# CALCULATION OF ELECTRON BEAM POTENTIAL ENERGY FROM RF PHOTOCATHODE GUN \*

Wanming Liu<sup>#</sup>, Illinois Institute of Technology, Chicago, IL, 60616, U.S.A

Haitao Wang, J. G. Power and Wei Gai, ANL, Argonne, IL 60439, U.S.A.

### Abstract

In this paper, we consider the contribution of potential energy to beam dynamics as simulated by PARMELA at low energies (10-30MeV). We calculate the potential energy of the relativistic electron beam using the static coulomb potential in the rest frame (the first order approximation as used in the PARMELA code). We found that the potential energy contribution to the beam dynamics could be very significant, particularly with the high charge beams generated by an RF photocathode gun. Our results show that when the potential energy is counted correctly and added to the kinetic energy from PARMELA, the total energy is conserved. Results of potential and kinetic energy simulations for short beams (  $\sim$ 1mm) at various charges (1-100nC) generated by a high current RF photocathode gun are presented.

## **INTRODUCTION**

During numerical simulations of the beam properties of the new Argonne Wakefield Accelerator photocathode gun[1], we observed a suspicious energy gain for highcharge, short-bunch electron beams in a drift space. The energy gain in the drift space is repeatable and proportional to the charge of the beam. After eliminating any possible cause of numerical errors, the energy gain from the drift space is still found to be significant.

Usually, the potential energy of a relativistic electron beam is negligible compared to its kinetic energy. Therefore, the observed energy gain in the drift space seemed unphysical and raised concerns about the accuracy of the PARMELA space charge calculation routine. Since we depend on PARMELA simulations in predicting the beam quality before we set up the diagnostic system and perform measurements, it would be troublesome if PARMELA did not calculate the space charge contribution correctly.

It was suggested to us [2] that the change in kinetic energy may be due to a corresponding change in potential energy. To answer the question about the accuracy of PARMELA space charge calculation and regain our confidence in the predicted beam qualities, we calculated the potential energy of a relativistic electron beam up to the first order approximation. By adding the potential energy of the beam together with its kinetic energy, the total energy of beam is conserved in drift space. The assumption about the potential energy of the relativistic beam turns out to be not true and the doubts about the PARMELA space charge calculation are dismissed.

## POTENTIAL ENERGY OF RELATIVISTIC BEAM

It is only possible to use a Lagrangian description of the interaction of two or more charged particles with each other at nonrelativistic velocities. The Lagrangian is supposed to be a function of the instantaneous velocities and coordinates of all the particles. When the finite velocity of propagation of electromagnetic fields is taken into account, this is no longer possible, since the values of the potentials at one particle due to the other particles depend on their state of motion at retarded times. Only when retardation effects can be neglected is a Lagrangian description of the system of particles possible [3].

## Lowest-Order Relativistic Corrections

If only the lowest-order relativistic corrections are desired, the interaction Lagrangian for two charged particles including lowest-order relativistic effects would be [2]:

$$L_{\rm int} = \frac{q_1 q_2}{r} \left\{ -1 + \frac{1}{2c^2} \left[ \vec{v}_1 \cdot \vec{v}_2 + \frac{(\vec{v}_1 \cdot \vec{r})(\vec{v}_2 \cdot \vec{r})}{r^2} \right\}$$
(1)

By summing up the interaction Lagrangian of all pairs of particles in the beam, we can get the potential energy of a relativistic beam up to its lowest order in the lab frame.

Equation (1) could be used to calculate the potential energy of a beam directly in the lab frame. It can be interpreted as the Coulomb potential plus the magnetic potential between two currents with relativistic corrections.

But the default time step output of the PARMELA simulation doesn't contain complete information about the velocities of particles [4]. There are some difficulties in applying equation (1) to obtain the potential energy of beam from PARMELA simulation without changing the output to a type which requires considerable disk space and massive IO operations.

### *Alternative approaches*

If we consider the potential of particles in the reference frame where the particle is at rest, the particle will see only the Coulomb force and thus the potential energy can be easily calculated using only the Coulomb potential in their rest frame

$$U_{1,2} = \frac{q_1 q_2}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}}$$
(2)

where (x1,y1,z1), (x2,y2,z2) are the coordinates of q1 and q2 in the rest frame of q1. By transforming this rest

<sup>\*</sup>Work supported by DOE

<sup>#</sup>wmliu@anl.gov

frame potential energy into the lab frame, we can then obtain the potential energy measured in lab frame. Equation (2) would be exact if the coordinates are measured simultaneously in the rest frame of q1 or q2. But we can only obtain the coordinates in lab frame from the code simultaneously at a given time. According to the special theory of relativity, the coordinates of particles measured simultaneously in the lab frame, when transformed into the rest frame of particles, will become coordinates measured at different time. This is where the approximation is required.

