ELECTRON BEAM STABILITY REQUIREMENTS FOR LINAC-RING ELECTRON-ION COLLIDERS *

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	eRHIC	ELIC	THERA	QCDE
$\xi_{x,y}$	0.007	0.006	0.0025	0.004
f_c [kHz]	78	250	0.28	0.1
σ_ξ/ξ	$9.5 \cdot 10^{-4}$	$6.3 \cdot 10^{-4}$	0.045	0.047

Table 1: Parameters of the linac-ring electron-ion colliders eRHIC, ELIC, THERA, and QCDE. The collision frequency f_c is defined as the number of collisions per second for one particular ion bunch.

Abstract

In recent years, linac-ring electron-ion colliders have been proposed at a number of laboratories around the world. While the linac-ring approach overcomes the beam-beam tuneshift limitation on the electron beam, it also introduces noise into the ion beam, via the beam-beam interaction with electron bunches of slightly fluctuating intensity and transverse size. The effect of these fluctuations is studied using a linearized model of the beam-beam interaction. Upper limits for the rms jitter amplitudes of electron beam parameters for various linac-ring electron-ion colliders are presented.

INTRODUCTION

In recent years, several linac-ring electron-ion colliders have been proposed at various laboratories, namely eRHIC at BNL [1], ELIC at Jefferson Lab [2], THERA at DESY [3], and the QCD Explorer QCDE at CERN [4]. Some design parameters of these machines are listed in Table 1.

A destinctive common feature of all these facilities is the fact that each electron bunch collides with the ion beam only once before it is dumped. This has the advantage that the beam-beam kick experienced by the electrons can practically be ignored. However, it also introduces noise into the ion beam via the beam-beam interaction with electron bunches of fluctuating intensity and transverse size. Assuming that these fluctuations do not exhibit any bunch-to-bunch correlations, this effect effect can be studied and upper limits on bunch-to-bunch intensity fluctuations and size variations can be established.

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THEORY

The beam-beamkick for round beams can be expressed as

$$\Delta z' = -2\frac{r_p}{\gamma}\frac{N_e}{r^2} z(1-\exp(-r^2/2\sigma^2)), \quad (1)$$

$$r^2 = x^2+y^2,\ z = x, y, \quad (2)$$

where $r_p = 1.54 \cdot 10^{-18}\,\mathrm{m}$, γ, N_e, and σ denote the classical proton radius, the Lorentz factor of the hadron beam, the number of electrons per bunch, and the electron rms transverse beam size, respectively. This expression can be linearized for $z \ll \sigma$ as

$$\begin{aligned}\Delta z' &= -2\frac{r_p}{\gamma}\frac{N_e}{r^2} z\left(1-\sum\frac{(-r^2/2\sigma^2)^n}{n!}\right)\\ &\approx -2\frac{r_p}{\gamma}\frac{N_e}{r^2} z\frac{-r^2}{2\sigma^2}\\ &= \frac{r_p}{\gamma}\frac{N_e}{\sigma^2} z\\ &= -\frac{4\pi\xi}{\beta^*}\cdot z\\ &= k\cdot z, \quad (3)\end{aligned}$$

which resembles a quadrupole kick. Introducing normalized coordinates z_N and z'_N, this finally becomes

$$\Delta z'_N \approx -4\pi\xi z_N. \quad (4)$$

At the interaction point (IP), where it is assumed that $\alpha = 0$, the action J before the quadrupole kick can be expressed as

$$J_i = z_N^2 + {z'_N}^2. \quad (5)$$

After experiencing the beam-beam kick with its fluctuating strength $\delta\xi$, the action is

$$\begin{aligned}J_f &= z_N^2 + (z'_N - 4\pi\delta\xi z_N)^2\\ &= z_N^2 + {z'_N}^2 - 8\pi\delta\xi z_N z'_N + (4\pi)^2(\delta\xi)^2 z_N^2. \quad (6)\end{aligned}$$

The change in action due to the kick is therefore

$$\begin{aligned}\Delta J &= J_f - J_i\\ &= -8\pi\delta\xi z_N z'_N + (4\pi)^2(\delta\xi)^2 z_N^2. \quad (7)\end{aligned}$$

