=1.4. The corresponding measurement result β=1.9, reaching the requirement of the microwave electronic gun. The warp between the simulation and the measurement is 26%. The results of measurement and simulation show that "Energy Method" is an effective method for the calculating of coupling factor β between cavity line and waveguide. The result is fairly accurate. The method can reduce process period and improve efficiency of the designing of the iris. It can also been applied for coupling calculation between other kinds of cavity lines and external circuits.

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[2] N. Kroll, D. Yu, Particle Accelerators, 1990, 34: 231-250
[3] Derun Li et al. Calculations of External Coupling to a Single Cell Rf Cavity. Proceeding of LINAC 1998