COHERENT SYNCHROTRON RADIATION FROM AN ELECTRON BEAM IN A CURVED WAVEGUIDE

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Abstract

The radiation emitted by a pulsed electron beam as it travels on a circular trajectory inside a waveguide is calculated using a 3D simulation. Forward-propagating wave equations for the fields in the waveguide are calculated by a perturbation of the Maxwell equations where the radius of curvature is large compared to the dimensions of the waveguide. These are integrated self-consistently with the distribution of charge in the beam to provide the complete fields (electric and magnetic) for all times during the passage of the beam through the waveguide and therefore are applicable to sections of any length or combinations thereof. The distribution of electrons and their momentum are also modified self-consistently so that the results may be used to estimate the effect of the radiation on the beam quality (emittance and energy spread).

INTRODUCTION

An electron bunch traveling along a circular trajectory can emit radiation coherently in as much as the wavelengths of interest are longer than the bunch length. This is called coherent synchrotron radiation (CSR). As the bunch travels along the arc, the CSR will move forward relative to the bunch because of the shorter path length taken by the radiation along the cord of the arc compared to the bunch. For high-brightness electron beam applications, this effect may adversely affect the transverse emittance by energy modulation that breaks the symmetry of an otherwise achromatic or isochronous bending system.

This paper presents a simulation method that can used to estimate possible beam quality degradation by CSR. It is based on a wave equation derived from Maxwell's equations as a perturbation valid when the width of the waveguide, x, is small compared to the radius of curvature, R, in a bend. This method is otherwise applicable to transient situations with waveguide boundary conditions, and is fully three-dimensional. This CSR wave equation is combined with a particle-in-cell simulation that provides a means to evaluate beam quality degradation. As an example, emittance growth in a four-dipole chicane bunch compressor is studied.

CSR WAVE EQUATION INTEGRATION

We can numerically solve for the radiation by taking a perturbation of Maxwell's equation for small x/R, using a local curvilinear coordinate system (Fig. 1). Note that the x-direction always remains on the same side of the reference trajectory, while the value of R changes sign as the curvature reverses (R starts as a positive value).

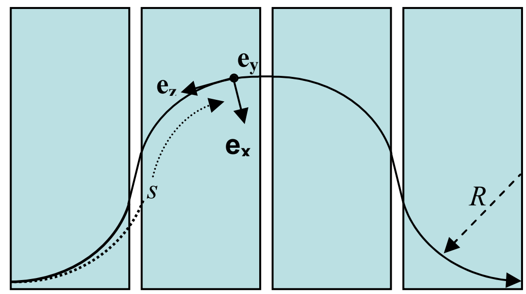

Figure 1. Four-dipole chicane reference trajectory and the local curvilinear coordinate system. The solid blocks are uniform field dipole bending magnets with bend radius *R*.

First, we make a Fourier transform our variables in the longitudinal coordinate, co-moving along the reference trajectory.

$$f(z,\mathbf{x}_\perp,s)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}F(k,\mathbf{x}_\perp,s)e^{-ik(s-z)}dk \tag{1}$$

With the resulting equation for the evolution of the CSR fields (here, modified to include finite γ) is [1]:

$$2ik\frac{\partial}{\partial s}\mathbf{E}_\perp(k)=-\left[\nabla_\perp^2+k^2\left(\frac{2x}{R}-\frac{1}{\gamma^2}\right)\right]\mathbf{E}_\perp(k)+4\pi\nabla_\perp\rho(k) \tag{2}$$

where:

$\mathbf{E}_\perp(k)$ = Fourier transform of perpendicular electric field vector,

γ = reference trajectory relativistic factor,

ρ = charge density, and

$\nabla_\perp$ = indicates grad operator in x-y plane.

The longitudinal field can be found from the perpendicular components using Poisson's equation:

$$\nabla\cdot\mathbf{E}(\mathbf{x})=-4\pi\rho(\mathbf{x})\,. \tag{3}$$

In integrating (2), we use a pseudo-spectral method instead of finite-differencing, which forces the transverse boundary conditions. Specifically, we split the integration step into two steps corresponding to the terms involving x and $\nabla_\perp$ respectively. The second operator is integrated by first transforming into in only sine or cosine terms, depending on the applicable boundary conditions, so that the transverse Laplacian transforms from $\nabla_\perp^2 \rightarrow -k_\perp^2$. In order to minimize the error, the split-step integration is sequenced as a ½ step in x-operator, followed by a full step in the $\nabla_\perp$ operator, followed by another ½ step in the x-operator.

BUNCH COMPRESSOR CHICANE LINEAR OPTICS

Because it is dispersive, particles with a higher (lower) energy will take a shorter (longer) total path through the chicane, pulse compression is accomplished by applying a linear energy chirp, such that the head of the bunch has a lower energy that the tail. For purposes of the current discussion, we will consider a four-dipole chicane, with parameters as in table 1.

