THE EFFECT OF MAGNETIC FIELD ERRORS ON DYNAMICAL FRICTION IN ELECTRON COOLERS*

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Abstract

A proposed luminosity upgrade to the Relativistic Heavy Ion Collider (RHIC) includes a novel electron cooling section [1], which would use ~55 MeV electrons to cool fully-ionized 100 GeV/nucleon gold ions. A strong (1-5 T) solenoidal field will be used to magnetize the electrons and thus enhance the dynamical friction force on the ions. The physics of magnetized friction is being simulated for RHIC parameters [2,3,4], using the VORPAL code [5]. Most theoretical treatments for magnetized dynamical friction do not consider the effect of magnetic field errors, except in a parametric fashion. However, field errors can in some cases dramatically reduce the velocity drag and corresponding cooling rate. We present two simple analytical models for the magnetic field errors. The simulated dynamical friction for the case of a perfect solenoidal field is compared with results from these new models. We simulate parameters for the existing cooler of the CELSIUS ring, because recent experiments [6] provide data that will later be used for code validation.

MOTIVATION

The semi-analytic simulation codes SIMCOOL [7] and BETACOOL [8] are being used [3,9] to design the proposed RHIC cooler. These codes rely on analytical expressions for the friction force on a single ion moving through a magnetized electron distribution. In particular, the formulas from Meshkov (DSM) [10] and Parkhomchuk (VP) [11] are generally used. The Meshkov result is a modification of previous work by Derbenev and Skrinsky (DS) [12]. The VP formula is an empirical generalization of the theoretical friction force for unmagnetized electrons. The semi-numerical results of Toepffer [13] are also interesting.

These calculations and parameterizations assume a constant electron density, a uniform neutralizing background, the absence of any perturbation from other ion trajectories, and a constant longitudinal magnetic field. The VP formula includes an effective velocity which may help to remove some of these assumptions in a parametric fashion. In this paper, we present work in progress, with the goal of understanding the effects of magnetic field errors. The DS and VP equations are reproduced elsewhere in this proceedings [4].

MAGNETIC FIELD ERRORS

We consider two simple models for errors in the field of a solenoid magnet, neglecting fringe field regions. Both models are current-free and divergence-free, for the most part. One is a sum of terms that vary sinusoidally along the longitudinal axis and exponentially in the transverse plane. The other is a piece-wise constant series of small tilts in the magnetic field direction. We assume for both models that, to lowest order, we have a constant magnetic field in the **z** direction.

The first model uses separation of variables to obtain:

$$B_x = \sum_i b_i \frac{k_{x,i}}{k_{z,i}} \exp\left(k_{x,i}x\right)\exp\left(k_{y,i}y\right)\sin\left(k_{z,i}z + \varphi_{z,i}\right) \quad (1a)$$

$$B_y = \sum_i b_i \frac{k_{y,i}}{k_{z,i}} \exp\left(k_{x,i}x\right)\exp\left(k_{y,i}y\right)\sin\left(k_{z,i}z + \varphi_{z,i}\right) \quad (1b)$$

$$B_z = B_0 + \sum_i b_i \exp\left(k_{x,i}x\right)\exp\left(k_{y,i}y\right)\cos\left(k_{z,i}z + \varphi_{z,i}\right) \quad (1c)$$

$$k_{z,i}^2 = k_{x,i}^2 + k_{y,i}^2 \qquad \lambda_i = 2\pi/k_{z,i} \quad (1d)$$

where B_0 is the idealized solenoid field, and we assume $b_i << B_0$ for all i. In general, the terms like $exp(k_{x,i}\,x)$ can be replaced by a sum of two exponentials, one with postive and one with negative argument, or equivalently by a sum of *sinh* and *cosh* terms. It's also possible to replace one (but not both) of the exponential terms (for every i) as a sum of sinusoidal terms. However, in this latter case, one must modify Eq. (1d) with an appropriate sign change.

The second model is motivated by the characteristics of the solenoid field in the electron cooler of the CELSIUS ring in Sweden. Figure 3 of Ref. [14] shows that, ignoring fringe fields, deviations of the magnetic field from that of an ideal solenoid can be approximated by a sum of piece-wise constant tilts, of the general form:

$$B_x = \sum_i b_{x,i} H\left(z - z_{x,i}\right) H\left(z_{x,i+1} - z\right) \quad (2a)$$

$$B_y = \sum_i b_{y,i} H\left(z - z_{y,i}\right) H\left(z_{y,i+1} - z\right) \quad (2b)$$

$$B_z = B_0 \quad (2c)$$

where $H(x)$ is the unit Heaviside function, and we assume $b_{x,i}$, $b_{y,i} << B_0$ for all i. This model has the disadvantage of (relatively small) violations of Maxwell's equations wherever $z=z_{x,i}$ or $z_{y,i}$. However, it would require many terms for Eq.'s (1) to reasonably model the characteristics of the magnetic field errors seen in [14], while Eq's (2) can do so simply and efficiently.

In order to apply these field errors in our VORPAL simulations, we must Lorentz transform them to the beam frame, which we take to be moving along the z-axis with

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velocity βc and relativistic factor γ. Using the standard Lorentz transforms of fields and coordinates [15] for the case where $\boldsymbol{E}=0$ in the lab frame, we obtain the following:

$$B_z' = B_z\left(x', y', \gamma\beta ct'\right) \tag{3a}$$

$$B_x' = \gamma B_x\left(x', y', \gamma\beta ct'\right) \qquad B_y' = \gamma B_y\left(x', y', \gamma\beta ct'\right) \tag{3b}$$

$$E_x' = -\beta c B_y' \qquad E_y' = \beta c B_x' \qquad E_z' = 0 \tag{3c}$$

where the primes indicate beam frame quantities. Strictly speaking, Eq.'s (3) should also include a *z'* dependence, but we can ignore this, because the simulations consider only a small domain with periodic boundary conditions.

