PROGRESS ON A VLASOV TREATMENT OF COHERENT SYNCHROTRON RADIATION FROM ARBITRARY PLANAR ORBITS*

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Abstract

We report on our progress in the development of a fully self-consistent Vlasov treatment of coherent synchrotron radiation (CSR) effects on particle bunches traveling on arbitrary planar orbits. First we outline our Vlasov approach and the approximation we are currently studying. Then we discuss recent numerical results for a benchmark model studied extensively with codes by several authors.

INTRODUCTION

Coherent synchrotron radiation (CSR) is expected to play an important and often detrimental role in various advanced accelerator projects, for instance in linac-based coherent light sources [1] and energy recovery linacs [2]. A large concern is that CSR may cause transverse emittance growth in a bunch compressor and microbunching. We propose a new method to study CSR effects based on a fully self-consistent Vlasov-Maxwell (VM) calculation of the phase space density. Our model can be applied to study CSR on particle bunches traveling on arbitrary planar orbits between parallel conducting plates. The plates represent shielding due to a vacuum chamber. The vertical distribution of charge is an arbitrary fixed function.

In the first section we recall briefly the main features of our VM treatment. A detailed description can be found in [3]. In the final section we present recent numerical investigations obtained in the Liouville-Maxwell approximation (LMA), where the bunch density is evolved under the fields produced by the unperturbed density (subject to external fields only). This study led us to a substantial improvement of our algorithm and to a better understanding of the approximations used.

THE MODEL

Our strategy is to solve the Vlasov equation for the phase space density on beam frame coordinates and to solve the Maxwell equations in the lab frame.

In the lab frame the spatial coordinates are (Z, X, Y) and the independent variable is $u = ct$. The particle orbits lie in planes Y = const between two infinite, perfectly conducting plates, at $Y = \pm g$. We have a reference orbit $\mathbf{R}_0(s) = (Z_0(s), X_0(s))$ where s is arc-length and a reference particle traveling on this orbit with constant speed βc so its trajectory is $\mathbf{R}_0(\beta u)$. Thus a point can also be specified in terms of Frenet-Serret coordinates relative to the orbit: $\mathbf{R} = (Z, X) = \mathbf{R}_0(s) + x\mathbf{n}(s)$ where $\mathbf{n}(s) = (-X_0'(s), Z_0'(s))$ is the unit normal vector. The corresponding unit tangent is $\mathbf{t}(s) = \mathbf{R}_0'(s) = (Z_0'(s), X_0'(s))$. After a change of independent variable from $u = ct$ to s through standard manipulations, a convenient set of dynamical variables for motion in horizontal planes consists of the "beam frame" phase space coordinates $(\mathbf{r}, \mathbf{p})$, where $\mathbf{r} = (z, x)$ and $\mathbf{p} = (p_z, p_x)$. Here $z(s) = s - \beta ct(s)$, where $t(s)$ is the time of arrival at arc-length s. The conjugate variable is the relative energy deviation $p_z(s) = (E(s) - E_0)/E_0$, with $E_0 = m\gamma c^2$ the energy of the reference particle and $p_x(s) = v_x(s)/\beta c$ where v_x is the velocity component along $\mathbf{n}$.

To solve the Maxwell equations in lab frame we must express the lab frame charge/current density in terms of the beam frame phase space density, $f(\mathbf{r}, \mathbf{p}, s)$. Define

$$\rho(\mathbf{r}, s) = Q\int d\mathbf{p} f(\mathbf{r}, \mathbf{p}, s),\ \tau(\mathbf{r}, s) = Q\int d\mathbf{p}\ p_x f(\mathbf{r}, \mathbf{p}, s), \tag{1}$$

where Q is the total charge and f has unit integral. To a good approximation the lab frame charge density ρ_L and the lab frame current density $\mathbf{J}_L$ are

