

# ENERGY CALIBRATION OF THE SPS WITH PROTON AND LEAD ION BEAMS

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## Abstract

The momentum of the 450 GeV/c proton beam of the CERN Super Proton Synchrotron was determined by a high precision measurement of the revolution frequencies of proton and lead ion beams. To minimize systematic errors the magnetic cycle of the SPS had to be rigorously identical for both beams, and corrections due to Earth tides had to be taken into account. This paper presents how the beam momentum was determined from the RF frequency for which the beams are centered in the machine sextupoles. The measured beam momentum is  $449.16 \pm 0.14$  GeV/c for a nominal momentum of 450 GeV/c, and the accuracy is limited by systematic errors.

## INTRODUCTION

In the past years the SPS machine was adapted to its role as injector for the LHC collider. As part of this effort the machine model was re-measured and the beam momentum at the extraction energy of 450 GeV/c was calibrated in order to obtain the best possible initial energy setting when the LHC will be commissioned.

## CALIBRATION PRINCIPLE

The speed of the particles  $\beta c$ , where  $c$  is the speed of light, is related to the RF frequency  $f_{RF}$  through

$$\beta c = \frac{C f_{RF}}{h} \quad (1)$$

where  $h$  is the harmonic number of the RF system,  $h = 4620$  for the SPS.  $C$  is the machine circumference. It follows from this equation that both the machine circumference and the revolution frequency must be known to determine the speed  $\beta$  and therefore the particle momentum.

The momentum calibration principle that is used here relies on the measurement of the revolution frequency of two ion species with different charge over mass ratio, and therefore different speed, that are injected into the same magnetic machine. Such a technique was used very successfully at LEP with protons and positrons [1].

The goal of the calibration is the determination of the beam momentum on the central orbit, where the beam is centered on average in the machine quadrupoles. On this orbit the momentum is determined by the dipole field. In practice the central RF frequency is measured by centering the beams in the machine sextupoles, where the transverse tune no longer depends on the setting of the chromaticity. For a sufficiently large number of sextupoles and

a correct alignment of the sextupoles with respect to the quadrupoles, the beam should be centered in the sextupoles and quadrupoles at the same time.

The speed  $\beta_p c$  of the proton beam is related to its momentum  $P$  and its rest mass  $m_p$  by

$$\beta_p^2 = \frac{P^2}{P^2 + (m_p c)^2} \quad (2)$$

An ion beam of charge  $Z$ , injected into the same magnetic machine and on the same orbit than the proton beam has a momentum  $P_i = ZP$ . The speed  $\beta_i c$  of the ions is

$$\beta_i^2 = \frac{P^2}{P^2 + (m_i c/Z)^2} \quad (3)$$

with  $m_i$  the ion rest mass. The two equations for  $\beta_p$  and  $\beta_i$  can be solved for the proton beam momentum  $P$ , yielding

$$P = m_p c \sqrt{\frac{\kappa^2 \mu^2 - 1}{1 - \kappa^2}} \quad (4)$$

with

$$\kappa = f_{RF}^i / f_{RF}^p, \quad \mu = \frac{m_i}{Z m_p} \quad (5)$$

$1/\mu$  is roughly the number of charges per nucleon of the ion. For  $\text{Pb}^{53+}$  ions  $\mu \simeq 4$ , and for fully stripped lead ions  $\text{Pb}^{82+}$ ,  $\mu \simeq 2.5$ .

Equation 4 can be approximated by

$$P \simeq m_p c \sqrt{\frac{f_{RF}^p}{2 \Delta f} (\mu^2 - 1)} \quad (6)$$

where the RF frequency difference  $\Delta f = f_{RF}^p - f_{RF}^i$  between the beams has been introduced. The measurement error  $\sigma_P$  on  $P$  is dominated by the term

$$\frac{\sigma_P}{P} \simeq \frac{\sqrt{\sigma_{f_{RF}^p}^2 + \sigma_{f_{RF}^i}^2}}{2 \Delta f} \quad (7)$$

with  $\sigma_{f_{RF}^p}$  and  $\sigma_{f_{RF}^i}$  the measurement errors on the central RF frequencies of the proton and ion beams.

## MACHINE PREPARATION AND MEASUREMENTS

As injector for the LHC the SPS accelerates high brightness proton beams from 26 to 450 GeV/c in approximately 20 seconds. The momentum calibration was performed at a nominal momentum of 450 GeV/c using proton and lead

ion beams. The settings of all magnets were kept rigorously identical for both beams, except for the chromaticity which had to be varied to determine the central frequency.

To maximize the frequency difference  $\Delta f$  for the calibration, the lead beam was not stripped in the injection transfer line and injected as  $\text{Pb}^{53+}$  into the SPS. The lifetime of  $\text{Pb}^{53+}$  in the SPS was 5.3 seconds at  $P_{\text{Pb}}/Z$  of 26 GeV/c, limited by the vacuum conditions. The lead ion source is composed of isotopically pure  $\text{Pb}_{208}$ .