Averaging ΔJ over many turns, the first term vanishes, and we get with $\langle z_N^2\rangle = J/2$

$$\begin{aligned}\frac{\langle\Delta J\rangle}{J} &= 8\pi^2\langle(\delta\xi)^2\\ &= 8\pi^2\xi^2\frac{\langle(\delta\xi)^2\rangle}{\xi^2}. \quad (8)\end{aligned}$$

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NUMERICAL EXAMPLES

The action (or emittance) growth rate τ_2^{-1} can be expressed as

$$\begin{aligned} \tau_2^{-1} &= \frac{1}{J}\frac{\mathrm{d}J}{\mathrm{d}t} \\ &= f_c \cdot \frac{\langle \Delta J \rangle}{J} \\ &= f_c \cdot 8\pi^2 \xi^2 \frac{\langle (\delta\xi)^2 \rangle}{\xi^2}. \end{aligned} \tag{9}$$

To limit the luminosity degradation due to emittance growth to tolerable numbers, the emittance growth time should be larger than the minimum acceptable time T_2,

$$\tau_2 > T_2. \tag{10}$$

This requirement provides an expression for the maximum relative rms jitter of the beam-beam parameter

$$\frac{\langle (\delta\xi)^2 \rangle}{\xi^2} < \frac{1}{T_2 f_c \cdot 8\pi^2 \xi^2}. \tag{11}$$

Results for the various linac-ring colliders are listed in Table 1, assuming a tolerable emittance growth time of $3600\,\mathrm{sec}$.

SIMULATION RESULTS

In the linearized case, the effect of fluctuations can be computed analytically, as demonstrated in Equation 9. However, study of the realistic nonlinear case requires numerical simulations.

As a first step, the simulation code was applied to the linear situation, and simulation results were compared with analytical calculations. Using eRHIC parameters (see Table 1) and a three percent relative rms jitter of the beam-beam parameter ξ, the evolution of the normalized particle action $J \cdot \beta/\sigma^2$ for this case shows perfect agreement of analytical results and simulations, as depicted in Figure 1.

In the nonlinear case, a three percent relative jitter for the beam-beam parameter ξ can occur due to 3% fluctuations of the electron bunch population, N_e, or 1.5% fluctuations in the electron beam size, σ_z. In contrast to the linear case where these fluctuations are equivalent, they are expected to act differently in the nonlinear situation, Equation 1.

The evolution of the normalized particle action for a three percent relative rms jitter of the electron bunch intensity N_e is shown in Figure 2. For small amplitudes below about one σ the agreement with the result of the linearized calculation is very good, while the linearized model overestimates the growth at amplitudes greater than one σ.

The overestimation at large amplitudes by linearization becomes even more apparent for a three percent relative rms jitter of the inverse squared electron beam size σ_z^{-2}, Figure 3. At amplitudes below one σ the agreement with the linearized model is again very good.

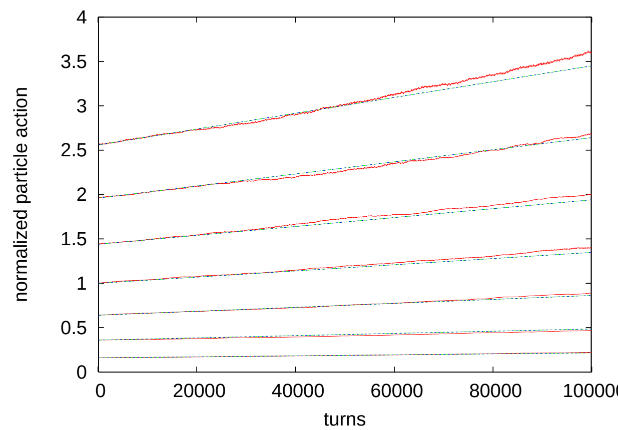

Figure 1: Normalized particle action $J \cdot \beta/\sigma^2$ vs. turn number for different initial values of J, with the beam-beam force linearized according to Equation 3. The straight, blue lines correspond to the analytical result, Equation 9, while the red lines show simulated data.