$$\begin{aligned}\rho_L(\mathbf{R}, Y, u) &= H(Y)\rho(\mathbf{r}, \beta u)\ , \\ \mathbf{J}_L(\mathbf{R}, Y, u) &= \beta c H(Y)\big[\rho(\mathbf{r}, \beta u)\mathbf{t}(\beta u + z) \\ &+ \tau(\mathbf{r}, \beta u)\mathbf{n}(\beta u + z)\big],\end{aligned} \tag{2}$$

where $\mathbf{r} = M^T(\beta u)(\mathbf{R} - \mathbf{R}_0(\beta u))$, $M = (\mathbf{t}, \mathbf{n})$, $\int H(Y)dY = 1$ and $H(Y)$ is an arbitrary fixed vertical distribution of charge. The derivation of (2) will be discussed elsewhere. We calculate the fields produced by $(\rho_L, \mathbf{J}_L)$, but averaged over the Y-distribution, for example,

$$\mathbf{E}(\mathbf{R}, u) := \langle \mathbf{E}(\mathbf{R}, \cdot, u)\rangle = \int_{-g}^{g} H(Y)\mathbf{E}(\mathbf{R}, Y, u)dY. \tag{3}$$

The averaged fields can be computed much more quickly, and we believe that it will produce nearly the same dynamics in the (Z, X) plane as the full fields. After imposing boundary conditions at the parallel plates by the method of images the averaging produces just a two-dimensional integral,

$$\begin{aligned}\mathbf{E}(\mathbf{R}, u) &= -\frac{1}{2\pi}\sum_{k=0}^{\infty} a_k \int_{-\infty}^{u-kh} dv \int_{-\pi}^{\pi} d\theta\ \mathbf{S}_E(\hat{\mathbf{R}}, v, k), \\ B_Y(\mathbf{R}, u) &= \frac{1}{2\pi}\sum_{k=0}^{\infty} a_k \int_{-\infty}^{u-kh} dv \int_{-\pi}^{\pi} d\theta S_B(\hat{\mathbf{R}}, v, k),\end{aligned} \tag{4}$$

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where $\mathbf{S}_E = \nabla\rho_L/\epsilon_0 + \mu_0\partial\mathbf{J}_L/\partial t$, $S_B = \mu_0(\nabla\times\mathbf{J}_L)_Y$, $\hat{\mathbf{R}} = \mathbf{R} + \sqrt{(u-v)^2 - (kh)^2}(\cos\theta, \sin\theta)$ and $a_k = (-1)^k(1-\delta_{k0}/2)$.

The Vlasov calculation is based on the equations of motion in the beam frame:

$$\begin{aligned} z' &= -\kappa(s)x, & p_z' &= F_z, \\ x' &= p_x, & p_x' &= \kappa(s)p_z + F_x, \end{aligned} \quad (5)$$

where the collective force is

$$\begin{aligned} F_z &= \frac{e}{\beta c E_0}\mathbf{V}\cdot\mathbf{E}(\mathbf{r},s), \\ F_x &= \frac{e}{E_0\beta^2}\Big[-X_0'(s)(E_Z(\mathbf{r},s) - V_X B_Y(\mathbf{r},s)) \\ &+ Z_0'(s)(E_X(\mathbf{r},s) + V_Z B_Y(\mathbf{r},s)) \\ &- \frac{\beta p_x}{c}\mathbf{E}(\mathbf{r},s)\cdot\mathbf{V}\Big], \\ \mathbf{V} &= \beta c\big(\mathbf{t}(s) + p_x\mathbf{n}(s)\big) \end{aligned}$$

These are the equations of standard linear optics perturbed by the collective force from CSR. The unperturbed version of (5) with $F_z = F_x = 0$ can be solved explicitly [5] in terms of the lattice functions $D(s),\ D'(s),\ R_{56}(s)$. This gives the transport map $\Phi(s|0)$ from $s = 0$ to arbitrary s, with inverse $\Phi(0|s)$. The Vlasov equation for the distribution function $g(\zeta_0, s) = f(\zeta, s)$ in the interaction picture is easily constructed as in [3], where $\zeta_0 = \Phi(0|s)\zeta$, $\zeta = (\mathbf{r}, \mathbf{p})$. Ultimately we will numerically integrate it using the PF method (method of local characteristics) [3].

