

# DEVELOPMENT OF A BEAM-BEAM SIMULATION CODE FOR $e^+e^-$ COLLIDERS \*

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## Abstract

BEPC will be upgraded into BEPCII, and the luminosity will be about 100 times higher. We developed a three dimensional strong-strong PIC code to study the beam-beam effects in BEPCII. The transportation through the arc is the same as that in Hirata's weak-strong code. The beam-beam force is computed directly by solving the Poisson equation using the FACR method, and the boundary potential is computed by circular convolution. The finite bunch length effect is included by longitudinal slices. An interpolation scheme is used to reduce the required slice number in simulations. The standard message passing interface (MPI) is used to parallelize the code. The computing time increases linearly with  $(n + 1)$ , where  $n$  is the slice number. The calculated luminosity of BEPCII at the design operating point is less than the design value. The best area in the tune space is near  $(0.505, 0.57)$  according to the survey, where the degradation of luminosity can be improved.

## INTRODUCTION

BEPCII is an upgrading scheme from BEPC. It is a double ring machine. Following the success of KEKB, the crossing scheme was adopted in BEPCII, where two beams collide with a horizontal crossing angle  $2 \times 11$  mrad. The design luminosity of BEPCII is  $1.0 \times 10^{33} \text{cm}^{-2} \text{s}^{-1}$  at 1.89 GeV, about 100 times higher BEPC [1].

The beam-beam interaction is one of the most important limiting factors determining the luminosity of storage ring colliders. Due to the complexity of the interaction, computer simulations are necessary to study it quantitatively. There have been various types of computer codes on this topic, such as weak-strong simulation [2] and strong-strong simulation [3, 4, 5]. Historically, the weak-strong simulation which is not self-consistent has been employed in order to simulate the effect in a reasonable computing time. The strong-strong simulation, which requires large amounts of computer resources, has recently become feasible due to the fast progress in computing power. Now the two dimensional simulation without finite bunch length effect can be done using a personal computer in a reasonable time. While a supercomputer is still necessary in the three dimensional strong-strong simulation. The strong-strong codes now have a reliable predictive capability of realistic beam-beam interaction and the simulation results show a

good quantitative agreement with the experimental observations [6].

In order to include the crossing angle effects, the beam-beam simulation has to take the bunch length into account. We have developed a new three dimensional strong-strong particle-in-cell (PIC) code SBBE, which was based on our previous work [7]. The code was written in standard C and parallelized with the standard message passing interface (MPI). The model and algorithm used in the code are explained in the following, and we also present the simulation results of beam-beam effects in BEPCII.

## MODEL AND ALGORITHM

The two colliding beams are both represented by macro-particles in SBBE. A macro-particle is treated as a single electron or positron dynamically. We initialize the macro-particles with the six-dimensional Gaussian distribution according to the optics parameters at the interaction point (IP) and the nominal emittance of the beam.

The one-turn map of macro-particles consists the following two parts:

1. Beam-beam interaction near IP. The bunch length effect is included by longitudinal slices. A slice interacts with the opposing slices one by one. When the interaction between two slices is considered, the potentials at two longitudinal points generated by a slice are computed directly and respectively by solving the Poisson equation. The potentials between the two points are calculated by linear interpolation [8]. The macro-particles in a slice are kicked by the opposing slices, and drift between two collision points near IP. When there is a crossing angle, the Lorentz Boost [9] is employed to treat the collision.
2. Transportation through the arc. Single particle dynamics in three dimensional space is taken into account. We only consider the linear map with synchrotron radiation, which is following Hirata's BBC [10].

### Beam-Beam Potential

Given a charge distribution  $\rho(x, y)$ , the potential  $\phi(x, y)$  generated by the beam satisfies the Poisson equation

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \phi(x, y) = -\frac{\rho(x, y)}{\epsilon_0}. \quad (1)$$

The solution of the potential  $\phi$  can be expressed as

$$\phi(x, y) = \frac{1}{2\pi\epsilon_0} \int dx' dy' G(x - x', y - y') \rho(x', y'), \quad (2)$$