At 450 GeV/c the closed orbit r.m.s in the SPS was 2.0 mm and 1.5 mm for the horizontal and vertical planes. The transverse tunes were set to  $Q_h = 26.18$  and  $Q_v = 26.14$ . The magnetic field in the reference dipole was measured with an NMR probe. The field was stable at  $2.0251 \pm 0.0002$  T during the two days of measurements.

The proton beam intensities corresponded to  $\sim 10^{11}$  protons per bunch. The total  $\text{Pb}^{53+}$  ion beam intensity was only  $\sim (3-5) \times 10^9$  charges.

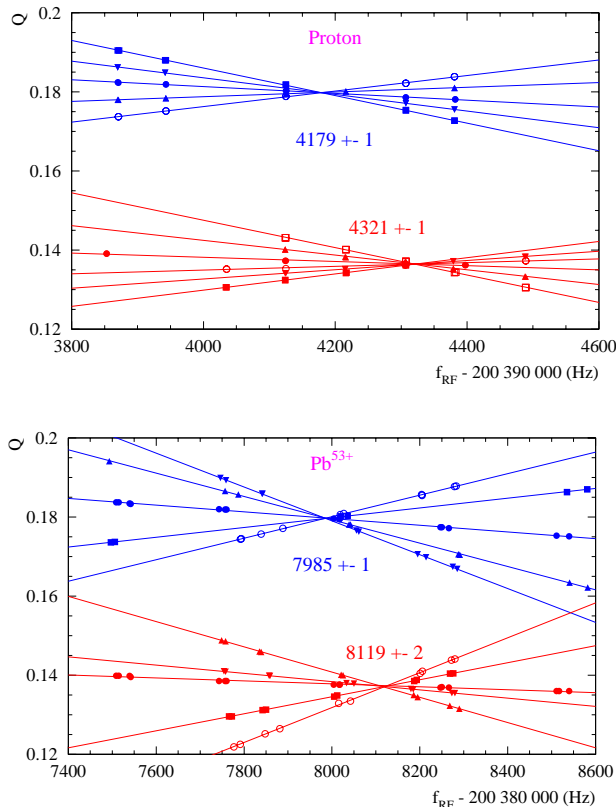


Figure 1: Tune dependence on RF frequency for different settings of the machine chromaticity for proton (top) and  $\text{Pb}^{53+}$  beams (bottom) at a proton momentum of 450 GeV/c. The RF frequency (and its error) that corresponds to the crossing of the lines is indicated for each measurement set (horizontal,  $Q \simeq 0.18$  and vertical,  $Q \simeq 0.14$ ).

## Central Frequency Measurements

The central RF frequency is obtained in the following way. For a number of different chromaticity ( $Q'$ ) settings (positive and negative), the tune is measured as a function of the radial position (RF frequency). For each  $Q'$  setting, the tune dependence on the RF frequency should be linear. The central RF frequency corresponds to the crossing point of all the lines obtained for all  $Q'$  settings, as can be seen in Figure 1. The measurement series is repeated twice, once for horizontal chromaticity changes where the horizontal tune is recorded, once for vertical changes where the vertical tune is recorded.

Table 1: Summary table of central frequency results for the proton beam (top), the lead beam (middle) and their difference (bottom). The frequency values are corrected for the predicted tidal shifts as explained in the text

Parameter	$Q'$ scan	Frequency (Hz)
$f_{RF}^p$	Horizontal	200'394'181.4 $\pm$ 1.0
	Vertical	200'394'321.2 $\pm$ 1.0
	Vert. - Hor.	139.8 $\pm$ 1.4
$f_{RF}^{pb}$	Horizontal	200'387'987.1 $\pm$ 1.2
	Vertical	200'388'120.8 $\pm$ 2.5
	Vert. - Hor.	133.7 $\pm$ 2.8
$\Delta f$ ( $f_{RF}^p - f_{RF}^{pb}$ )	Horizontal	6'194.3 $\pm$ 1.6
	Vertical	6'200.4 $\pm$ 2.7
	Vert. - Hor.	6.1 $\pm$ 3.1

The chromaticity is corrected in the SPS using 108 lattice sextupoles, 54 LSD type (vertical focusing) and 54 LSF type (horizontal focusing) magnets. The LSD magnets are grouped in two families, the LSF in three families. The total number of SPS lattice quadrupoles is 216, i.e. there is only one sextupole for two quadrupoles. Since horizontal chromaticity changes are mainly performed using the LSF sextupoles, while the vertical chromaticity is mainly varied using the LSD family, the central frequencies obtained from the two planes are dominated by the alignment of the corresponding family. Differences in central frequency as determined from horizontal or vertical  $Q'$  scans are an indication for the size of the alignment errors between sextupole families.

