ON THE VLASOV – MAXWELL EQUATIONS*

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Abstract

There are many interesting physical questions based on the solution to Vlasov – Maxwell equations (VME). However, a solving procedure turns out to be very difficult and hard. But it is often preferable, on physical ground, to a common point of view. Such point of view maybe a structure of some solution. We define and discuss the notation of structure for the distribution function and prove, the strucrure of the Lorentz force represent the structure of the one. At the time of the discovery of the integrable systems the question of VME integrability had been considered. Moreover, as an example, we consider, in the framework of this approach, the relation of integrability and dispersion with a spectra of a Vlasov operator.

INTRODUCTION

A new approach to studying a nonlinear bunched beam dynamics based on the self-consistent Vlasov – Maxwell equations and arguments from optimal control theory is considered. An interesting property of Maxwell equations first stated by V.I. Zubov is employed. The property consists in the following. Given a specified beam motion in R^3 there exist electric and magnetic fields which realize this motion. This property along with some mathematical aspect of optimal control theory with a given quality criterion allows to construct in certain cases a solution to the problem of focusing and acceleration for charged particle beams. The approach can be regarded as a development of the known algorithm by R.C. Davidson.

There are many interesting physical questions which based on of the solution Vlasov – Maxwell equations (VME). A lot of analitycal investigations and computer experiments (see, for example [1] and references therein) are devoted to the study of this problems. However, the procedure of solve is very difficulty and hard. But it is often preferable, on physical grounds, to a common point of view. Such point of view maybe a structure of some solution. The authors consider the structure of the Lorentz force and its relation to the structure of VME. We define and discuss the notation of structure for the distribution function and prove, the structure of the Lorentz force represent the structure of the one. At the time of the discovery of the integrable systems the question of VME integrability had been considered. Moreover,as a example, we consider, by means this concept, the relation integrability and dispersion with a spectra of a Vlasov operator.

The main result consist in the following: the Vlasov distribution function has a form of a Hamilton function in angle – action variables.

BASIC CONCEPTS

As well known [2], the average of acceleration, in the Vlasov sense, may be written down as follows

$$\langle v^{\cdot} \rangle = \frac{\int\limits_{\infty} v^{\cdot} f(t,r,v,v^{\cdot})dv^{\cdot}}{\int\limits_{\infty} f(t,r,v,v^{\cdot})dv^{\cdot}} = qE(t,r) + \frac{1}{c}[v \times H]. \quad (1)$$

This has become a standard notation in this problem. Here vector v is a nonrelativistic velocity, E and H are electric and magnetic fields.

The right - hand side of the equation (1) determines some structure for the function

$$f(t,r,v) = \int\limits_{\infty} f(t,r,v,v^{\cdot})dv^{\cdot}.$$

But what kind of structure is it? This question is considered below.

Now, we are in a position to state the main results of this paper. Let us define a function $f(t,r,v)$ as follows

$$\begin{aligned} f &= \int \sum_{1}^{N} \alpha_j f_j e^{iw_j} dv^{\cdot} \quad (2) \\ f_j &= f_j(r,v^{\cdot}), j \in \overline{1,N}; F_i = \int\limits_{\Theta} v_j^{\cdot} f_j dv_j^{\cdot} \end{aligned}$$

where $\alpha_j, j = \overline{1,N}$ some number and may by chosen arbitrarily, Θ is domain of the existence of $v^{\cdot}$.

As is known, the Vlasov equation have the form

$$\partial_t f + v\partial_r f + (qE + \frac{1}{c}[v \times H])f = 0. \quad (3)$$

Substituting f from (2) into (3), we find that this reasoning yields a simple equation

$$\begin{aligned} \sigma_N f_j &= \int\limits_{\Omega} \sum_{k \le -N}^{N} (1 - \frac{|k|}{1+N}) e^{ik\circ r} e^{-ik\circ p} f_j(p) dp, \\ \sum_{k \le -N}^{N} &= \sum_{\alpha \le -N}^{N} \sum_{\beta \le -N}^{N} \sum_{\gamma \le -N}^{N}, k = (\alpha,\beta,\gamma) \end{aligned}$$

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where N is sufficiently great number and sign $\circ$ is a scalar product. Differentiating this equation with respect to t we get

$$i\omega_j v \circ \sigma_N f_j + (qE + \frac{1}{c}[v \times H]) \circ \sigma_N f_j = \quad (4)$$

$$\int_\Omega \left| \sum_{k \le -N}^{N} (1 - \frac{|k|}{1+N}) ik \circ v e^{ik\circ r} e^{-ik\circ p} f_j(p) \right|^3 dp.$$

The equation (5) have an uniquely determined solution if and only if the condition

$$\int_\Omega \sum_1^N (qE + \frac{1}{c}[v \times H]) \circ f_j^0 f_j^o dp = 0$$

is true.