NUMERICAL STUDIES AND DEVELOPMENT OF THE METHOD

To date we have studied the Vlasov equation in the LMA using particle simulations on (5) for a benchmark bunch compressor [4] taking an initial Gaussian distribution with linear energy chirp without shielding. The calculations are quite delicate and the most costly part of the calculation, the number of field calculations, needs to be minimized. Thus we feel the need to proceed in small steps toward a fully self-consistent calculation. We have focused on the effect of CSR through the p_z equation, ignoring the self fields in the p_x equation. In [3] we studied the mean relative energy loss and the standard deviation of the relative energy deviation by solving the equations of motion in the beam frame using a 4th order variable step Runge-Kutta, the field calculation being done for each particle at each step. Here we report on a much more effective and accurate strategy: (1) we solve the equations of motion in the beam frame interaction picture using the lattice functions and Euler's method and (2) we evaluate the field on a grid adapted to the charge density in the beam frame and then use interpolation to find the field at an arbitrary point, thus minimizing the number of field evaluations. The choice of an adapted grid is related to the fact that the unperturbed charge density in the beam frame is strongly stretched and tilted along the chicane. In figure 1 (left frame) we show E_Z in coordinates tilted so that a uniform mesh can be used for interpolation. This is a typical case and a 20×20 grid (400 field calculations) seems satisfactory. Thus we are able to study accurately not only moments but reduced distributions as well (charge/current densities, energy distribution etc.), since we can follow millions of particles through the chicane in minutes. Figures 1 (right frame) and 2 (left frame) show the charge density at two positions using 50 million particles (which took about 10 hours on a work station after the field calculation); note the excellent resolution. At 7.5m the density is in good agreement with the unperturbed density, whereas at 15m it is not. This study of the charge/current densities is very important to us for two reasons, first by comparing with the unperturbed densities we can tell whether or not the CSR is a small perturbation and second it will be helpful when we get to the self-consistent case.

In figure 2 we show the calculations of the mean relative energy loss $\langle p_z\rangle$ (right frame), in figure 3 (left frame) the standard deviation of the relative energy deviation and in figure 3 (right frame) the normalized transverse emittance (x-emittance) for comparison with [4]. The mean relative energy loss and standard deviation of the relative energy deviation are in reasonable agreement with the benchmark results, given that we are not doing a self-consistent calculation, while the x-emittance from the 3rd magnet on is not. There are two possible explanations. It could be that the LMA is simply not a good approximation. However we believe it is more likely that the self forces in the p_x equation must be included. We are beginning that study, however the calculation of F_x is delicate since it involves a small contribution obtained by subtracting large terms. Once this study is completed, we intend to integrate the Liouville equation in the interaction picture using the PF method and also study the VM case with particles. This should be good preparation toward integrating the Vlasov equation in the interaction picture with the PF method.

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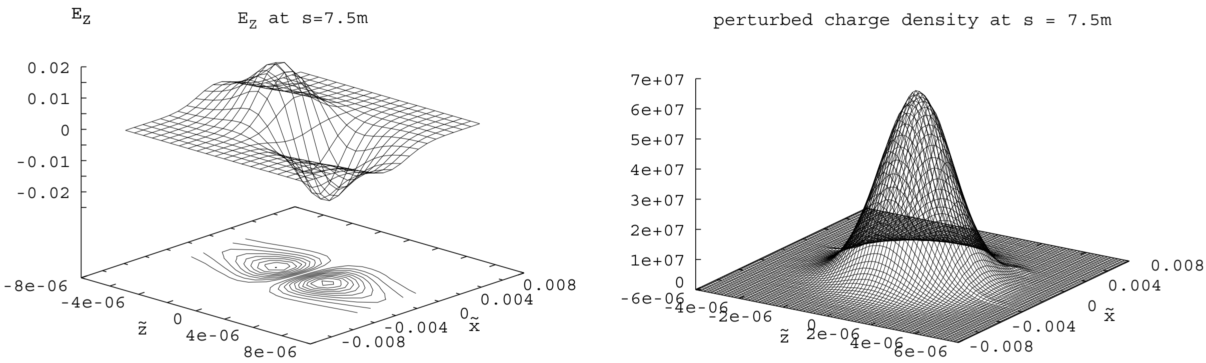

Figure 1: Left: E_Z at s = 7.5m (end of 3^{rd} magnet of the chicane). Right: Perturbed charge density at s = 7.5m.