Table 1. Parameters for four-dipole chicane bunch compressor

Parameter	Value
Bend radius	1.2 m
Magnet length	40 cm
Waveguide height	6 cm
Waveguide width	10 cm
Chirp	0.16 cm^{-1}
Beam radius, rms	1.0 mm
Pulse length, initial, rms	1.0 mm
Pulse length, final, rms	0.3 mm
Beam energy	250 MeV
Bunch charge	1 nC
Normalized x-emittance, initial	1 mm-mrad

To first order, a change in energy relative to the reference particle will change $x' \equiv dx/ds$ by at the rate

$$\frac{dx'}{ds} = \frac{1}{R}\frac{\Delta\gamma}{\gamma}. \tag{4}$$

Relative to the reference trajectory, a particle at $x(\Delta\gamma/\gamma)$ and $x'(\Delta\gamma/\gamma)$, will change its z-coordinate at the rate:

$$\frac{dz}{ds} = \sqrt{y'^2 + x'^2 + (1 + x/R)^2} - 1. \tag{5}$$

Finally, the rate of change in relative energy from the longitudinal electric field is:

$$\frac{d(\Delta\gamma/\gamma)}{ds} = \frac{eE_z}{\gamma mc^2}. \tag{6}$$

The charge density in (2) and (3) can be computed from a cloud-in-cell interpolation of an ensemble of macro-particles, in this case 100,000 particles are used to model the 6.25 x 10^9 electrons in a 1 nC bunch. The macro-particles are integrated along deviations from the reference trajectory based upon first-order perturbations in energy, which are in turn adjusted by the longitudinal electric field. This is accomplished by a leapfrog scheme as follows: (1) update energies and positions, then (2) update fields and angles, x' and y' on alternating steps, staggered by one half-step.

RESULTS

Using the four-dipole design, (2) was integrated with and without pulse compression (i.e. energy chirp applied to pulse). As an example of the complexity of the difficulty in calculating CSR in a waveguide from theory, the longitudinal electric field, taken at the mid-plane (y=0) of the waveguide as a function of x and z is shown (Fig. 2). In this coordinate system, recall that negative z is ahead of the reference point and x is positive away from the initial curvature. The field is normalized to the steady-state longitudinal electric field for a Gaussian pulse [2]:

$$E_0 = \sqrt{\frac{2}{\pi}}\frac{Ne}{(3R^2\sigma^4)^{1/3}} \tag{7}$$

where:
N = number of electrons in bunch,
e= charge of electron,
σ= pulse length (rms) of Gaussian bunch,

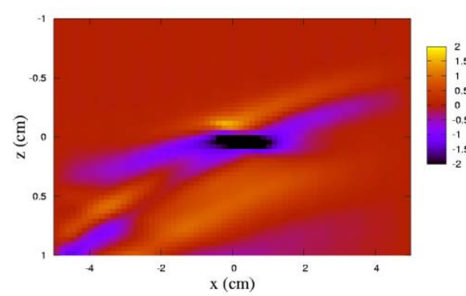

Figure 2. Longitudinal electrical field,, normalized to E_0 (7), at midplane (y=0) of rectangular waveguide (10 by 6 cm) as a function of x and z, midway through compression from 1 mm to 0.3 mm in a four-dipole chicane.

In order to evaluate the possible effect on emittance, the projected phase space of the macro-particle ensemble at the end of the chicane can be analyzed and the emittance computed from

$$\varepsilon_n \equiv \beta\gamma\sqrt{<x^2><x'^2> - <xx'>^2}, \tag{8}$$

where the <> brackets indicate an average of the ensemble of particle comprising the pulse. The projected x-x' phase space is shown for the case without compression (Fig. 3). The emittance growth is not significant primarily due to cancellation between the four legs of the chicane.

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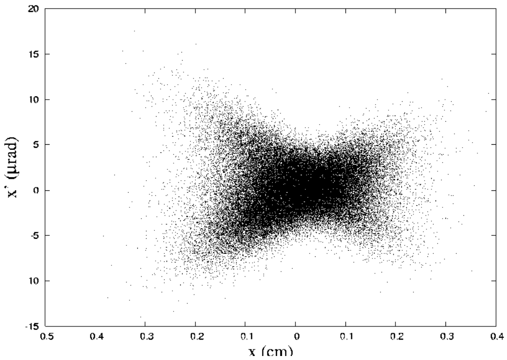

Figure 3. Phase space plot at end of chicane with no chirp. The projected normalized x-emittance growth is approximately 0.6 mm-mrad.

However, we can see a significant growth in the projected x-emittance for the compressed case (Fig. 4).

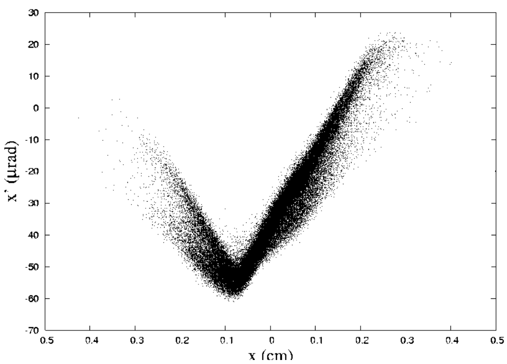

Figure 4. Phase space (x-x') at end of chicane with compression from 1.0 to 0.3 mm. The projected normalized x-emittance growth is approximately 16 mm-mrad

CONCLUSION

The model shows several promising features over previous methods of calculating CSR. Specifically, it takes the boundary conditions into account explicitly, is fully three-dimensional, includes space charge, and is valid in transient conditions. Furthermore, because it does not neglect the formation time, which in this example is 32 cm out of 40 cm in each section, it may be suitable for the study of potential instabilities.

This model does however have limitations as well, the most notable of which is the assumption of periodicity implicit in the use of the longitudinal Fourier transform. Finally, it lacks a realistic model for attenuation due to finite resistivity of the walls. Research continues in these areas as well as improvement in the theory of CSR.

ACKNOWLEDGEMENTS

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