## *Transformation of Potential Energy Between the Rest Frame and the Lab Frame*

According to special theory of relativity, the 4momentum of particles between two reference frames is related by

$$\begin{bmatrix} E^{r} \\ P_{x}^{r} c \\ P_{y}^{r} c \\ P_{z}^{r} c \end{bmatrix} = \begin{bmatrix} \gamma_{t} & 0 & 0 & -\beta_{t} \gamma_{t} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta_{t} \gamma_{t} & 0 & 0 & \gamma_{t} \end{bmatrix} \begin{bmatrix} E \\ P_{x} c \\ P_{y} c \\ P_{z} c \end{bmatrix}$$
(3)

from which we then have

$$E^{r} = \gamma_{t} E - \beta_{t} \gamma_{t} P_{z} c \tag{4}$$

and

$$P^{r}{}_{z}c = -\beta_{t}\gamma_{t}E + \gamma_{t}P_{z}c$$
<sup>(5)</sup>

Assume that the particle has an energy gain of  $\Delta E^r$ and the momentum changes by  $(\Delta P_x^r, \Delta P_y^r, \Delta P_z^r)$  after a space charge impulse. By transforming this changed energy momentum vector back into lab frame, we then have

$$E + \Delta E = \gamma_t (E^r + \Delta E^r) + \beta_t \gamma_t (P^r_z + \Delta P^r_z) c (6)$$

By summing up (6) for all particles, we obtain

$$\sum_{i=1}^{N} (E_i + \Delta E_i) = \sum_{i=1}^{N} [\gamma_t (E^r_i + \Delta E^r_i) + \beta_t \gamma_t (P^r_z + \Delta P^r_z)c]$$
(7)

Substituting (5) into (7), we then have

$$\sum_{i=1}^{N} \Delta E_{i} = \sum_{i=1}^{N} \gamma_{t} \Delta E^{r}_{i} + \beta_{t} \gamma_{t} \sum_{i=1}^{N} \Delta P^{r}_{z,i} c \qquad (8)$$

In the rest frame of the particles, the momentum should be conserved as there is no external force and the relativistic effect from the relative motion between particles is negligible. Taking into consideration momentum conservation in their rest frame of the particles, equation (8) becomes

$$\sum_{i=1}^{N} \Delta E_i = \sum_{i=1}^{N} \gamma_i \Delta E^r{}_i \tag{9}$$

When there is no external force applied to the system, the total kinetic energy gain should only come from the reduction of potential energy and thus we have the transform relation between potential energy in the rest frame and lab frame:

$$U = \gamma U^r \tag{10}$$

Here  $U^r$  is the Coulomb potential energy of particles in their rest frame, and U is the potential energy of particles in the lab frame which includes contributions from both Coulomb and magnetic potentials.

### NUMERICAL RESULTS

In this section, we use equation (2) to calculate the potential energy of each particle pair in their rest frame and then translate the potential energy back into the lab frame using equation (10). We then sum up the lab frame potential energy of all the particles to obtain the potential energy of the beam. The kinetic energy of beam is obtained by summing up the kinetic energy of each particle in the beam. The total energy of beam is then obtained by adding the beam potential energy to the beam kinetic energy.

### Results for the AWA Photocathode Gun

Results from PARMELA simulations with beam



Figure 1. PARMELA simulation result for 1nc beam for our new photocathode gun

intensities of 1 nc, 33 nc and 100 nc for our new photocathode gun are given in this section.



Figure 2. PARMELA simulation result for 33nc beam for our new photocathode gun

Figure 1 shows the result for a 1 nc beam. The potential energy is very small compared to the kinetic energy and thus there is no observed kinetic energy gain in the drift space.

Figure 2 shows the result for a 33 nc beam. The potential energy of the beam is about 2% of its kinetic energy. The beam kinetic energy changes by 0.44% while drifting from 1 m to 3 m, but the total energy only changes by 0.03%.

Figure 3 shows the simulation result for a 100 nc beam. As shown in the figure, the potential energy is about 3.8% of beam kinetic energy. The kinetic energy increases by about 1% while drifting from 1 m to 2.7 m. The total energy changes by only about 0.16%.

From figure 1 to figure 3, notice that the kinetic energy



Figure 3: PARMELA simulation result for 100nc beam from our new photocathode gun

gain in the drift space increases with the amount of charge in the beam which agrees with Coulomb's law. The total energy is conserved after accounting for the potential energy.

## Further Validation

As shown above, the total energy of beam is conserved in a drift space for our new photocathode gun. For further validation, we studied results for beams with different initial energies.

Figure 4 shows the result of a beam with 4 MeV initial



Figure 4: PARMELA simulation result for a monoergic 4 MeV beam.

energy. As shown in the plot, the potential energy of the beam is about 10% of the kinetic energy at the beginning. After drifting for 250 cm, the beam kinetic energy

increased by about 8.8% while the total energy changed only by about 0.38%.

Figure 5 shows the result for an 8 MeV monoergic beam. The initial potential energy is about 7.2% of the initial kinetic energy. After drifting for a distance of 300 cm, the kinetic energy increased by 5.2% while the total



Figure 5: PAMELA simulation result for beam with 8MeV mono initial energy.

energy only changed by about 0.6%.

Figure 6 shows the result for a beam with 16 MeV initial energy. In this figure, the beam kinetic energy increased by about 2% while the total energy changed only about 0.1% after drifting a distance of 300cm.



Figure 6: PARMELA simulation result for a beam with mono initial energy of 16MeV

#### **SUMMARY**

The kinetic energy gain in drift spaces observed in PARMELA simulations is results from the conversion of the potential energy of the beam. The potential energy of a high charge low energy beam is not negligible. After the beam potential energy is counted, the total energy is conserved. Doubts about the accuracy of this aspect of PARMELA simulation should be dismissed.

## REFERENCES

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