

## SPACE-CHARGE EFFECTS NEAR A CATHODE\*

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### Abstract

RF photocathode guns are excellent sources of high brightness electron bunches. To mitigate space charge effects downstream of the gun it is often desirable to produce electron bunches with a uniform density distribution. Our goal is to understand how the longitudinal bunch profile is determined by the emission dictated by the laser pulse intensity and the effects of the self-field dynamics near the cathode. The particle-in-cell simulation XOOPIC is used to simulate beam dynamics near the cathode and the results are compared to those obtained from linear theory. Studies are focused on flat-top and Gaussian bunch profiles.

### INTRODUCTION

Electron bunch dynamics in electron guns has been studied extensively (see [1]- [3] and references therein). It is well-known that, for high currents, space-charge effects dominate and prevent further emission by forming a virtual cathode. At lower currents, space-charge modifies the beam distribution function.

We consider the temporal evolution of a bunch under influence of both external and self fields. The emitting cathode is assumed to be flat and the geometry of the system is cylindrical (see Fig. 1). The bunch evolution is analyzed in the approximation of low-density bunches, i.e. with current density lower than the critical value at which a virtual cathode can form. These densities are typical of those in applications [4], which avoid the virtual cathode regime. Near the cathode, where, to first approximation, the motion is non-relativistic, there is a large change in density as the bunch accelerates. Space-charge effects are treated analytically as perturbations to the motion in the hypothetical situation where noninteracting particles are accelerated by external fields. Another important approximation is that the electrons are emitted along (homogeneous) magnetic field lines. This "large B" approximation allows (by an average over gyro-motion around magnetic field lines) us to restrict to the case where electrons move only in the axial direction. The electric and magnetic fields are allowed to vary radially, axially and in time. Emission is assumed to be azimuthally symmetric, as is the RF cavity. The subsequent evolution is assumed, as well, to be cylindrically symmetric.

With the above approximations, the continuity equation

can be written in terms of current density as

$$\frac{\partial j}{\partial t} + v \frac{\partial j}{\partial z} = \frac{j}{v} \frac{\partial v}{\partial t} + A\delta(z), \quad (1)$$

and the momentum equation becomes

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial z} = -\frac{e}{m} [E_{\text{ext}}(z, r) + E_{\text{self}}(z, r, t)] + B\delta(z), \quad (2)$$

where  $j(z, r, t) = -en(z, r, t)v(z, r, t)$  is current density,  $A\delta(z)$  represents the source of electrons at the surface of the flat cathode, and  $B\delta(z)$  is responsible for an "instant" kick that each electron gets when it leaves the cathode.

The self-field of the electrons in the non-relativistic case can be written in terms of a Green's function in cylindrical coordinates, defined in the  $z > 0$  half-space. We assume that cathode emission depends only on external parameters (such as the laser pulse), and not directly on the local self-fields. This implies that the electron energy and current density on the cathode are known. A further simplification, which is valid close to the cathode, is that the external field RF is time-independent. This approximation is valid when the bunch length is much smaller than the RF period.

### FLAT-TOP PROFILE EVOLUTION

The external field in the cathode region is assumed to be homogeneous with magnitude  $E_0$ . In the following we use the dimensionless variables:  $\chi = z/r_b$ ,  $\rho = r/r_b$ , where  $r_b$  is beam radius; we also use  $\tau = t/\sqrt{r_b/a}$ , where  $a = -eE_0/m$  is the acceleration; and current density  $\tilde{j} = j/j_0$ , where  $j_0$  is the magnitude of emitted current density. It is convenient to change variables from  $z$  to  $\xi = \sqrt{a/r_b} \int_0^z \frac{dz'}{v^{(0)}(z')}$  and set the normalized velocity  $w = v/\sqrt{ar_b}$ , where  $v$  is the velocity. For parameters of practical interest, the initial energy  $mv_0^2/2 \sim 1$  eV with

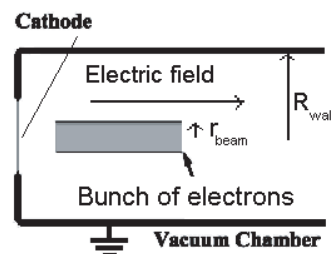


Figure 1: A Schematic view of the cylindrical cathode used in the simulation and theory.