The tune data are shown in Figure 1 for the two beams. The crossing points of the chromaticity curves are determined with an accuracy of  $\pm 1-2$  Hz. The results are given in Table 1. The frequency difference between protons and lead ions is approximately 6.2 kHz. A systematic effect is apparent in the data: the central frequencies determined from vertical and horizontal  $Q'$  scans differ by approximately 140 Hz. Fortunately this shift does not significantly affect the frequency difference that is relevant for the calibration.

Table 2: Beam parameters obtained from the central frequency measurements, averaged over the data of the two planes. The errors are determined from the systematic difference between the data from the two planes

Parameter	Value
Proton momentum $P$ (GeV/c)	$449.155 \pm 0.136$
Central orbit length $C$ (m)	$6911.5662 \pm 0.0024$

## BEAM MOMENTUM

Large accelerators are subject to radial deformations due to Earth tides, an effect that has been clearly demonstrated for LEP, where the tidal distortions contributed significantly to the beam energy fluctuations [2]. For the SPS the predicted tidal deformations induce changes of the central frequency of up to  $\pm 5$  Hz. Tide corrections ranging from  $-2.7$  to  $+2.8$  Hz have been applied to the measured central frequency values given in Table 1 because the measurements were performed over a period of 2 days.

The proton momentum  $P$  and the central orbit length are obtained from equations 1 and 4 using the particle masses and fundamental parameters [3]. The differences observed between the planes must be taken into account as systematic errors, although for the frequency difference  $\Delta f$  relevant for  $P$ , the results agree within 2 standard deviations, which is acceptable. A conservative approach is adopted and the systematic difference is taken into account by averaging the results for the two planes. Half the difference between the measurements for the two planes is used as systematic error and is propagated to all derived quantities.

The systematic difference of the central frequencies in Table 1 leads to an uncertainty of 2.4 mm on the length of the central orbit that also contributes an additional uncertainty of 0.08 GeV/c to the momentum. This uncertainty is added quadratically to the other error.

Systematic effects due to the settings of the horizontal orbit correctors are negligible. The field integral of the orbit correctors was less than  $10^{-5}$  of the total field. Effects due to orbit lengthening [4] are also negligible.

The result for the beam momentum for a nominal settings of 450 GeV/c is given in Table 2. The beam momentum is  $449.16 \pm 0.14$  GeV/c, which is  $-0.19 \pm 0.03\%$  lower than the nominal momentum. The accuracy on the beam momentum of 0.03% is dominated by the systematic effects. The intrinsic accuracy is 0.01–0.02%. A more detailed account of the data analysis and of the results is given in Reference [5].

## CENTRAL FREQUENCY UNCERTAINTIES

The observed differences in the central frequency obtained from vertical and horizontal  $Q'$  scans correspond to a systematic 0.7 mm shift of the average magnetic center be-

tween the LSF and LSD sextupole families at 450 GeV/c. This number largely exceeds the expected alignment accuracies of 0.2 mm or better. Statistically a systematic alignment difference of less than 0.1 mm is expected. Following this observation the central frequency was remeasured as a function of beam momentum [6]. The dependence of the horizontal-vertical central frequency difference is shown in Figure 2, where a steep momentum (resp. sextupole magnet field) dependence is observed. The effect vanishes at injection energy. A mechanical movement of the sextupole has been excluded, leaving a possible shift of the magnetic center of all (or some) sextupoles.

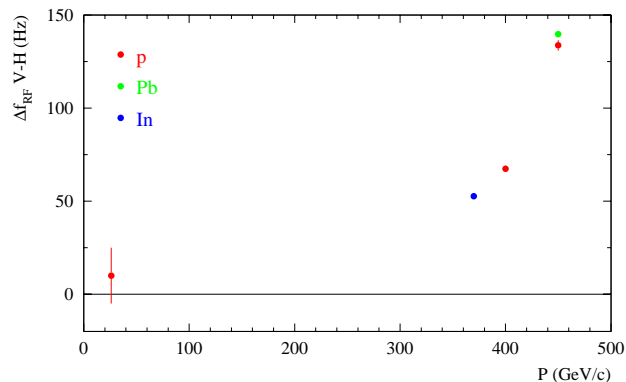


Figure 2: Momentum dependence of the measured central RF frequency difference between the horizontal and vertical chromaticity scans.

## CONCLUSION

The SPS central beam momentum was determined in view of the LHC commissioning. The measured beam momentum is  $449.16 \pm 0.14$  GeV/c for a nominal momentum of 450 GeV/c. This momentum value was confirmed during a test of the new transfer line to the LHC in the fall of 2004 [7].

An unexpected difference in the machine circumference was observed for measurements with the different sextupole families, indicating systematic difference in the average radial position of the magnetic centers of the sextupoles. The large difference is not understood, and measurements indicate a strong dependence of the magnetic field.

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