TRANSVERSE STABILITY STUDIES OF THE SNS RING

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Abstract

Detailed studies of the transverse stability of the SNS ring have been carried out for realistic injection scenarios. For coasting beam models and single harmonic impedances, analytic and computational results including phase slip, chromaticity, and space charge are in excellent agreement. For the dominant extraction kicker impedance and bunched beams resulting from injection, computationally determined stability thresholds are significantly higher than for coasting beams.

INTRODUCTION

Because the 248 meter SNS ring will operate at the extremely high beam intensity of 1.5×10^{14} 1 GeV protons, transverse instabilities are a concern. In response, we have undertaken a broad study of transverse stability in SNS. Our approach has been twofold: we studied analytic coasting beam models [1,2] in the SNS parameter regime and applied the results of these studies to benchmark the transverse stability model in the ORBIT code. With this confirmation of the accuracy of ORBIT, we then carried out stability calculations for realistic bunched beams obtained during injection. For comparison with the coasting beam results, these latter calculations were first carried out with single harmonic impedances. Finally, transverse stability was calculated for the full injection process using the most recent measurement of the extraction kicker impedance [3], which is dominant in the ring. We now present the results of these studies.

COASTING BEAM TRANSVERSE STABILITY OF SNS

The analytic formulation used in the coasting beam study was discussed in detail in Ref. [2]. In the present work we introduce one additional effect, namely, chromaticity. This changes the equation for the slowly varying part of the dipole moment to

$$\frac{\partial d_s(\delta,\tau)}{\partial\tau} + i\Delta(\delta)d_s(\delta,\tau) = \chi\int_{-\infty}^{\infty} g(\delta)d_s(\delta,\tau)d\delta, \qquad [1]$$

where $\delta = \frac{\Delta E}{E_0}$ is the fractional energy deviation, $\tau = \frac{\omega_0 t}{2\pi}$ is the time in units of the ring period, $\Delta(\delta) = \frac{\delta}{\beta^2}2\pi[(n+\nu_b)\eta+\xi]$, $\chi = \frac{-Nr_p Z_\perp(n\omega_0+\omega_b)}{\gamma\beta\nu_b Z_0}$, and $g(\delta)$ is the beam energy distribution. The remaining quantities in these equations are the relativistic factors β and γ, the mode number n, the betatron tune ν_b, the phase slip factor η, the chromaticity ξ, the number of protons N, the classical proton radius r_p, the transverse impedance $Z_\perp$, the ring frequency ω_0, the betatron frequency ω_β, and the impedance $Z_0 = 377\Omega$. Following the analysis of Ref. [1], we assume a time dependence $d_s \propto e^{-2\pi i\Omega\tau}$ where Ω is complex. Instability occurs when the imaginary part of Ω is positive and the stability boundary lies on the real axis in the Ω plane. Then, manipulating Eq. [1], we obtain the dispersion relation

$$\begin{aligned} h(n) &\equiv -\frac{Nr_0 Z_\perp(n+\nu_b)\beta^2 E_0}{2\pi i\gamma\beta\nu_b Z_0\left|(n+\nu_b)\eta+\xi\right|} \\ &= \frac{1}{\displaystyle\int_{-\infty}^{\infty}\frac{g(\Delta E)d(\Delta E)}{\Delta E - \frac{\beta^2 E_0}{\left|(n+\nu_b)\eta+\xi\right|}\Omega}}. \\ &\equiv \frac{1}{f(\Omega)+ig(\Omega)} \end{aligned} \qquad [2]$$

By plotting $\frac{1}{f(\Omega)+ig(\Omega)}$ as evaluated from the integral in Eq. [2] in the complex plane for real values of Ω we determine a stability diagram. By comparing this with the *h(n)* as evaluated from the first line in Eq. [2], we can determine the stability as a function of the parameters in the equation. In particular, we concern ourselves with the beam energy distribution *g(ΔE)*, the mode number *n*, the value of the impedance, and the chromaticity ξ.