SIMULATION DETAILS

We have developed, within the VORPAL code, a 3D algorithm capable of directly simulating from first principles the magnetized (or unmagnetized) friction force and diffusion coefficients for ions in an electron (or positron) cooler. This algorithm uses molecular dynamics techniques (i.e. simulating every particle in the problem) to explicitly resolve close binary collisions and thus capture the friction force and the diffusion tensor with a bare minimum of physical assumptions [2,16]. This required generalization of the 4th-order algorithm of Makino & Aarseth [17] to accommodate charged particles in a magnetic field.

Parameters Used for the CELSIUS Cooler

Relevant physical and numerical parameters are shown in Table 1. Most physical quantities are not known precisely. The longitudinal rms velocity of the electrons in the cooler is $\Delta_{e,\|}$~3000 m/s. In the absence of any model for magnetic field errors, we attempt to capture the associated effects in a parametric fashion by increasing this rms velocity to an effective value of $\Delta_{eff,\|}$=9000 m/s.

These periodic electrostatic simulations are conducted in the beam frame, where all particle motion is non-relativistic, within a domain (typically a cube) just large enough to capture the largest relevant impact parameters. The electrons interact only with the ions (typically eight of them), and not with each other, which reduces the run-time by factors of order 10^5, at the expense of losing correct e- shielding dynamics. For RHIC parameters, the interaction time in the cooler is shorter than the plasma period, so this approximation is justified. For CELSIUS parameters, this approximation should be validated.

Cluster Computing with VORPAL

The ion dynamics during a single pass through the cooler is strongly dominated by diffusion [2], making it very difficult to extract the average velocity drag due to dynamical friction with any precision. Diffusion cannot be suppressed through the numerical trick of using correlated e-/e+ pairs [2], because for CELSIUS parameters the friction due to positrons is very different. Hence, the only recourse is to repeat a given simulation many times, each time with different seeds for generating the initial electron positions and velocities. This allows one to quote the average friction force (or velocity drag) with meaningful statistical error bars.

Managing 100's of VORPAL simulations for each parameter set is made tractable through use of the TaskDLiSH software package, which provides task farm functionality for executing shell scripts on a cluster. TaskDLiSH, a simplified implementation of TaskDL [18], works by first creating a "tuple space," in which tasks are placed. The tasks here are simply shell scripts, which launch VORPAL with slightly modified input parameters.

Table 1: Simulation Parameters for CELSIUS Cooler

Symbol	Meaning	Value	Units
B_0	solenoid field	0.1	T
L_{sol}	solenoid length	2.5	m
β	proton bunch velocity / c	0.308	
τ_{lab}	interaction time (lab frame)	2.7×10^{-8}	s
τ_{beam}	interact. time (beam frame)	2.6×10^{-8}	s
Δt	largest time step	2.6×10^{-12}	s
dt_{min}	smallest time step	8.0×10^{-14}	s
ω_{pe}	e- plasma frequency	4.1×10^{8}	rad/s
Ω_L	e- Larmor frequency	1.8×10^{10}	rad/s
r_L	e- Larmor (gyro-) radius	7.9×10^{-6}	m
$L_{x,y,z}$	sim. domain dimensions	6.0×10^{-4}	m
n_e	e- number density	5.4×10^{13}	m^{-3}
N_e	# of simulated e-'s	1.2×10^{3}	
$\Delta_{e,\perp}$	transverse rms e- velocity	1.4×10^{5}	m/s
$\Delta_{e,\|}$	long. rms e- velocity	3.0×10^{3}	m/s
$\Delta_{eff,\|}$	effective long. rms e- vel.	9.0×10^{3}	m/s

Once the tuple space is created, distributed "workers" launch the tasks on remote. Workers then reserve and process tasks independently. New tasks may be added to the tuple space at any time, based on the results of previously executed tasks, and workers may be managed (e.g. decommissioned) by placing directed tasks into the workspace. TaskDLiSH provides efficient scheduling of cluster resources and automated load balancing, because workers poll for tasks when they are ready. Fault tolerance is automatic, because the failure of one worker or task does not impact other workers or tasks.

STATUS & FUTURE PLANS

We have conducted many simulations for CELSIUS parameters, using the higher effective rms electron

velocity $\Delta_{eff,\|}$=9000 m/s, with no magnetic field errors. From one to five thousand ion trajectories were generated in each case, in order to extract from the diffusion dominated dynamics a reasonable average value for the velocity drag. This work will be presented elsewhere.

We have begun to repeat some of these simulations with the expected rms electron velocity $\Delta_{e,\|}$=3000 m/s, including magnetic field errors as described by the "first model" of Eq.'s (1) or the "second model" of Eq.'s (2). For the second model, we are using a series of terms taken directly from the mapped field errors of the CELSIUS solenoid. For the first model, we are at present considering only a single sinusoidal term, symmetric in x and y, with an amplitude (b_1=0.001*B_0) and wavelength (20 cm) that are roughly consistent with the CELSIUS solenoid measurements.

Initial results indicate that the second model will suppress the velocity drag (i.e. reduce the friction force and thus increase the cooling time) more strongly than the first model with a single term. We don't present details here, because the simulation results must be examined further and many more ion trajectories must yet be generated. Also, it remains an open question how to properly choose the number of sinusoidal terms, much less their amplitudes, wavelengths and degrees of asymmetry.

In future work, simulation results will be validated through comparison with recently obtained data [6]. Also, the understanding and expertise gained during study of CELSIUS parameters, will later be applied to the parameters of the proposed RHIC cooler.

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