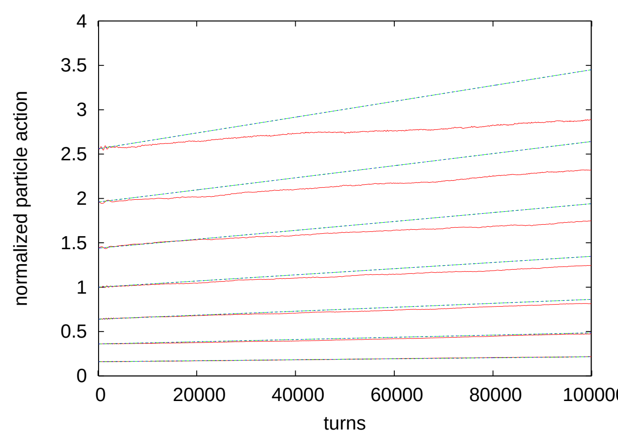

Figure 2: Normalized particle action $J \cdot \beta/\sigma^2$ vs. turn number for different initial values of J. The analytical result for the linearized beam-beam force is indicated by blue lines, while the red lines show simulation data for the nonlinear beam-beam force with fluctuating electron beam intensity N_e. The rms width of the fluctuation is set to $\sqrt{\langle (N_e - \langle N_e \rangle)^2 \rangle}/\langle N_e \rangle = 0.03$.

CONCLUSION

An analytic expression for the maximum tolerable rms jitter of the beam-beam parameter due to bunch-to-bunch fluctuations of the electron beam parameters in electron-ion colliders has been derived, using a linearized approximation of the beam-beam force. Simulations show that this approximation is justified for small particle amplitudes, while it significantly overestimates the resulting growth of particle action for particles beyond one σ. However, this depletion of the beam core determines the associated luminosity degradation.

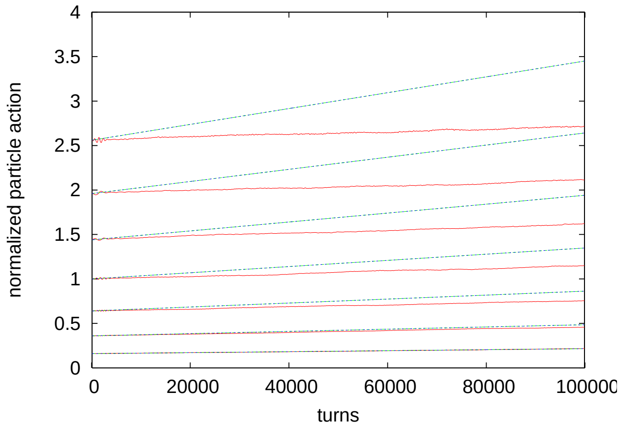

Figure 3: Normalized particle action $J \cdot \beta / \sigma^2$ vs. turn number for different initial values of J. The red lines show simulated data for the nonlinear beam-beam force with fluctuating electron beam size σ_z. The magnitude of the fluctuation is set to $\sqrt{\langle(\sigma_z^2 - \langle\sigma_z^2\rangle)^2\rangle} / \langle\sigma_z^2\rangle = 0.03$.

The calculated stability requirements for eRHIC and ELIC are quite tight, with $\sigma_\xi / \xi < 0.001$, while the corresponding numbers for THERA and QCDE are significantly more relaxed, $\sigma_\xi / \xi < 0.05$.

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REFERENCES

[1] eRHIC ZDR, C-A/AP/142

[2] L. Merminga, Y. Derbenev, ELIC: An Electron-Light Ion Collider at CEBAF, in: ICFA Beam Dynamics Newsletter No. 30, April 2003

[3] THERA: Electron-Proton Scattering at a cms energy of $\approx$ 1 TeV, TESLA Technical Design Report, Part VI, Chapter 2, http://tesla.desy.de/new_pages/TDR_CD/PartVI/append.html

[4] D. Schulte, F. Zimmermann, QCD Explorer Based on LHC and CLIC, LHC Project Note 33, CLIC Note 589