Several cases were considered. In all we used the SNS lattice with tunes of $\nu_x = 6.23$ and $\nu_y = 6.20$, although in a few cases we also considered a uniform approximation to SNS. Variations on tracking included MAD first order matrices and fully symplectic tracking. Symplectic tracking was carried out both for natural chromaticity and for zero chromaticity obtained by adjusting the sextupole fields. We also did the calculations both with and without space charge forces, which constitute an imaginary contribution to the transverse impedance. We examined the modes $n = 10$, which is in the dominant peak of the bunched beam spectrum, and more recently $n = 25$, to benchmark ORBIT at shorter wavelength. Initially we studied a coasting KV beam with zero energy spread (a delta function distribution) and $N = 3\times10^{14}$ protons, which is typical of SNS with longitudinal bunching included. For the delta function distribution, the analytic stability diagram predicts that all positive real impedances lead to instability, regardless of imaginary impedance values (space charge), and that negative real impedances result in stability. The ORBIT calculations, performed for $n = 10$ under the variety of assumptions discussed above were completely consistent with the analytic results. The delta function distribution can be considered a limiting case of the Lorentz distribution at zero energy spread. As a second case, we considered the Lorentz distribution, for which the dispersion relation predicts a straight line stability diagram with the stability boundary at some positive real impedance, again independent of the imaginary value. ORBIT calculations for the Lorentz distribution confirmed, for the most part, the analytic stability results. For several cases with differing single particle transport (linear MAD and fully symplectic alternatively with natural and with zero chromaticity) in which space charge was ignored, the agreement was precise for both $n = 10$ and for $n = 25$. All cases with the impedance set above threshold, even by only a few percent, proved to be unstable while all cases with the impedance set below threshold were stable. Due to Landau damping, increasing the energy spread or the chromaticity stabilized the beams by increasing the threshold. For cases with space charge included and $n = 10$ the results were less consistent. For the linear MAD transport ORBIT obtained precise agreement with the analytic model, but symplectic tracking led to lower threshold estimates – about 60% of the analytically predicted impedance values. Of all the cases studied thus far, with or without space charge, chromatic effects, or energy spread, these two are the only ORBIT calculations that conflict with analytic predictions. We have yet to resolve this issue.

As a final extension of the coasting beam calculations, we constructed "SNS coasting beams" as follows: Using ORBIT, we injected a beam of 1.5×10^{14} protons over 1060 turns into the SNS ring. The dynamics included transverse painting, symplectic tracking, space charge, the ring RF focusing, and the longitudinal and transverse impedances from the extraction kickers, which dominate the ring. Two cases were considered: one with a painted energy distribution due to energy corrector and energy spreader cavities in the HEBT, and the other without those cavities. In both cases, the peak distribution at the longitudinal center of the bunch was used to generate a coasting beam of the same shape and intensity. The resulting energy distributions were fit by simple functions that could be used in Eq. [2] to provide stability diagrams. For the case with the HEBT cavities the distribution was well represented by the sum of a rectangular distribution and a rational function, while for the case with no HEBT RF cavities the distribution was well represented by the sum of rectangular and Gaussian contributions, as shown in Fig. [1]. Although we treated both cases analytically, the HEBT RF cavities have been removed from the SNS design. Consequently we concentrate on the case without cavities here. The bunch factor for this case in the ORBIT injection simulation was 0.4, so we used $N = 3.75\times10^{14}$ protons in the calculations and coasting beam simulations here.

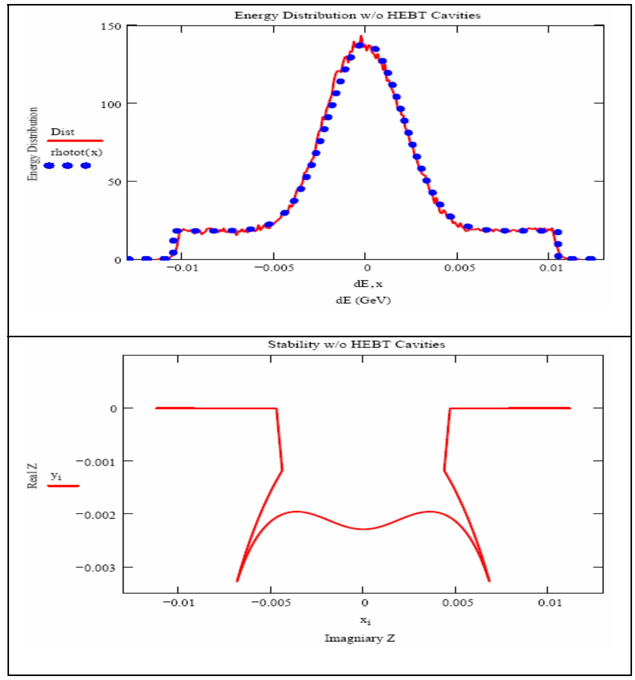

FIGURE 1. Top) Energy distribution of "SNS coasting beam" without HEBT RF cavities. Red curve is from simulation and blue curve is fit using Gaussian plus rectangular distribution. Bottom) Stability diagram for given energy distribution.