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where  $G(x, y)$  is the Green function

$$G(x, y) = -\frac{1}{2} \ln[x^2 + y^2]. \quad (3)$$

The transverse dimension of the beam is generally much smaller than that of the pipe near IP. Therefore the open boundary condition is assumed. The two dimensional field solver is following Cai [5], and we compute the boundary potential by a FFT method. We represent the potential  $\phi(x, y)$  by its values at discrete set of points

$$\begin{aligned} x_j &= x_0 + jh_x & (j &= 0, 1, \dots, J) \\ y_l &= y_0 + lh_y & (l &= 0, 1, \dots, L) \end{aligned}, \quad (4)$$

where  $h_x$  and  $h_y$  are horizontal and vertical grid spacing respectively. We write  $\phi_{j,l}$  for  $\phi(x_j, y_l)$ . The points where  $j = 0, j = J, l = 0$  or  $l = L$  are boundary points. Here we describe the method by introducing how to compute the potential  $\phi_{0,l}$  where  $l = 0, \dots, L - 1$ . Following (2),  $\phi_{0,l}$  can be written as

$$\begin{aligned} \phi_{0,l} &= \frac{1}{2\pi\epsilon_0} \sum_{m=1}^{J-1} \sum_{n=1}^{L-1} C_{m,n} G_{m,|l-n|} \\ &= \frac{1}{2\pi\epsilon_0} \sum_{m=1}^{J-1} \phi_{0,l}^m \quad (l = 0, \dots, L - 1), \end{aligned} \quad (5)$$

where  $C_{m,n}$  is the charge on point  $(x_m, y_n)$ ,  $G_{m,n}$  is the value of Green function  $G(mh_x, nh_y)$ , and  $\phi_{0,l}^m$  is defined as

$$\phi_{0,l}^m = \sum_{n=1}^{L-1} C_{m,n} G_{m,|l-n|} \quad (l = 0, \dots, L - 1). \quad (6)$$

$C_{m,n}$  and  $G_{m,n}$  are extended respectively as

$$C_{m,n} = 0 \quad (L \leq n \leq 2L - 1), \quad (7)$$

$$G_{m,n} = \begin{cases} \text{arbitrary} & (n = L) \\ G_{m,2L-n} & (L < n \leq 2L - 1) \end{cases}. \quad (8)$$

With the extended series,  $\phi_{0,l}^m$  can be rewritten as a formalism of circular convolution

$$\phi_{0,l}^m = C_{m,l} \otimes G_{m,l} \quad (l = 0, \dots, 2L - 1). \quad (9)$$

The value of  $\phi_{0,l}^m$  in (9) is equal to that in (6) for  $0 \leq l \leq L - 1$ , and meaningless for  $L \leq l \leq 2L - 1$ . The discrete Fourier transforms (DFT) of  $C_{m,l}$  and  $G_{m,l}$  are evaluated as

$$\hat{C}_{m,\hat{k}} = \sum_{l=0}^{2L-1} C_{m,l} \exp(-i\frac{\pi}{L}lk) \quad (k = 0, \dots, 2L - 1), \quad (10)$$

$$\hat{G}_{m,\hat{k}} = \sum_{l=0}^{2L-1} G_{m,l} \exp(-i\frac{\pi}{L}lk) \quad (k = 0, \dots, 2L - 1). \quad (11)$$

The DFT of  $\phi_{0,l}^m$  can be expressed as

$$\hat{\phi}_{0,\hat{k}}^m = \hat{C}_{m,\hat{k}} \hat{G}_{m,\hat{k}} \quad (0 \leq k \leq 2L - 1). \quad (12)$$

The solution of  $\phi_{0,l}^m$  is obtained by the inverse DFT

$$\phi_{0,l}^m = \frac{1}{2L} \sum_{k=0}^{2L-1} \hat{\phi}_{0,\hat{k}}^m \exp(i\frac{\pi}{L}lk) \quad (0 \leq l \leq 2L - 1). \quad (13)$$

In the technique, we compute  $\phi_{0,l}^m$  in (5) by three times FFT instead of computing it directly by (6). Since the dimension of mesh and grid number are both constant during the course of tracking,  $G_{j,l}$  is constant and only two times DFT is needed in fact. The potentials on other boundaries can be computed by a similar method. After the boundary potentials are known, the problem is converted into a Dirichlet one of the Poisson equation. The five-point difference scheme is used to approximate the two dimensional Laplacian operator

$$\frac{\phi_{j-1,l} + \phi_{j+1,l} - 2\phi_{j,l}}{h_x^2} + \frac{\phi_{j,l-1} + \phi_{j,l+1} - 2\phi_{j,l}}{h_y^2} = -\frac{\rho_{j,l}}{\epsilon_0}. \quad (14)$$

The FACR (Fourier analysis and cyclic reduction) method is used to solve equations (14), then we can obtain the potentials on the inside grid points.

### Parallel Scheme

The longitudinal boundaries of slices are chosen so that the number of macro-particles in each slice is uniform. Due to the synchrotron oscillation, the  $z$  value of a particle varies each turn. That is to say the particles in one slice may enter the others of the same bunch. For high energy electron or positron storage ring, the synchrotron tune is generally very small ( $\sim 0.01$ ), that is to say the oscillation frequency is very low. It is reasonable for us to assume that it is impossible for a particle in one slice to jump into not-adjacent ones the next turn when the slice number is not very large. A slice exchanges macro-particles with its adjacent ones at IP and before collision each turn in our code.