Example The most convenient for the kinetic description of a beam particle behavior in an electrostatic field is the Vlasov – Poisson system (VPS):

$$\partial_t f + v\partial_x f + E\partial_v f = 0, \quad (5)$$

$$\nabla E = -4\pi\rho. \quad (6)$$

Here $f = f(t, x, v)$ is a distribution of particles in a phase space $\{x, v\}$ depending on the time t; $x \in R^3$, $v \in R^3$, $E = E(t, x)$ is the electric field, and

$$\rho = q\int f(t, x, v)dv, \quad j = q\int v f(t, x, v)dv$$

are the charge density and the current density respectively.

The VPS problem consists in proving the existence of a C^1 or L^p solution $f(t, x, v)$ for all $t \ge 0$ where $f(0, x, v) = \xi(x, v)$ is a given function.

The report goal is to show a new approach for the numerical simulation of beam dynamics based on the universality of Maxwell equations (V.I. Zubov [4]) and L-moment problem (M.G. Krein [5]). Here we only briefly describe how the L-problem reduces questions of choice of needed field $E(x)$ to an approximation problem.

Let us represent the solution f as a sum

$$f = \sum C_k \psi_k e^{\lambda_k t}, \quad k = 1, 2, ...$$

where $\psi_k = \psi_k(x, v)$, C_k and λ_k are some constants.

This reasoning yields a simple equation

$$\lambda_k\psi_k + E\partial_v\psi_k = \int_{\Omega_x}\int_{\Omega_v} v\partial_x\Phi(x-y; v-u)\psi_k(y, u)dudy$$

where $\Phi(x - y; v - u)$ is a Chezaro kernel. The calculation can be carried out and a solution will be a unique one iff

$$\int_{\Omega_x}\int_{\Omega_v} E\partial_v\psi_k \cdot \psi_k^* dudy = 0. \quad (7)$$

Here ψ_k^* is a conjugate function to ψ_k, and Ω_x, Ω_v are space and velocity volumes where the motion is carried out.

Now we will develop the object in view some simple situation, somehow, in order to construct the $E(x)$ we can use relation (7). The L-moment problem for a continuous medium the physical system is considered at define mesh point $\{\Theta_\alpha\}, \alpha = 1, 2, ..., M$ may be written down as follows

$$\sum_{\alpha=1}^N E(\Theta_\alpha)\eta_k(\Theta_\alpha) = 0, \qquad k = 1, 2, ..., N,$$

$$\sum_{\alpha=1}^N E(\Theta_\alpha)\Omega_\alpha = 1$$

Here $\{\Theta_\alpha\}$ is sets of the special given random points, $N \le \infty$, $\eta_k(\Theta_\alpha) = E\partial_v f_0$, and f_0 is some optimal process, such that

$$\partial_t = 0 \Rightarrow \partial_x f_0 = -\partial_v f_0.$$

This technique allows to provide a precise numerical calculation of the dynamics of charged particles in beams. The L-moment method allows studying the detailed characteristics of bunched beams, taking into account a distribution of particles, real self and external fields, construct optimal fields, and others.

It is obvious that the function $E(x)$ is such that the known equations are valid.

More precisely, the function $E(x)$ given above must be such that the following conditions

$$\nabla E(x) = 4\pi q \int f(t, x, v)dv, \quad (8)$$

$$\text{rot}E(x) = 0 \quad (9)$$

are satisfied. This system is overdetermined and for this reason we consider the following approach (S. Krein).

The second equation in the system (**??**), (**??**) is unresolved for any vector field E. Indeed, let $\text{rot}E = \varpi$, $\varpi \in L^2$, $\int f dv \in L^2$ and from here we have got the following condition $\text{divrot}E = 0$ thus we go to equation $\text{div}\varpi = 0$. But all fields in L^2 form a subspace $S \subset L^2$. Thus the system (5), (6) may be resolved in the subspace $S \times L^2$ of the space $L^2 \times L^2$ only. But an orthogonal supplement to a subspace S in L^2 consists of gradient functions which equal zero on the boundary $\partial\Omega$. In this connection we shall associate some scalar function P. This reasoning yields the following system

$$\nabla E(x) = 4\pi q \int f(t, x, v)dv, \quad (10)$$

$$\text{rot}E(x) = -\text{grad}P \quad (11)$$

with condition $P_{\partial\Omega} = 0$ and other condition on the boundary $\partial\Omega_x : \beta E_{\partial\Omega_x} = \alpha$. The system (10), (11) is the elliptic one, and it can be resolved [6].

ENDNOTE

In this paper we propose a new approach scheme to solving Vlasov – Maxwell problems on the base of control theory. From a mathematical point of view this means that

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the solutions to the nonlinear evolutionary wave equations have got simplification in the description by use some results of control algorithms.

In particular, this is due to the presence of the universality property of the Maxwell equations. A lot of analytical investigations and computer experiments are devoted to studying these equations. Our earlier works are devoted to these investigations and also include, in some cases, new analytical results. In this report we briefly present an approach to study some problems of beam focusing and acceleration.

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