The stability diagram resulting from the "SNS Coasting beam" is shown at the bottom of Fig.1. The axes correspond to imaginary (horizontal) and to real (vertical) impedances, respectively. The stability diagram is valid for different values of phase slip factor, chromaticity, intensity, and mode number, but the scales depend on all these factors. For $n = 10$ and the SNS case considered here, at zero chromaticity 0.001 on either axis in Fig. [1] corresponds to 11.2 kΩ/m and for natural chromaticity 0.001 represents 105.5 kΩ/m.. In comparison, the extraction kicker impedance is $Z_R \sim 30$ kΩ/m, $Z_I \sim 50$ kΩ/m (peaking in the vicinity of n = 10) and the space charge impedance is $Z_I \sim 3.5$ MΩ/m. The analytic instability thresholds from the stability diagram in Fig. [1] have been compared with computational ORBIT results for a real impedance of $n = 10$ and several cases. The results show that ORBIT is in good agreement with the analytic predictions, as shown in Table 1. The cases are as follows: 1) Linear MAD tracking, no space charge; 2) symplectic tracking, corrected chromaticity, no space charge; 3) symplectic tracking, natural chromaticity, no space charge; 4) Linear MAD tracking, with space charge; and 5) bunched beam, natural chromaticity, no space charge. The ORBIT threshold for Case 2 is slightly high, probably because the chromaticity correction failed to completely zero the chromaticity. Case 4 shows the destabilizing effect of space charge for coasting beam

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cases. Finally, Case 5 was calculated using a bunched beam and, comparing with the otherwise equivalent Case 3, we see that the bunched beam is much more stable. Thus, coasting beam studies, although providing the opportunity to benchmark against analytic calculations, may not be too relevant for SNS.

Table 1. SNS Coasting Beam Stability Results

Case	Analytic Threshold (kΩ/m)	ORBIT Stable (kΩ/m)	ORBIT Unstable (kΩ/m)
1	25.6	25	30
2	25.6	30	40
3	242	200	300
4	0	0	10
5		800	1000

BUNCHED BEAM TRANSVERSE STABILITY OF SNS

Because SNS will operate with bunched beams, and because coasting beam predictions differ significantly from bunched beam calculations, we carried out several bunched beam simulations with ORBIT. These were done for 1060 turn injection of 1.5×10^{14} protons assuming no HEBT RF cavities. The calculations included transverse injection painting, the ring RF longitudinal focusing, the extraction kicker longitudinal and transverse impedances, and variations on the single particle transport and presence of space charge forces. In all cases, thresholds were obtained in terms of impedances by multiplying the extraction kicker impedance by varying coefficients. The results are shown in Table 2. The cases are as follows: 1) Linear MAD tracking, no space charge; 2) symplectic tracking, corrected chromaticity, no space charge; 3) symplectic tracking, natural chromaticity, no space charge; 4) Linear MAD tracking, with space charge; 5) symplectic tracking, corrected chromaticity, with space charge; and 6) symplectic tracking, natural chromaticity, no space charge. The results of Cases 3 and 6 show, as in the coasting beam calculations, that chromaticity provides significant stabilization. We also see from Cases 1 and 2 that, if space charge is neglected, SNS at zero chromaticity is predicted to be unstable at the extraction kicker impedance. The relevant rows are Cases 4-6, which are the same as Cases 1-3, respectively, except for the inclusion of space charge. Unlike the coasting beam case in Table 1, for which space charge is strongly destabilizing, the effect of space charge on the SNS bunched beam is stabilizing to the zero chromaticity case and very mildly destabilizing at natural chromaticity. Most important, we see that SNS should be stable with at least a factor of 2 to spare over the extraction kicker impedance. At this time, we have no analytic model to treat the bunched beam case, but we are developing a formulation to provide an approach to this problem

Table 2. SNS Bunched Beam Stability Results

Case	ORBIT Stable ×Z	ORBIT Unstable ×Z
1	0.5	0.6
2	0.6	0.8
3	5	7
4	1.5	2
5	2	3
6	3	4

CONCLUSIONS

There are several conclusions to be made from these studies. The first is that, for coasting beams, the ORBIT code benchmarks very well with analytic results of instability thresholds, including the effects of phase slip, chromaticity, and space charge. These beams are stabilized due to Landau damping by increasing the energy spread and/or the chromaticity. For coasting beams, space charge effects tend to be destabilizing as the imaginary space charge impedance shifts the beam away from the stabilizing Landau damped portion of the stability diagram. However, coasting beam results are found to be more unstable than those of bunched beams for otherwise similar cases. The coasting beam model predicts instability for SNS ring energy distributions and intensities, while realistic simulation with 3D space charge shows the beam is stable, even for zero chromaticity. Unlike coasting beams, bunched beams are not significantly destabilized by space charge effects. The greater stability of bunched beams is due to several factors including the coupling of many modes and the spread of betatron tunes along the longitudinal coordinate due to vacuum chamber and bunch factor effects. Finally, real bunched beam dispersion relations will be required to describe our particular SNS Ring situation.

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