The computing time increase linearly by a factor of  $n^2$ , where  $n$  is the number of slices in one bunch. A supercomputer is necessary for the three dimensional beam-beam simulation. The standard message passing interface (MPI) is used to parallelize the code. It's natural to represent one slice with one MPI node: the macro-particles' data in a slice is stored in the corresponding node. When two slices collide, their corresponding nodes assign the charges of macro-particles to grid points respectively, exchange the smoothed charge distribution on grid, and then the beam-beam force experienced by a slice can be computed by its corresponding node. Since a slice needs to exchange macro-particles with the adjacent ones, the computing time increases linearly by a factor of  $(n + 1)$  not  $n$ . The parallel scheme is shown in Figure 1. The scheme is not scalable as Qiang's [11], while it is efficient and suitable for small or medium-scale computer clusters.

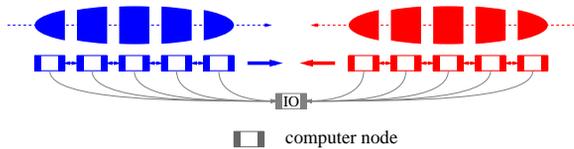


Figure 1: Parallel scheme. One MPI node is used to represent one slice. The IO node is used to collect data and save them to a file.

## SIMULATION RESULTS

We studied the beam-beam effects in BEPCII by simulation using the code. The design parameters of BEPCII is shown in Table 1. In the simulation, 300,000 macro-

Table 1: Design parameters of BEPCII

$E$	1.89 GeV	$\epsilon_x/\epsilon_y$	144 / 2.2 nm
$C$	237.53 m	$\sigma_z$	1.5 cm
$N_b$	93	$\sigma_e$	$5.16 \times 10^{-4}$
$I_b$	9.8 mA	$\nu_x/\nu_y$	6.53 / 7.58
$\xi_y$	0.04	$\nu_s$	0.034
$\theta_c$	$2 \times 11$ mrad	$\tau_x/\tau_y$	31553 / 31553 turn
$\beta_x^*/\beta_y^*$	1 m / 1.5 cm	$\tau_s$	15777 turn

particles are used to represent one bunch which is divided into 5 slices. The transverse plane is divided into  $128 \times 256$  grids with total size of  $30\sigma_x \times 60\sigma_y$ .

### Tune Survey

It's well known that the operating point strongly affects the beam-beam interaction. Here we investigate how the tunes affect the luminosity and try to find the best area in the tune space. By experience, the strong-strong simulation was done in the area ( $0.505 \leq \nu_x \leq 0.545$ ,  $0.55 \leq \nu_y \leq 0.595$ ) with a step of 0.005. Figure 2 shows the contour plot of luminosity versus tune. The best area is

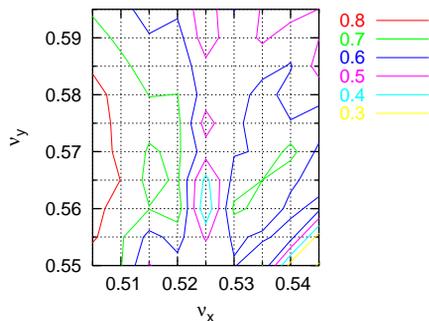


Figure 2: Luminosity versus transverse tune. The luminosity is normalized by the design value.

near (0.505,0.57) where the luminosity is about 80% of the design value.

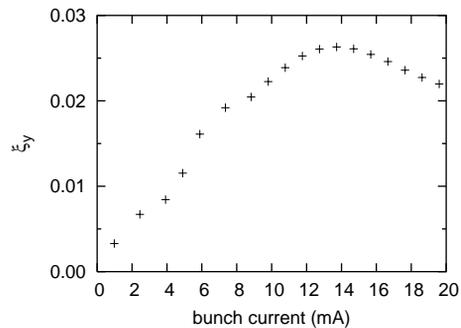


Figure 3: Beam-beam parameters versus bunch current.

### Beam-Beam Limit

The saturation phenomenon of the beam-beam parameter is called the beam-beam limit, which means that the luminosity varies linearly with bunch current. Here the parameter is calculated using the luminosity as follows

$$\xi_y = \frac{2r_e\beta_y}{N\gamma} \frac{L}{f_{rep}}. \quad (15)$$

Figure 3 shows the simulation results for various bunch currents. As seen in the figure,  $\xi_y$  reaches maximum ( $\sim 0.025$ ) near 13mA, and tapers when the current is further increased.

## CONCLUSION

We have developed a new parallel strong-strong beam-beam code, which is used to study the effects in BEPCII. The simulation results show that the calculated luminosity is less than the design value. Further study need be done to optimize the machine.

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