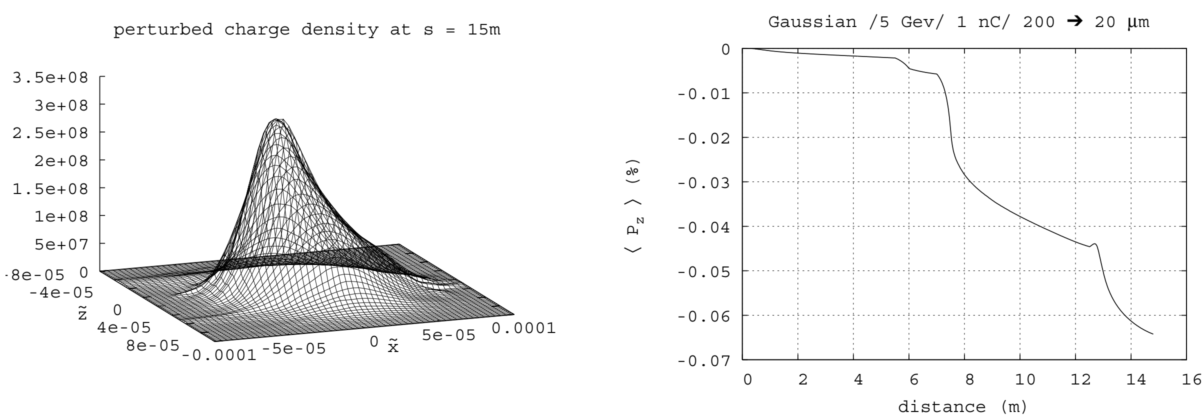

Figure 2: Left: Perturbed charge density at s = 15m (end of chicane). Right: Mean of the relative energy loss.

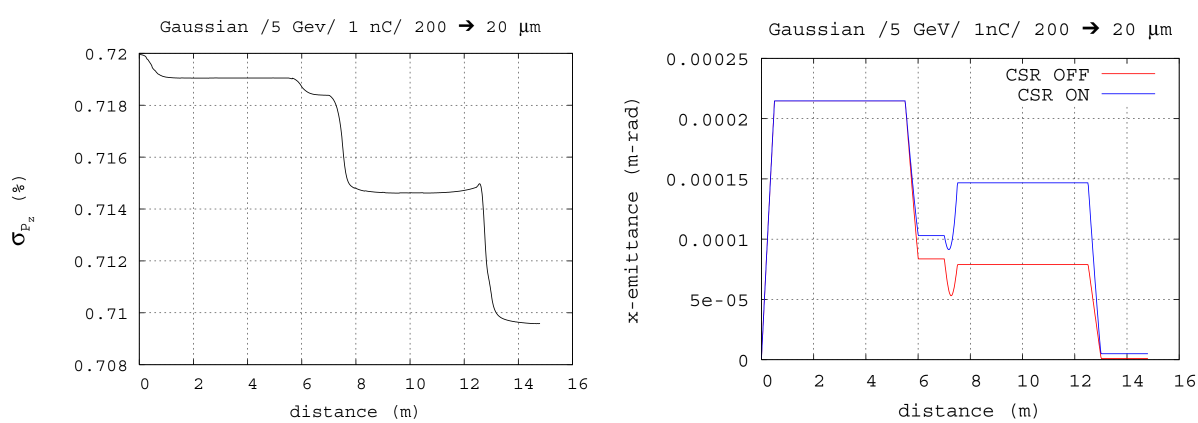

Figure 3: Left: Standard deviation of the relative energy deviation. Right: Normalized transverse emittance (x-emittance).