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$w_0 \approx 0.006$ . The normalized system of equations is then

$$\frac{\partial w}{\partial \tau} + w \frac{\partial w}{\partial \chi} = 1 + \alpha \int_0^{2\pi} d\phi' \int_0^1 d\rho' \rho' \int_0^\infty d\chi' \frac{\bar{j}}{w} \frac{\partial \bar{G}}{\partial \chi} \quad (3)$$

$$\frac{\partial \bar{j}}{\partial \tau} + w \frac{\partial \bar{j}}{\partial \chi} = \frac{\bar{j}}{w} \frac{\partial w}{\partial \tau}, \quad (4)$$

where

$$\alpha = \frac{-j_0 r_b}{4\pi\epsilon_0 E_0 \sqrt{\frac{e}{m} E_0 r_b}} = \frac{|j_0| r_b^2}{I_A} \left( \frac{mc^2}{eE_0 r_b} \right)^{3/2} > 0. \quad (5)$$

The self-field is assumed to be small, so that we can lin-

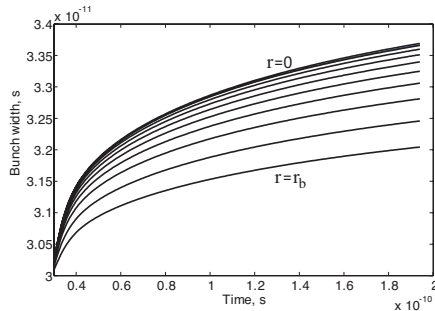


Figure 2: Bunch lengthening at various radial positions as a function of time. The initial current is  $I_0 = 32$  A. The lines correspond to radial positions starting at  $r = 0$  with increments of  $0.1r_b$

earize the velocity about its value due to the external field:

$$w(\chi, \rho, \tau) = w^{(0)}(\chi) + w^{(1)}(\chi, \rho, \tau). \quad (6)$$

The linearization allows us to reduce Eqs. (3)-(4) to the generic form

$$\left( \frac{\partial}{\partial y_1} + \frac{\partial}{\partial y_2} \right) f(y_1, y_2) = F(y_1, y_2), \quad (7)$$

$$f(y_1, y_2 = 0) = 0. \quad (8)$$

This form of equation has a solution, found by using a Laplace transformation, that is given by

$$f(y_1, y_2) = \int_0^{y_2} ds F(y_1 - s, y_2 - s). \quad (9)$$

Assume that emission starts at  $\tau = 0$ , and a constant current density is emitted for  $0 < \tau < \Lambda_0$ . We write the current density as

$$\bar{j} = \left[ 1 + \bar{j}^{(1)}(\xi, \rho, \tau) \right] \times [\Theta(\tau + g_1(\tau, \rho) - \xi) - \Theta(\tau + g_2(\tau, \rho) - \xi - \Lambda_0)], \quad (10)$$

where  $\Theta(x)$  is the step-function. The boundaries of the bunch are modified by the functions  $g_1(\tau, \rho)$  and  $g_2(\tau, \rho)$ ,

whose physical meaning is to modify the arrival time of the head and rear of the bunch to the point  $\xi$ .

These functions reflect bunch expansion. We substitute the ansatz (10) into Eqs.(3)-(4), separate terms with  $\delta$ -functions (arising from derivatives of step-functions) and then linearize and solve using (9) to obtain Fredholm integral equations of the second kind, which are solved by iteration. An analytical solution may be derived if we neglect the radial dependence of the current density, as in Ref. [6]. In this paper, however, we take explicit account of this dependence.

## COMPARISON OF THEORY AND PIC SIMULATIONS

### Flat-top profile

The analytical solutions were tested by comparing with results obtained using the XOOPIC [7] particle-in-cell code. The system is cylindrical with a flat cathode at 1 V potential and grounded walls.

Simulations were performed using  $E_0 = -60$  MV/m for  $r_b = 1$  mm beam radius (which are close to those in RF-guns [4]). Self-field effects appear at distances of  $z \sim r_b$ . Even though, for  $E_0 = 60$  MV/m, electron energy gain at  $z \sim r_b$  is not very high ( $\Delta E \sim (0.06 - 0.12)mc^2$ ), the relativistic corrections for the drift time to reach  $z \sim r_b$  is 4%. This is seen in Figs. 3-4.

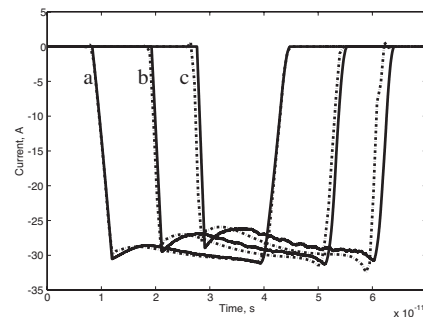


Figure 3: The current profile through fixed surfaces located at  $z/r_b = 0.2, 0.6, 1$  (a,b,c). Here  $E_0 = 60$  MV/m,  $I_0 = 32$  A. The current pulse at the cathode was assumed to be a 30ps flat-top. Analytical results (dash-dotted lines) and simulation results (solid lines) are plotted. The self-field interactions lead to the perturbations of initially flat profile and the bunch lengthening.

Our model (effectively) uses interacting cylindrical layers of electrons—electrons don't change their radial position. Thus, every layer (of width  $dr$ ) can have a different length. Of course, in systems that are not strongly magnetized, there will be radial motion that will smear this effect after one betatron period. The effect of bunch lengthening for a flat-top profile is calculated by using  $g_1(\tau, \rho)$  and

$g_2(\tau, \rho)$  from Eq. (10)

$$\Lambda(\tau, \rho) = \Lambda_0 + g_1(\tau, \rho) - g_2(\tau, \rho) \quad (11)$$

Electrons on axis "feel" the strongest field and, thus, lengthening is larger on axis. Fig. 2 shows the change of the bunch length for different radial positions. The well-known theory of bunch lengthening effects due to space-charge, developed in Ref. [6], is applicable only far away from the cathode when bunch density is constant along the bunch and its length is much bigger than its radius. Neither of these assumptions are valid in the case of short bunches near the cathode, considered here.

In Fig. 3 we plot the the temporal current profile at various spacial locations. Note that non-relativistic theory agrees with simulations near the cathode, but starts to disagree as soon as relativistic corrections are significant (these corrections are obvious from the plots by checking the arrival time of the beam head). The high-frequency structure forms near the cathode where density changes drastically. Upon reaching relativistic velocities, the profile shape changes due to bunch lengthening (and, thus, lowering of the current). No new structures are observed away from the cathode (our model does not include coherent synchrotron radiation or wakefields, which of course do add structure to the current. Here we note that flat top bunches can generate current profile perturbations, through self-field interactions, as they are emitted from the cathode. This is not the case with Gaussian bunches, which do not have the perturbations seen in Fig. 3

### Gaussian profile

We next consider an emitted current which is constant out to radius  $r_b$  and Gaussian in time. The zero'th order approximation corresponds to propagating a Gaussian pulse

$$j^{(0)}(\xi, \tau) = j_0 \exp \left[ -(\tau - \xi - \tau_{\text{center}})^2 / \sigma^2 \right] \quad (12)$$

where  $\tau_{\text{center}}$  is position of the maximum and  $\sigma$  is the width. The solution of Eqs.(3)-(4) has a Gaussian shape with a lower magnitude and a larger  $\sigma$ . This result has been tested using OOPIC. In Fig. 4 currents through fixed surfaces are plotted as a function of time as obtained from simulations and from numerical solution of our linearized problem. The shapes remain Gaussian and do not have the bumps seen in Fig. 3).

### CONCLUSIONS

An approximate theory for the temporal evolution of the current profile near the cathode has been developed and compared to PIC simulations. The key assumptions that were made to simplify the theoretical investigations are non-relativistic dynamics, a time-independent accelerating field and strong magnetization (or no radial motion). We find that, for flat-top longitudinal distributions, the initial dynamics near the cathode can impart structure on the

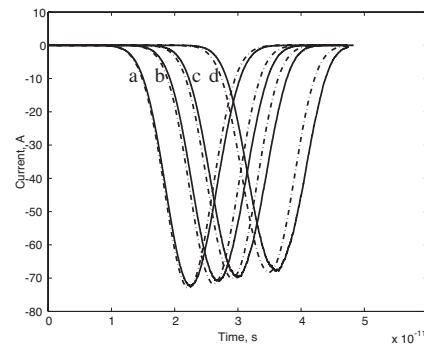


Figure 4: Current temporal profiles through fixed surfaces located at  $z/r_b = 0.2, 0.6, 1, 2$  (a,b,c,d). Here  $E_0 = 60$  MV/m,  $I_0 = 80$  A. Analytical results (dash-dotted lines) and simulation results (solid lines) are plotted. The self-field interactions lead to the current magnitude decrease and lengthening.

bunch through the self-fields. For flat-top distributions, the structure induced near the cathode due to self-field may later act as a seed various other, undesirable, instabilities. Simulations are in good agreement with the theory. Though this theory has been developed to analyze perturbations for flat-top and Gaussian longitudinal profiles, it is readily extendable to allow for investigation of arbitrary current profiles.

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